Uncertainty and Statistical Risk

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ABSTRACT

All measurements are subject to uncertainty and the measurement result is complete only when accompanied by a statement of uncertainty associated with the measured value. This is the case for most statistical indicators, from the conjunctural predictions, to the measurement of household's expenditure and income, to the calculation of some GDP components, to determining of the price indexes, to determining the unobservable economy, to determine the population's perception regarding quality of life etc., up to voting intentions, to list only a few areas. The uncertainty, in the field of measurement, must be materialized by a statistical indicator, which expresses a certain fact, the distance /closeness to the true value of the size subject to the measurement process. Uncertainty also appears as the result of human ignorance, and its form of manifestation is the variability which, exceeding certain admissible limits, can generate what we commonly call a risk, namely to make an erroneous decision in a situation where necessary information is distorted precisely because of too much variability. The risk in making decisions is present in all human activities, from where the vastness of the problem as a research field. In the paper we propose the exposure of uncertainty measurement procedures and the link between uncertainty and risk. The statistical modeling of risk has as a starting point the assumption that risk can be assimilated with the possibility of suffering a certain loss. Because the possibility is expressed quantitatively, by probability, the risk appears as a probability function in the occurrence of an unwanted phenomenon. Finally, certain Taguchi risk applications are presented, regarding the relationship between this risk and the potential index of a process.

Key words: uncertainty, errors of III and IV type, *statistical risk, Taguchi method*

1. CHARACTERISTICS MEASUREMENT

Deriving from the measurement science, metrology, this issue has evolved into engineering sciences but it is also indispensable in the socio-human field, with the essential distinction that there is no measure unit (like in Physics) in the social sense.

A statistical indicator can be the numerical expression of an economic category but also the correspondent of a variable. The statistical indicator is used to numerically express the sizes, structures, interdependencies or changes in time of some social-economic phenomena. The overwhelming majority of the statistical indicators are obtained by statistical processing, according to standardized methodologies of the observation data, measured on units (individuals, families, companies and so on) of a survey.

It is common knowledge that there are many types of measurements (besides the classical physical ones), such as measuring customer satisfaction, poverty, inflation, environmental degradation, the health of the population, school performance, the volume of national or regional production, measuring profitability and so on.

All measurements are subject to uncertainty and the measurement result is complete only when accompanied by an indication of the associated uncertainty. This uncertainty has a probabilistic basis and reflects the incomplete knowledge of quantity value (Eisenhart 1963).

One of the simplest decision-making problems under uncertainty is the acceptance /rejection of a statistical assumption, a hypothesis that may be true or false. Uncertainty occurs precisely because of sampling: it works with a part (or parts) of a certain "whole" (population, batch of products etc.) and not with the whole collectivity, so the decision to accept /reject the hypothesis is based on the examination of only that part, which is the sample. In most situations, indicators that reflect customer satisfaction, poverty and so on are obtained based on data collected through surveys.

2. LITERATURE REVIEW

We will not dwell on the common definitions and common sense of the term uncertainty (according to Dex: "lack of certainty, uncertainty, doubt, hesitation"; or according to Macmillan's Dictionary of Modern Economics: "the fact that something is not known or has not been decided; uncertainty about/over/as to; a degree of uncertainty (some uncertainty); uncertainty surrounds something (people are very uncertain about it)", and we will tackle the formal definitions and concepts of statistical uncertainty offered by the profile standards, as official regulatory documents.

Thus, in ISO 14253-1: 2013 we find the general definition of uncertainty: "a non-negative parameter characterizing the dispersion of the quantitative values being attributed to a measurand, based on the information used". For the measured values ISO 20988: 2007 it states the following definition: "uncertainty (of measurement) - parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand". ISO 13843:2017 defines the standard uncertainty: "uncertainty of the result of a measurement expressed as a standard deviation" and ISO 17123-1: 2014 establishes expanded measurement uncertainty: "half-width of a symmetric coverage interval centered around the estimate of a quantity with a specific coverage probability.” The target value in this area is recommended through ISO/IEC Guide 99: 2007 “target measurement uncertainty: measurement uncertainty specified as an upper limit and decided on the intended use of measurement results”. Same standards classify uncertainty measurement in two main types: “Type A evaluation, evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions” (VIM 2012). In the case of type A, it is assumed that the distribution of the observed values is of Gaussian type, with the average *m* and the standard deviation equal to the standard deviation of the mean σ. Type B evaluation of measurement uncertainty is the evaluation of a component of measurement uncertainty (standard uncertainty) determined by means other than a Type A evaluation of measurement uncertainty (ISO 17123-1: 2014). In this case, for an assessment of uncertainty, the only available information in this case is that X lies within a specified range [a, b]. Measurement uncertainty is often considered as the standard deviation of the probabilities distribution that could be attributed to a measured quantity. The relative uncertainty is the uncertainty of measurement relative to the magnitude of a single choice for the value to be measured. This particular unique single choice is usually called the measured value, which may be optimal in a well-defined sense (e.g., mean, median, or modal value). Thus, the relative measurement uncertainty is the uncertainty of measurement divided by the absolute value of the measured value, when the measured value is not zero. The uncertainty should not be confused with the estimate attached to a result of measurements that characterize the range of values within which the average is supposed to be.

All measurements are subject to uncertainty and the measurement result is complete only when accompanied by a statement of associated uncertainty. This uncertainty has a probabilistic basis and reflects the incomplete knowledge of the quantity value (Petrescu, 2004).

Measurement uncertainty is often considered as the standard deviation of the knowledge status probabilities distribution over the possible values that could be attributed to a measured quantity. The relative uncertainty is the measurement uncertainty in relation to the magnitude of a single choice for the value for the measured quantity, when this choice is nonzero. The development of mathematical possibilities of uncertainty representation is a concern that Helton and Oberkampf (2004) dedicates an interesting study.

According to Eisenhart (1963), measurement is the assignment of numbers to certain material objects in order to represent the relationships between them and certain specific (particular) properties of the objects in question. The author distinguishes between the *measurement method* and the *measurement process*. Finkelstein (1982) develops the basic principles of measurement in various branches of science, including in the socio-human field, and Murphy (1961) emphasizes that a measurement method does not become a measurement process until this process is in the state of statistical stability, meaning that measurements become the "product" of an identifiable statistical universe. This means that we can allocate a well-defined statistical distribution to these values that represent the measurand. We usually "hope" that this repartition be the normal one, because the parameters of this statistical law, m and σ^2, are exactly the average and respectively the theoretical dispersion of the measured quantity. As well known, they are estimated by $\bar{x}$ (arithmetic mean) and by the indicator $s^2 = (n-1)^{-1}\sum(x_i - \bar{x})^2$, where x_i are the observed values of the variable X. In the case of the normal law (Gauss-Laplace), the average value can be taken as an acceptable reference value, constituting a substitute of the so-called "true value" of the measurand, which is unknown (see also ISO 5725-1 page 6, section 3.5).

Hunter (1980) considers that a measurement operation is in a state of statistical stability if there are quantitative measures of repeatability and reproducibility. By "repeatability" he understands a measure of variability between the values observed in the same laboratory and by "reproducibility", a measure of variability between two or more laboratories. The term "laboratory" is generic (conventional), the laboratory may also be a public opinion polling organization. The two concepts are defined by the aforementioned standard as representing *fidelity* under repeatability and reproducibility conditions. Quantitative measures proposed by Hunter are the associated standard deviations (S_R) and the so-called repeatability /reproducibility limits (*r* sau *R*), defined as , where $|d|$ is the absolute value of the difference between two results of an observation obtained under repeatability /reproducibility conditions. An assessment version of uncertainty evaluation and of prediction chances is performed by Helton and Oberkampf (2004), based on the uncertainty of input information and its propagation in prediction models to the uncertainty of the final solutions, in case of using Monte-Carlo simulation methods.

Hoffman and Hammonds (1994) tackled the issue of uncertainty propagation within predictive models, making a clear distinction between the uncertainty generated by the lack, respectively information insufficiency and the uncertainty generated by the variability of the input data.

Stigler (1986) develops in a monumental book the issue of mathematical methods in measuring and modeling uncertainty. Stigler's emphasis is upon how, when, and where the methods of probability theory were developed for measuring uncertainty in experimental and observational science, for reducing uncertainty and as a conceptual framework for quantitative studies in social sciences. He describes the scientific context in which the different methods evolved and identifies the problems (conceptual or mathematical) that retarded the growth of mathematical statistics and the conceptual developments that allowed major breakthroughs. Oberkampf et al (2001) deal with the subject of uncertainty and Jacquin (2010) develops an intriguing approach of predictive uncertainty.

Among the interesting applications of uncertainty determination, can be mentioned the ones of Valcan (2013) to determine the uncertainty in measuring the value of phosphorus in water, or Beck and Krueger (2016) discuss the issue of integrated global climate change models, combining the representations of the economic system and climate, models that have become important tools in supporting policy makers on climate matters. Riahi et al (2007) present three versions of scenarios in economic and social development and the consequences on greenhouse gas emissions and analyze the feasibility of each of them. Conclusions focus on uncertainties and costs. Rui et al. (2016) develops the problem of uncertainty and its measurement on a specific domain - the resilience, by non-probabilistic indicators and metrics to which they associate the uncertainty dimension. Georgescu -Roegen (2000) addresses the issue of uncertainty and choice based on a paper of Armstrong (1948) which, for the measurement of uncertainty, assigns a binary system, assigning 0, respectively 1 to complete uncertainty, respectively to complete certainty, then 0.5 to neutral uncertainty, repeating the process on successive intervals. The author draws the attention to the distinction between uncertainty and probability and the need to eliminate confusion between the two categories. The author also highlights the issues related to the nature of expectations and uncertainty, certainty and quasi-certainty, between expectation and subjective belief. Ferson et al. (2007) approach the characterization of measurements that include epistemic uncertainties in the form of intervals. It examines the application of the basic description and the determination of some algorithms to make inferences on the data observed. Bachmann et al (2010) raise the level of macroeconomic approaches using microdata based on business environment making uncertainty indicators based on both ex-ante, as well as on ex-post forecast errors. Jurado et al (2015) exploit a rich literature and volume of observational data in order to provide direct econometric estimates of variable macroeconomic uncertainty over time. Cox and Harris (2006) approach some developments of previous material on some automatic calculation of uncertainty programs. The software architecture is harmonized with a reference work and regulation in the field: “Evaluation of measurement data. Guide to the expression of uncertainty in measurement” JCGM 100:2008. A critical analysis of the literature on decision-making risks and the measurement of uncertainty up to the level of 2010 is carried out by Zio and Pedroni (2012).

Another area of interest, closely related to uncertainty, is that of risk. Kaplan and Garrick (1981) define the risk and offer a calculation version based on Bayes’ theorem, differentiating the relative risk, the relativity of the risk, the acceptability of the risk. The presentation of the different risk types (financial, technical, bankruptcy, so on) and the calculus methods are developed by Isaic-Maniu (2006). A development of the risk topic in a technical field - making composite materials with a risk specificity, is performed by Taguchi, Ionesiet et al (2012), Walls et al. (2016) and Montewka et al (2014) address the issue of risk in determining reliability.

3. MEASURING UNCERTAINTY

It is known that one of the simplest decision-making problems under uncertainty is the acceptance / rejection of a statistical assumption, a hypothesis that may be true or false. Among the sources of uncertainty we can mention: an incomplete, partial information on a certain entity, lack of information, inadequate interpretation of information, mis-attribution of causality. The uncertainty in the field of measurement gains a concrete outline through a statistical indicator, through a formula in the end, which expresses a certain fact, the distance / closeness to the true value of the physical size subjected to the measurement process. The uncertainty can not be separated from the human factor. Statistics, among its many meanings, is defined as follows: "the science of decision-making under uncertainty". Referring to the human factor, Cox (1957) states "the observer is a part of what he observes, the thinker is a part of what he thinks. We can not passively observe the statistical universe as mere spectators, because we ourselves are part of this universe". Uncertainty appears as the fruit of human ignorance, and its form of manifestation is variability, which exceeding certain admissible limits, can generate what we commonly call a risk: to make an erroneous decision in a situation where, the necessary information is distorted, precisely because of this exaggerated variability.

Statistically, the standard uncertainty defined in ISO 20988: 2007 as the uncertainty of the result of a measurement, expressed by a standard deviation, is basically an estimation of the internal variability of a set of observations $\{x_1, x_2 \dots x_n\}$ carried out on a measurand.

"Measurement", in this context represents the whole set of operations where experimental data are obtained $x_i, i = 1,2,3 \dots n$ (the term "measurement" is commonly used as a synonym for the x_i measured or the experimental value).

The standard deviation is calculated using the well-known formula ($s = \sqrt{s^2}$) where

$$s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2 \qquad [1]$$

with $\bar{x} = \frac{1}{n}\sum x_i$ which is not an unbiased estimator of the theoretical standard deviation (σ), but s^2 is such an estimator for the variance σ^2.

In order to obtain an unbiased estimator, we will have to remind that the distribution of the statistic is chi (not chi-square) with *(n-1)* degrees of freedom, and that its mean value *E(s)* is computed as:

$$E(s) = \sigma \cdot \sqrt{\frac{2}{(n-1)}} \cdot \Gamma\left(\frac{n}{2}\right) / \Gamma\left(\frac{n-1}{2}\right) \qquad [2]$$

where $\Gamma(\bullet)$ is Euler's famous Gamma function (see Patel et al, 1996, pp. 118). Therefore, the non-stacking coefficient for s will be precisely the inverse of the coefficient that multiplies the expression under the radical. However, in practice we do not work with the modified estimator. When calculating the natural variation range of the measurand, or when we want to build confidence intervals - either for the average (μ) or variance (σ^2) - we use the statistics $\bar{x}$ and s, or we rely on the fact that the statistic $(n-1)s^2/\sigma^2$ has a chi-square (χ^2) distribution with *(n-1)* degrees of freedom, as demonstrated by Helmert in 1876 (Sheynin, 1995, pp. 88). Similarly, we make use that the statistics $\bar{x}$ and s^2 are independent and $\bar{x}$ and $w = x_{max} - x_{min}$ (the data amplitude x_1, x_2, ...,x_n) are independent and that the statistic $t = \sqrt{n}(\bar{x} - \mu)/s$ has a Student distribution with *(n-1)* degrees of freedom. We also know that the average has an approximately normal distribution with the same mean (μ) with the x generating variable but with the adjusted variance σ^2/n, where n is the number of values of which $\bar{x}$ was evaluated.

The size $\frac{s^2}{n}$ is named in the dedicated standard (VIM 2012) as the experimental variance of the mean and is a measure of the uncertainty of $\bar{x}$. As is well-known, statistical applications use the expressions $\mu - 3\sigma/\sqrt{n}$, respectively $\mu + 3\sigma/\sqrt{n}$, as limits for the mean of a process, where μ is estimated through the mean of the group's mean and σ by an average amplitude, either directly through s. According to the known theory:

$$Prob\left\{|x - E(x)| \leq 3 \cdot \sqrt{Var(x)}\right\} \approx 0.9973 \qquad [3]$$

if x is a normal variable then outside the boundaries there will be about 0.27% of the variable's values; thus, on average, one of each 370 subgroup's means will be out of bounds when the process is in control.

Since the average is an almost normal variable, even if the measurand x proves to be another statistical law, we can say that a quantitative measure of the average uncertainty is the interval [$\bar{x} - 3s/\sqrt{n}, \bar{x} + 3s/\sqrt{n}$]. The composite standard uncertainty is also defined as a standard deviation, but in this case, the measurand is considered a (differentiable) function of certain input measures x_1, x_2, ...,x_n, $\tilde{O} = f(X_1 X_2,...,X_N)$, every such measure having its own variability, given by $s(x_i)$ its standard deviation. The measurand Y is indirectly evaluated, through measuring X_i, $i = \overline{1,N}$ elements and the composite standard uncertainty noted, in ISO/IEC Guide 99: 2007, but also in Charles et al (2017), with $u_c(y)$ the positive square root of the expression:

$$u_c^2(y) = \sum_{i=1}^{N} \left[\frac{\vartheta f}{\vartheta x_i}\right]^2 \cdot u^2(x_i) \qquad [4]$$

where $u(x_i)=s(x_i)$ the standard deviation of X_i component estimated in x_i and the values of X_i are considered uncorrelated and the form of the function *f* is not significantly nonlinear.

In the case of correlated variables and nonlinear distribution, the above formula is completed accordingly. In many situations, the relation $Y=f(X_1, X_2, \ldots\ldots, X_n)$ can be linearized (by logarithm, inversion or other operations). For example in case of polytropic functions: $Y = c \cdot X_1^{a_1} \cdot X_2^{a_2} \cdot \ldots \cdot X_N^{a_N}$, $X_i \geq 0, a_i \in R, c > 0$, and thus, by reverting, a linear function is obtained in the new variables. The process is well-known in the theory of regression and correlation, where linearization plays an important role in facilitating the estimation of function parameters.

Extended uncertainty is defined according to (VIM 2012), as a size that defines a range around the result of a measurement, a range in which a high fraction of the distribution of the values that can reasonably be attributed to the *Y* measurand. This uncertainty is also called global uncertainty and is denoted as *U*. In quantitative terms, *U* is expressed as $U = K \cdot u_c(y)$, where *K* is an expansion factor which, according to the VIM 2012 document, can usually take values between 2 and 3, and $u_c(y)$ is the composite standard uncertainty. Thus, the result of a measurement can be expressed in the form $Y = y \pm U$, where *y* is the best estimate of the *Y* measurand, the range *[y-U, y+U]* being the domain considered to incorporate an important fraction of the values of this *Y* measurement.

If we consider that $y = \bar{y}$ (the mean being the best situation of *Y*) and considering that $u_c(y)$ is a standard deviation s_y, the interval $\left[\bar{y} - K \cdot s_y, \bar{y} + K \cdot s_y\right]$ is a range of natural tolerances of (P, γ) type, where *P* is the proportion of values found in this interval, γ being the likelihood with which this situation happens:

$$Prob\left\{\int_{L_i}^{L_s} f(y)dy \geq P\right\} = \gamma \qquad [5]$$

where: $L_i = \bar{y} - K \cdot \bar{s}_y$, $L_s = \bar{y} + K \cdot \bar{s}_y$, $K = K(n, -P, \gamma)$, $0 < P, \gamma < 1$; *f(y)* being the probability density function of *Y* measurand. Thus, what is the explanation of the fact that *K* factor is preferred with values between 2 and 3.

If *Y* is normally distributed, then we know that the range $\left[\bar{y} - 2s, \bar{y} + 2s\right]$ contains about 95.45% of the *Y* values and if we take *K=3* then $\left[\bar{y} - 3s, \bar{y} + 3s\right]$ contains about 99.73% of the values.

The measure γ is the *probability of coverage*, or even the *confidence* with which we assert that the respective interval $\overline{y} \pm K \cdot s$ contains the proportion *P* of the values for measurand distribution *Y*.

4. THE ERRORS AND THEIR ASSOCIATED RISKS

Typical statistical errors in the hypothesis verification process are Neyman and Pearson's well-known Type I and Type II, usually noted with α and β, on which we no longer insist, only extend to other possible interpretations beyond the classical assumption, for example in the field of computer security: Type I errors (or false positives) that classify the authorized users as imposters; Type II errors (or false negatives) that classify imposters as authorized users.

Superior type errors appeared in statistical specific literature later. Type III errors, which according to Raiffa (1970) are the correct resolution of a false problem. Continuing with the example of computers area, the term "false positive" can also be used when antivirus software incorrectly classifies a harmless file as a virus. Incorrect detection may be due to a heuristics or an erroneous signature of viruses in a database. Similar problems can occur with antitrust or antispyware software. Type IV errors were proposed by Marascuilo and Levin (1970) which they defined in a way similar to Mosteller as the mistake of "misinterpreting a correctly rejected hypothesis". None of these last two proposed risk categories met with broad acceptance from specialists. Depending on uncertainty, the risk is characterized by the possibility of being quantified by probability laws, although it is also dependent on "fluid" elements, such as uncertainty and loss, which can not always be expressed numerically. Here is a brief presentation to some of the definitions of risk. *Mic Dicţionar Enciclopedic*, page 809: "Risk: danger, possible inconvenience". It then identifies a number of concretizations, with examples such as "contractual risk" - the debtor sustains the damaging consequences of the issuance of the creditor of his obligations to him as a result of the debtor's failure to fulfill his obligations because of causes that are not attributable to him. The notion of work risk appears - the bearing by an owner or the holder of a direct operative administration right, of damages resulting from the loss or destruction of work by major force or in case of unforeseeable circumstances; the notion of insured risk is also mentioned. DEX (1998) states: "The risk represents the possibility to reach a danger, to face a tribulation or to bear a loss; possible danger". Merriam - Webster's Collegiate Dictionary (1994) also offers historical details in the sense that the term "risk" and its derivatives began to be used in English.

The risk may or may not be manifested by the action of the so-called risk factors which, when operating effectively, can cause various losses. It should also be said that despite the fact that there is a risk in a given situation, it may not be manifested, that is to say, it does not produce effects (sometimes the decision-maker acts randomly and may indeed have the chance of not suffer loses, damages etc.). A risk-taking action is said to be risky or dangerous (hazardous). Over time, new notions and new activities have been created in the field of management, thus emerging a new risk management activity as a technique of evaluation, minimization and prevention of accidental loss in a business, insurance, safety measures or other appropriate elements, and obviously a new "risk manager" specialist. The wide range of risks is given by the fact that besides Nature, man created a contradiction that includes the economy as a whole, as well as industry, finance, commerce and so on, the natural environment being increasingly anthropogenic, natural components have been increasingly modified by human activities. The human activities contain the germs of environmental destruction, and in addition, these are activities dedicated to the satisfaction of many non-vital needs (Georgescu-Roegen, 2000). The difficulty arises because the uncertainty and the risk being in their structure impregnated with the random element, immediately attracts the statistical-probabilistic arsenal. Moreover, by minimizing both the risks and the possible losses from the effective risk manifestation, we need the operational research methods.

Therefore, the mathematical modeling of risk starts from the assumption that risk can be assimilated to the possibility of suffering a certain loss. Since the possibility is quantifiable by probability, the risk appears as a function of the probability of occurrence of an unwanted phenomenon, but also of the adverse effects of this event on which we did not anticipate the production. The effects in turn are manifested - combined: money losses, drops in performance, delays in executing operations and so on.

The uncertainty occurs precisely because of the sampling: it works with a part (or parts) of a "whole" (population, lot, firms etc.) and not with the whole collectivity, the decision to accept /reject the hypothesis based on the examination of only that part, which is the sample.

Among the versions of economic risk we can mention: the risk of indebtedness, the financial risk, the settlement risk, the bankruptcy risk. The latter risk requires the theoretical construction of a function (analytical relation) that can estimate the probability that a particular enterprise, company etc.) to record losses (sufficiently large) that would not allow it to pay its various invoices to utilities (gas, water, electricity) or to be unable to repay any credits or loans contracted on the market. The economic risk is seen in conjunction with the so-called "political" and "country" risks. These can not be expressed by a formula, being a composite indicator, using a conventional “scoring system” based on various factors taken into account: legislative stability, economic democracy and the level of official corruption – an issue of special interest on the side of investors.

Country risk is defined in summary as a risk of default, non-recovery, non-repayment of capital, or situations where a particular political regime in a country tends to nationalize the assets of private companies, usually foreign. Various specialized publications as *The Economist* or *Euromoney* have tried since the 1980s to construct a synthetic indicator that would allow the classification of countries according to this country risk, in ascending order: the last places are occupied of countries with a maximum risk in the perception of those interested. Statistically, the curve is modeled using random variables. Thus, for example, the technical risk (Isaic-Maniu 2006) is determined from the reliability index. Formally, if *T* is the variable representing the time-to-failure of the object, then one may write:

$$T: R(T_0) = Prob\{T \geq T_0\} \qquad [6]$$

The complementary value of reliability is precisely the technical risk of falling down a component or a system:

$$Prob\{T \leq T_0\} = F(T_0) = 1 - R(T_0) = 1 - Prob\{T \leq T_0\} \qquad [7]$$

Here $R(t)$, in our case for $t=T_0$, stands for the reliability function associated with the variable *T*. Consequently, the complement of *R* is the so-called non-survival (or non-reliability) function $F(t)=1-R(t)$, which represents from statistical point of view, the distribution function (*df*) of *T*. Here $F(t_0)$ is hence the probability that the system operates less than a desired time t_0. If the reliability $R(t_0)$ is low, consequently this technical risk is high. More adequate to define this technical risk seems to be hazard rate (or failure rate) function which may be called also „the danger of failure”:

$$z(t) = \frac{f(t)}{1-F(t)} = \frac{f(t)}{R(t)} = \frac{dF(t)}{R(t)} = -\frac{dR(t)}{R(t)} \qquad [8]$$

A high value of $z(t)$ means a low level of reliability ($z(t)$ is expressed usually in failures/hour).

Excessive risk treatment, especially of financial placements, is performed by Radulescu and Radulescu (2006). If the variable *X* is the profit of an investment, the loss suffered by an insured, or the exceedance of the alert threshold of the concentration of a pollutant in the air, then the associated risk is:

$$R_1(X) = Var(X) = M\left[(X-m)^2\right] \qquad [9]$$

where $m = M(X)$ is the average value of variable X.

Furthermore,

$$R_2(X) = M\left[(T-X)_+^2\right] \quad [10]$$

where T is a so-called "disaster threshold" (Radulescu and Radulescu 2006, p. 199) and it is obvious that the investor wants to have a profit placed always above the threshold.

If X is interpreted as exceeding the concentration of a pollutant relative to a certain maximum allowable T level, then the risk of pollution is:

$$R_3(X) = M\left[(X-T)_+^p\right],\ p \in \{1,2,...\} \quad [11]$$

The notation originates in *Lower Partial Moment* indicator – the partially lower order alpha moment in relation to T threshold:

$$LPM_\alpha(T,X) = M\left[(T-X)_+^\alpha\right] = \int_{-\infty}^{T} (T-x)^p\, dF_X(x) \quad [12]$$

where: $F_X(x)$ is the distribution function of X variable.

If T equals the mean $m = M(X)$ and $\alpha = 2$, then we obtain the so called semivariance:

$$SV(X) = M\left[(m-X)_+^2\right] \quad [13]$$

The expressions $R_2(X)$ and $R_3(X)$ are nothing but average risks associated to quadratical form $L(x,T) = k(x-T)^2$ used by Gauss since 1809 (Kackar, 1985).

5. TAGUCHI RISK AND AN APPLICATION

The expression $L(x,T) = k(x-T)^2$ was reactivated by Taguchi (Alexis 1999): T is the target value of measurable characteristic X, and $L(x,T)$ is the loss quality function. Then:

$$R_T(x) = M\left[L(x,T)\right] = \int_D L(x,T) f(x) dx \quad [14]$$

is *Taguchi risk*, meaning the average value of the variable $L(x,T)$ (Voda 2009) where $f(x) = F_X^{'}(x)$ and D is the definition domain of X, usually $[0,+\infty)$.

In our case:

$$\begin{aligned} M\left[L(x,T)\right] &= k\left[M(X^2) - 2TM(X) + T^2\right] = \\ &= k\left[M(X^2) - M^2(X) + \left(M^2(X) - 2TM(X) + T^2\right)\right] = \\ &= k\left[Var(X) + \left(M(X) - T\right)^2\right] \end{aligned} \quad [15]$$

If the variabile is normally distributed $X \sim N(\mu,\sigma^2)$, then the Taguchi risk becomes: $M\left[L(x,T)\right] = k\left[\sigma^2 + (\mu - T)^2\right]$, and its estimation becomes:

$$\hat{R}_T(x) = k\left[s^2 + (\overline{X} - T)^2\right] \qquad [16]$$

where $\overline{X} = n^{-1}\sum x_i$, and $s^2 = (n-1)^{-1}\sum (x_i - \overline{X})^2$

The more symmetrical is the distribution of the characteristic, and its mean on the observation data $(\overline{X})$ is closer to the target value (T), the lower the associated risk is.

Taguchi (Alexix 1999) assumes that if the mean $\overline{X}$ is very close to the target value *T*, then the standard deviation changes as follows:

$$s_1 = s\left(\frac{T}{\overline{X}}\right) \qquad [17]$$

Certainly, if $T \approx \overline{X}$, then one can suppose that $s_1 \approx s$, and the loss function is:

$$M\left[L(x,T)\right] \approx ks_1^2 = k\left[s\frac{T}{\overline{X}}\right]^2 = kT^2\left(\frac{s}{\overline{X}}\right)^2 \qquad [18]$$

If $\overline{X} = T$ thus, in the exact version, derived from relation [14] we have:

$$\hat{R}_T(x) = ks^2 = k\overline{X}^2\left(\frac{s}{\overline{X}}\right)^2 \qquad [19]$$

Both in forms [18] and [19] occurs the coefficient of variation $s/\overline{X}$, Taguchi thus motivating his idea to use this indicator as a performance indicator of a process (in the broadest sense, not just as a technological process). Appreciation and introduction of the coefficient of variation, as a performance indicator of the process, was disputed, considering that the importance given by Taguchi to the inverse of this indicator - the signal to noise ratio or the perturbation coefficient, is exaggerated and often unconvincing. It can not be judged the performance of a process, for example, only from the point of view of the variance coefficient or its inverse.

Formula [17] shows that if $T = 0$, then $s_1 = 0$, that would mean that all values are identical, which is a limited case. Consequently, we believe that Taguchi's hypothesis [17] can only be valid if $T \neq 0$. In case $T = 0$ we should start from formula (16):

$$\hat{R}_T(x) = k(s^2 + \overline{X}^2) \qquad [20]$$

that we can rewrite:

$$\hat{R}_T(x) = k^2 s^2\left[1 + \left(\frac{\overline{X}}{s}\right)^2\right] = ks^2\left[1 + \left(SN(X)\right)^2\right] \qquad [21]$$

thus highlighting the indicator *SN* (X) - signal to noise ratio. In order to mimimize risk $\hat{R}_T(x)$ a function of two variabiles $\overline{X}$ and s^2 has to be minimized.

A measurable feature, which generate frequent fallouts to the flywheel from the constructional component of a compressor, has fixed tolerances as follows: Lower Specified Limit LSL=263.48 mm and Upper Specified Limit USL=263.68 mm. The target value is T=263.58 mm, exactly the middle of the [LSL-USL] interval. If the performance level is fixed to Cp=2 and the process mean is $\overline{\mathrm{x}} = 263.58$ mm with standard deviation s=0.011 mm, then the Taguchi index has the estimated value $C_{pm} = \dfrac{USL - T \cdot LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \hat{C}_{pm} = 0.40$ that is a very weak potential index of the process. In this case, we can calculate

Taguchi risk for $\mathrm{LSL} - \mathrm{USL} = 12 \cdot \mathrm{s}$ with formula [18] and it results: $M[L_T(\mathrm{x})] = k \cdot \left(\dfrac{0.2}{6} \cdot \dfrac{1}{0.403}\right)^2 = k \cdot 0.00684 \approx 0.007 \cdot k$. Constant k is expressed in monetary units.

If the deviation d is exactly 1, then $k = A_d$ representing the cost for a non-conforming unit of product (with charactersitic values greater than USL or lower than LSL). The higher the Taguchi risk is, the bigger the cost for the defective unit. Thus:

Cost /defective unit	Taguchi risk /unit
1 m.u.	0.007mm
2 m.u.	0.014 mm
3 m.u.	0.021 mm
4 m.u.	0.028 mm
5 m.u.	0.035 mm

Here the m.u. symbolize the chosed monetary unit (ROL, EUR, USD and so on). The relation between the monetary unit and the risk being linear ($R = k \cdot u$), as the unit of defective product is cheaper, the associated average loss, also expressed in monetary units, is lower.

6. CONCLUSIONS

One of the most common decisions under uncertainty is the acceptance /rejection of a statistical hypothesis, which may be true or false. The uncertainty is generated, in this case, by the fact that only a part of a population is involved, and that the assertions are based on the indicators obtained from the observation data of a sample, not from the whole population. It is the case of most situations of determining the statistical indicators, from the conjunctural predictions, to the expenditures and incomes of the households, to the determination of some components of the GDP, to the intentions of voting - to list only a few of the domains. The risk in decisions making is present in all human activities, hence the vastness of the problem, as a research field. The difficulty is amplified by the fact that the uncertainty and the risk are impregnated by the random factor, but this draws the statistical probabilistic tools in dealing with the issue. The statistical modeling of the risk starts from the assumption that the risk can be assimilated to the possibility of suffering a certain loss. Because the possibility is expressed quantitatively by probability, the risk appears as a probability function in the occurrence of an unwanted phenomenon. The uncertainty and the risk are modeled by random variables. In many cases, the variace is used as a measure of risk, but in the case of less symmetric distributions the result is inconclusive, in which case the Taguchi risk can assess the loss.

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