Identify Relative importance of covariates in Bayesian lasso quantile regression via new algorithm in statistical program R

Fadel Hamid Hadi Alhusseini
Department of Statistics and Informatics, University of Craiova, Romania

Taha al Shaybawee
Faculty of Economics and Business Administration, Al-Qadiseya University, Iraq

Fedaa Abd Almajid Sabbar Alaraje
Department of accounting, University of Craiova, Romania

ABSTRACT

In this paper, we propose a new algorithm to determine the relative importance of covariates by Bayesian Lasso quantile regression for variable selection assigning new formula of Laplace distributions for the regression parameters. Simple and efficient Markov chain Monte Carlo (M.C.M.C) algorithm was introduced for Bayesian sampler. Simulation approaches and two real data set are used to assess the performance of the proposed method. Both simulated and real data sets show that the performs of the proposed method is quite good for Identify Relative importance of covariates.

Keywords: *Bayesian lasso quantile regression, Prior distributions, posterior distributions, MCMC algorithm, Relative importance.*

JEL Classification*: C21, C11, C52,*

INTRODUCTION

Lasso is an important approach for coefficients estimation and variables selection in widely regression models. Lasso procedure has become applicable with many fields of statistics. Lasso method was proposed by (Tibshirani, 1996) for coefficient estimation and variable selection in the classical regression model. The Bayesian Lasso estimate can be expressed as a posterior mean when assign Laplace prior distribution for the parameters of covariates (Park and Casella, 2008). But the using of Laplace prior distribution directly is a hard matter to computing the posterior distributions and inefficient algorithm. So, we should use a simplified formula for Laplace prior distribution. Most researchers whose working on the field of the regularization Bayesian Lasso method used a scale mixture of normal (S.M.N) priors. Also, those researchers are putting conditions on the algorithms to make some of coefficients estimate equal zero exactly. But indeed, these coefficients are closed from zero. In this paper, we construct a new algorithm to identify the relative importance of covariates in quantile regression model. This paper is arranged as follows: Introduction of quantile regression model is shown in Section 2. In Section 3, we presented the Bayesian lasso quantile regression. In section 4, simulation study is implemented to evaluate the of the proposed techniques for the Identify Relative importance of model covariates. In section 5, real data considered to evaluate the proposed technique. In Section 6, the briefly conclusion.

2. QUANTILE REGRESSION MODEL

The quantile regression model has become very public since the seminal work of Koenker and Bassett (1978). It has been applied in different fields of studies, for instance: ecological studies (Cade and Noon, 2003), agricultural economics (Kostov and Davidova, 2013), growth chart (Wei et al., 2006), et cetera. The quantile regression model has good features compared to the mean regression model. Whereas, quantile regression is robust against to outliers, data (Koenker and Geling, 2001). Also the quantile regression model is efficient even though the assumptions of error distribution are not achieving. The quantile regression has the big ability to accommodate non-normal errors. The modeling of the quantile regression model has been written as:

$$y_i = x_i^T \beta_\theta + \varepsilon_i, \qquad \theta \in (0,1), \tag{1}$$

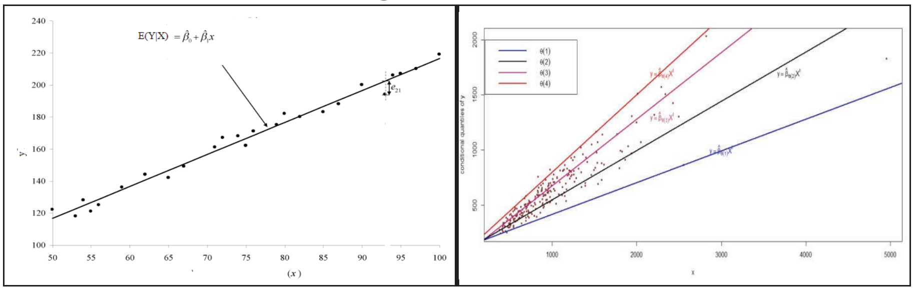
Where y_i $[i = 1,2, \dots \dots n]$ is the response variables.

x_i^T $[i = 1,2, \dots \dots$ is a vector of explanatory variables (covariates).

β_θ is vector of unknown parameters and θ is the quantile level.

θ is belong to open interval 0<θ <1, there are infinite points within this interval. This mean there are infinite quantile regression lines, each quantile model belongs to specific quantile level. Therefore, the quantile regression has high flexibility to provide us a full picture about the relationships between the response variables and covariates. Unlike classical regression model which is estimated only one regression line conditional mean of the response variable (y) given x, $E\ (y|x)$. This clear from two following figures:

Regression lines estimated via classical regression model and quantile regression model



The figure at right is represented regression line estimated to classical regression model. Sometimes, the classical regression model cannot give us a complete picture about the relationship between a response variable and covariates. The figure at left give us imagine that the quantile regression model can give us a complete picture about the relationship between a response variable and covariates whereas a many quantile regression lines are estimated. Each line belongs to specific quantile level. Therefore, the quantile regression can provide us a complete picture about the relationship between a response variable and covariates. (Koenker and Bassett, 1978) showed that the quantile regression parameters can be estimated by

$$\min_{\beta_\theta} \sum_{i=1}^{n} \rho_\theta(y_i - x_i^T \beta_\theta) \quad (3)$$

where $\rho_\theta(u)$ is the loss function (check function) defined by $\rho_\theta(u) = u\{\theta - \mathrm{I}(u \leq 0)\}$.

Unfortunately, the loss function is not differentiable at the 0. So that, there is no solution to Equation 2 (Koenker, 2005). (Koenker and D'Orey, 1987) are providing us a solution to this problem via a linear programming algorithm. Also (Yu and Moyeed (2001)) are proposed another technique for coefficient estimation of quantile regression by Bayesian approach. Li et al. (2010) proposed the Bayesian Lasso for coefficient estimation and variable selection of linear quantile regression model.

3. BAYESIAN LASSO QUANTILE REGRESSION

The Lasso quantile regression is proposed by (Li and Zhu, 2008). Where, the classical lasso quantile regression model has been written as

$$\min_{\beta_\theta} \sum_{i=1}^{n} \rho_\theta(y_i - x_i^T \beta_\theta) + \lambda \parallel \beta_\theta \parallel, \tag{3}$$

where λ is the shrinkage parameter, Also the equation (3) is not differentiable at 0, but possible, achieving parameters estimation through (rq.fit.lasso) function (Koenker, R. (2005)) within Package 'quantreg' (2013). But there is another method for achieving parameters estimation of the lasso quantile regression model is a Bayesian lasso quantile regression which implements variables selection and coefficient estimation to quantile regression model simultaneously. Li et al. (2010) proposed the Bayesian Lasso for linear quantile regression model by putting a Laplace prior which takes the form $p(\beta_j|\lambda) = {}^{\lambda}/_{2}\ exp\{-\lambda|\beta_j|\}$. On the j_{th} linear quantile regression model coefficient. More specifically, they put a scale mixture of normal prior distributions on β_θ. And exponential prior distributions for the variance parameters assuming they are independent.

$$p(\beta_j|\lambda) = {}^{\lambda}/_{2}\ exp\{-\lambda|\beta_j|\} = \int_0^\infty \frac{1}{\sqrt{2\pi u}}\ e^{-\frac{\beta_T^2}{2u}}\ \frac{\lambda^2}{2}\ e^{-\frac{\lambda^2}{2}u} \tag{4}$$

Equation (4) has shown a reformulation of the Laplace distribution, where this formulation provides simple computation and efficient MCMC algorithmic. Li et al. (2010) proposed the Bayesian Lasso for the linear quantile regression model through using equation (4) as a Laplace distribution prior. From these prior distributions of quantile regression parameters and the likelihood function of asymmetric Laplace distribution which belongs to random error of quantile regression model. We will obtain the posterior distributions of the coefficients of quantile regression model. In this section, we introduced a new procedure for implementing variable selection in Bayesian lasso quantile regression via uploading the MCMC algorithm. From known, the coefficients estimated via Bayesian approach is achieved through many thousands of iterations. At each iteration a new estimator will be generated according to the proposed algorithm (Gramacy, R. B., & Lee, H. K. H. (2008)). We proposed to calculate probability values for all independent variables by comparing the coefficient estimates on the open interval (-0.05 , 0.05). When the coefficient estimate is outside the open interval (-0.05 , 0.05), at the probability value will be set greater than 0.5 and that means the independent variable has a high relative importance in the model. On the contrary, when the coefficient estimate is in this interval (-0.05 , 0.05), at the probability value will less than 0.5 and that means this independent variable has a lower relative importance in the model. See (Reed, C 2011, Alhamzawi, R., 2016, Alhusseini 2017).

4. SIMULATION STUDY

In this section, the relative importance of covariates in our model is identified by the probability approach via following true model:

$$y_i = x_i^T \beta_\theta + \varepsilon_i,$$

We simulate 5 covariates from a multivariate normal with mean zero and covariance with $(\Sigma_x)_{hl} = 0.5^{|h-l|}$. The residuals $\varepsilon_i, i = 1, \dots \dots \dots, 100$ are generated from two different distributions a stander normal distribution *N* (0,1), and mixture normal distribution 0.3 N (1,1) + 0.7N (2,2). Via two simulation example first very sparse case and second dense case. In this simulation study via our algorithm will estimate thousand coefficients of covariates via thousands of iterations. Each estimated coefficient will compared with an open interval (-0.05,0.05) for computing its probability value. If this estimated coefficient outside the open interval (-0.05,0.05) by a probability value greater than 0.5, this means the covariate has strong relative important in our model. But If this estimated coefficient inside the open interval (-0.05,0.05) by a probability value less than 0.5 this means the covariate has weak relative important in our model.

4.1 First simulation example (very sparse case).

The coefficient of covariates is $\beta = (5,0,0,0,0)^T$. The number of observations is n=100. Three quantile levels are considered low quantile level: $\theta_1 = 0.20$, middle quantile level $\theta_2 = 0.60$ and high quantile level $\theta_3 = 0.90$. Table no. 1 presents the probability values of covariates via three quantile levels and two different residuals

Distributions:

Table 1: show the probability values of covariates for the simulation very sparse case:

		$\varepsilon_i \sim N\,(0,1)$		$\varepsilon_i \sim 0.3\,\mathrm{N}\,(1,1)\, +\, 0.7\mathrm{N}\,(2,2)$	
Quantile level	**covariates**	Probability values of covariates outside (-0.05,0.05)	Probability values of covariates inside (-0.05,0.05)	Probability values of covariates outside (-0.05,0.05)	Probability values of covariates inside (-0.05,0.05)
$\theta_1 = 0.20$	X_1	**0.867**	**0.133**	**0.877**	**0.123**
	X_2	**0.246**	**0.754**	**0.261**	**0.739**
	X_3	**0.891**	**0.109**	**0.865**	**0.135**
	X_4	**0.830**	**0.170**	**0.788**	**0.212**
	X_5	**0.805**	**0.195**	**0.815**	**0.195**
$\theta_2 = 0.60$	X_1	**0.851**	**0.149**	**0.848**	**0.152**
	X_2	**0.299**	**0.701**	**0.292**	**0.708**
	X_3	**0.831**	**0.169**	**0.838**	**0.162**
	X_4	**0.735**	**0.265**	**0.769**	**0.231**
	X_5	**0.791**	**0.209**	**0.774**	**0.226**
$\theta_3 = 0.90$	X_1	**0.856**	**0.144**	**0.868**	**0.132**
	X_2	**0.243**	**0.757**	**0.251**	**0.749**
	X_3	**0.884**	**0.116**	**0.874**	**0.126**
	X_4	**0.810**	**0.190**	**0.821**	**0.179**
	X_5	**0.166**	**0.834**	**0.176**	**0.824**

From results listed in table 1 at $\varepsilon_i \sim N\,(0,1)$ and a low quantile level $\theta_1 = 0.20$. We see four covariates (X_1, X_3, X_4, X_5) have strong relative important in our model (quantile regression model at $\theta_1 = 0.20$). Therefore, these covariates are very active in constructing our model. But there is one covariate (X_2) has weak relative importance in the quantile regression model at $\theta_1 = 0.20$. So, this covariate is inactive in constructing our model. We can cancel it. Also from the results listed in table 1 at $\varepsilon_i \sim 0.3\,\mathrm{N}\,(1,1)\, +\, 0.7\mathrm{N}\,(2,2)$ and a low quantile level $\theta_1 = 0.20$. We see four covariates (X_1, X_3, X_4, X_5) are very active in our model. But, there is one covariate has weak relative important in our model.

From result showed in table 1 at $\varepsilon_i \sim N\,(0,1)$ and a middle quantile level $\theta_2 = 0.60$. There are four covariates (X_1, X_3, X_4, X_5) have strong relative important in quantile regression at $\theta_2 = 0.60$, and one covariate has weak relative importance in our model. Also the same results are appeared with at $\varepsilon_i \sim 0.3\,N\,(1,1)\ +\ 0.7N\,(2,2)$. Where, there are four covariates (X_1, X_3, X_4, X_5) have strong relative important in our model . But there is on covariate has a weak relative important in our model. But at high quantile level there are three covariates (X_1, X_3, X_4) have strong relative important in quantile regression at $\theta_3 = 0.90$. This means, these covariates are important to constructing our model. Also from results, there are two covariates (X_2, X_5) have a weak relative important in quantile regression at high quantile level $\theta_3 = 0.90$. We can delete them from our model.

4.2 Second simulation (sparse case).

The coefficient of covariates is $\beta = (0.85,0.85,0.85,0.85,0.85)^T$. Also the number of observations are n=100. Three quantile levels are considered low quantile level: $\theta_1 = 0.20$ middle quantile level $\theta_2 = 0.60$ and high quantile level $\theta_3 = 0.90$. Table no 2 presents the probability values of covariates via three quantile levels and two different residual distributions

Table 2: show the probability values of covariates for the simulation sparse case:

		$\varepsilon_i \sim N\,(0,1)$		$\varepsilon_i \sim 0.3\,N\,(1,1)\ +\ 0.7N\,(2,2)$	
Quantile level	**covariates**	Probability values of covariates outside (-0.05,0.05)	Probability values of covariates inside (-0.05,0.05)	Probability values of covariates outside (-0.05,0.05)	Probability values of covariates inside (-0.05,0.05)
$\theta_1 = 0.20$	X_1	**0.762**	**0.238**	**0.662**	**0.338**
	X_2	**0.892**	**0.108**	**0.710**	**0.280**
	X_3	**0.925**	**0.175**	**0.716**	**0.284**
	X_4	**0.456**	**0.544**	**0.423**	**0.577**
	X_5	**0.869**	**0.131**	**0.713**	**0.287**
$\theta_2 = 0.60$	X_1	**0.746**	**0.254**	**0.707**	**0.293**
	X_2	**0.903**	**0.097**	**0.669**	**0.331**
	X_3	**0.682**	**0.318**	**0.646**	**0.354**
	X_4	**0.353**	**0.647**	**0.347**	**0.653**
	X_5	**0.783**	**0.217**	**0.685**	**0.315**
$\theta_3 = 0.90$	X_1	**0.801**	**0.199**	**0.694**	**0.306**
	X_2	**0.739**	**0.261**	**0.685**	**0.315**
	X_3	**0.664**	**0.336**	**0.724**	**0.276**
	X_4	**0.283**	**0.717**	**0.281**	**0.719**
	X_5	**0.055**	**0.945**	**0.385**	**0.615**

From table 2 at $\varepsilon_i \sim N\ (0,1)$ and a low quantile level $\theta_1 = 0.20$. We see four covariates (X_1, X_2, X_3, X_5) have high relative important in our model (quantile regression model at $\theta_1 = 0.20$). Therefore, these covariates have big ability in constructing our model. But there is one covariate (X_4) has weak relative importance in quantile regression model at $\theta_1 = 0.20$. Therefore, this covariate is a weakness in constructing our model. We can ignore it. From the results which are belong to quantile regression a low quantile level $\theta_1 = 0.20$ and $\varepsilon_i \sim 0.3\ \mathrm{N}\ (1,1)\ +\ 0.7\mathrm{N}\ (2,2)$. We see four covariates (X_1, X_2, X_3, X_5) are very active in our model. But ,there is one covariate (X_4) has a weak relative importance in our model.

From result showed in table 2 at $\varepsilon_i \sim N\ (0,1)$ and a middle quantile level $\theta_2 = 0.60$. There are four covariates (X_1, X_2, X_3, X_5) have a strong relative important in quantile regression at $\theta_2 = 0.60$, and one covariate has a weak relative importance in our model. Also the same results are appearing with at $\varepsilon_i \sim 0.3\ \mathrm{N}\ (1,1)\ +\ 0.7\mathrm{N}\ (2,2)$. Where, there are four covariates (X_1, X_2, X_3, X_5) have strong relative important in our model. But there is one covariate has a weak relative important in our model. But at a high quantile level, there are three covariates (X_1, X_2, X_3) have a strong relative important in quantile regression at $\theta_3 = 0.90$. This means, these covariates are important to constructing our model. Also from results, there are two covariates (X_2, X_5)have a weak relative important in quantile regression at high quantile level $\theta_3 = 0.90$. We can delete them from our model.

5. REAL DATA

In this section, we will used prostate cancer data which are first analyzed by Stamey *et al.* (1989) It was also analyzed using the model selected in the quantile regression model by Alhamzawi, R (2015). These data are existing within “bayesQR” package in R. The sample size of prostate cancer data was of 97 observations, the response variable is the level of prostate antigen referred to as (lpsa) and there are eight covariates.

These covariates are X_1 logarithm of cancer amount, referred to as (lcavol), X_2 the logarithm of the weight of the prostate, referred to as (left), X_3 age , X_4 the logarithm of the volume of benign enlargement of the prostate, referred to as (lbph), X_5 seminal vesicle invasion, referred to as (svi), X_6 logarithm of Capsular penetration in prostate cancer, referred to as (lcp), X_7 Gleason score, referred to as (gleason) and X_8 percentage of Gleason scores 4 or 5, referred to as (pgg45).

We applied a Bayesian lasso quantile regression for identifying of relative importance of covariates in our model at two quantile levels ($\theta_1 = 0.45\ and\ \theta_2 = 0.85$). Where, the relative importance of covariates is summarized in table 3

Table 3 shows the probability values to covariates of prostate cancer data

	Quantile level at $\theta_1 = 0.45$		Quantile level at $\theta_2 = 0.85$	
covariates	Probability values of covariates outside (-0.05,0.05)	Probability values of covariates inside (-0.05,0.05)	Probability values of covariates outside (-0.05,0.05)	Probability values of covariates inside (-0.05,0.05)
lcavol	**0.865**	**0.135**	**0.725**	**0.275**
lweight	**0.675**	**0.325**	**0.882**	**0.118**
age	**0.768**	**0.232**	**0.634**	**0.366**
lbph	**0.283**	**0.717**	**0.361**	**0.639**
svi	**0.875**	**0.125**	**0.732**	**0.268**
lcp	**0.182**	**0.818**	**0.310**	**0.690**
gleason	**0.247**	**0.753**	**0.185**	**0.815**
pgg45	**0.536**	**0.464**	**0.137**	**0.863**

The results in Table (3) at quantile level $\theta_1 = 0.45$, there are contains five important covariates (**lcavol**, **lweight** , **age**, **svi** , **pgg45**) in quantile regression model at quantile level ($\theta_1 = 0.45$). Where, these covariates have a big importance in constructing our model. Because of, these independent variables have a Probability value greater than 0.5. Also, at the same results in a table (3) contains three unimportant independent variables (**lbph**, **lcp, Gleason)** in our model at the quantile level ($\theta_1 = 0.45$). Where, these covariates have a lower relative importance in constructing our model. We can ranked these independent variables in our model as follows:

- First rank is assigned to **svi** by a Probability value is **0.875**,that is greater than 0.5. This means, **svi** is main covariate in quantile regression model at quantile level ($\theta_1 = 0.45$).
- Second rank is assigned **lcavol** with a Probability value is **0.865**, that is greater than 0.5. This means, **lcavol** is the main covariate in the quantile regression model at the level ($\theta_1 = 0.45$).
- Third rank is assigned to **age** by a Probability value is **0.768**, that is greater than 0.5. This means, **0.768** is the main covariate in the quantile regression model at the level ($\theta_1 = 0.45$).
- Fourth rank is assigned to **lweight** by a Probability value is **0.675** ,that is greater than 0.5. This means, **lweight** is the main covariate in the quantile regression model at the level ($\theta_1 = 0.45$).
- Fifth rank is assigned to **pgg45** by a Probability value is **0.536**, that is greater than 0.5. This means, the **pgg45** is the main covariate in the quantile regression model at the level ($\theta_1 = 0.45$).

These five covariates have a higher relative importance of modeling the relationship with the level of prostate antigen referred to as (lpsa). But the following three covariates have a lower relative importance of modeling the relationship with the level of prostate antigen referred to as (lpsa).

- Sixth rank is assigned to **lbph** by a Probability value is **0.283**, that is less than 0.5. This means, the **lbph** is the secondary covariate in the quantile regression model at the level ($\theta_1 = 0.45$).
- Seventh rank is assigned to **gleason** by a Probability value is **0.247**,that is less than 0.5. This means, the **gleason** is secondary covariate in quantile regression model at level ($\theta_1 = 0.45$).
- Eighth rank is assigned to **lcp** by a Probability value is **0.182** ,that is less than 0.5. This means, **lcp** is secondary covariate in quantile regression model at level ($\theta_1 = 0.45$).
- We can exclude these three independent variables from our model (quantile regression model at the level ($\theta_1 = 0.45$).

Also from the results listed in table 3 at quantile level $\theta_2 = 0.85$, there are four active covariates are arranged as follows:

1- **Lweight** has a probability value is **0.882** ,that is greater than 0.5. Therefore, **Lweight** is a strong in structure of our model (quantile regression model at level ($\theta_2 = 0.85$)

2- **Svi** has a probability value is **0.732** ,that is greater than 0.5. Also, **svi** is a strong in structure of our model (quantile regression model at level ($\theta_2 = 0.85$)).

3- **lcavol** has a probability value is **0.725**, that is greater than 0.5. Also, **lcavol** is a strong in structure of our model (quantile regression model at level ($\theta_2 = 0.85$)).

4- **Age** has a probability value is **0.634**, that is greater than 0.5. Also, **Age** is a strong in structure of our model ((quantile regression model at the level ($\theta_2 = 0.85$)).

These four covariates above have a high relative importance in constructing the quantile regression model at the level ($\theta_2 = 0.85$)). Therefore, we must use them in our model.

At the same time, there are four unimportant covariates have a probability value less than 0.5. Therefore, we can exclude the following four unimportant covariates from our model

- **lbph** has a probability value is **0.361**,that is less than 0.5. It considers modest in our model. We can exclude it from our model.
- **lcp** has probability value is **0.310** ,that is less than 0.5. It considers modest in our model. We can exclude it from our model.
- **Gleason** has probability value is **0.185**,that is less than 0.5. It considers modest in our model. We can exclude it from our model.
- **pgg45** has probability value is **0.137**, that is less than 0.5. It considers modest in our model. We can exclude it from our model.

Therefore, we cannot depend on these four unimportant covariates for assessing the relationship with the response variable. The level of prostate antigen referred to as (lpsa).

6. CONCLUSION

Each covariate has a specific influence in specific regression models, this clear from simulation studies and real data. Some covariates have a big relative importance in studying model, and other covariates have a weaker relative importance in studying the model. In simulation study at first simulation example. We see the covariate X_2. Has a weak influence in the quantile regression model at low and middle quantile levels. But at a high quantile level, there are two covariates (X_2, X_5) have a weak relative important in our model. Therefore, we can delete them from our model quantile regression model at the high quantile level. Also in second simulation example.We see the covariate (X_4) Has a weakness in the quantile regression model at three quantile levels. So, these weak covariates, we can cancel them from our model. And depend on strong So, these weak covariates in constructing our model.From real data, we see five covariates have big relative importance in our model (quantile regression model at quantile level $\theta_1 = 0.45$). But the rest three covariates have a weaker relative importance in our model (quantile regression model at quantile level $\theta_1 = 0.45$). Ana also from real data in quantile regression model at quantile level $\theta_2 = 0.85$, there are four covariates have a strong in constructing our model. But rest four covariates have simple influence in our model

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