Simulation Process, Theoretical Paradigm And Operational-Strategic Realy As Tool For Managerial Decision Making

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ABSTRACT

The present study is concerned with the concept of simulation and the development of simulation models, transposed into mathematical formalism from engineering sciences into economic context, as a powerful and effective tool for managerial decision making. The applications under consideration involve deterministic systems with continuous time and states, as well as with discrete ones.

There presented simulations are of type G/G/1 activity, considered to be representative for models of modern business-type of problems, as well as paradigms, concepts and the mathematical formalism from engineering sciences, which have been successfully applied to economic organization-type problems, such as the Pollaczek – Khinchin formula, Lindley's equation, the Wiener-Hopf equation.

Finally, two classical, representative methods for simulations are briefly and synthetically discussed, the Monte-Carlo method and the Metropolis method, together with the methodology of implementation via the specialized software Microsoft Excel, and with the convergence of the simulation processes.

Key Words*: Simulation, modeling, business model, mathematical formalism.*
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INTRODUCTION

The complexity of the managerial decision, aimed at consolidating the market position and subsequently the development of a modern economic organization, requires increasing the profit rates, which demands establishing appropriate business models, innovative, cross-disciplinary concepts, adapted from other areas of fundamental and applied research.

We propose here a methodology based on the concept of simulations, which has been extensively used in fundamental research in the engineering sciences.

The explosive growth of digital resources provides this paradigm new, ample dimensions, as the modern simulations models are nothing else but virtual information structures (Bratley et al., 1983). Simulations, together with the development of models, play a great role in the process of design, implementation, and exploitation of complex technological systems. In the case of an economic organization system, these are described through the multitude of its components, the strategic-operational strategic relations among them, and the multiple variables that define the state of the system at any point in time, the simulation process is used to reproduces a given process, via some model elaborated based on the original process, at some point in time when the original process is not present.

The situations when simulations are recommended can be identified for the following types of economic processes: activities with significant financial allocations and with large numbers of operational alternatives for distribution chains, optimal decisions, high risk economic-industrial processes – air transportation, industrial-economic processes involving dangerous substances, chemical industry, slow agricultural processes, and rapid industrial, chemical, or physical processes. It was stated that *"the simulation of a given economic process (operational-strategically identifiable) is designed based on a second economic process, distinct from the one under consideration, which however follows the same rules encapsulated in analogous equations that are applicable for both models."*(Heche et al, 2003)

Rather than deterministic systems (for which the current states are uniquely and continuously determined by the previous states), via differential equations, it is discrete, stochastic systems that contributed to the development of the simulation paradigm.

Summarizing, simulation amounts to reproducing, with some degree of accuracy, an economic phenomenon over some well defined time horizon.

A system is said to be of "continuous time" if the time variable takes real numerical values within some interval (not necessarily bounded or open).

A system is said to be of "discrete time" if the set of all possible values of the time variable forms an ordered, countable sequence, possibly finite.

The set of time values can be thought of a spatial structure on the real axis, obtained by sampling the continuous time axis at regular time intervals.

Another possible definition, which can be useful in processes based on iterative algorithms, is that of a discrete, zero measure set, that is a subset of some larger set (the real axis), similarly to the classification of the time variable, the set of state of the system can be classified as continuous or discrete.

The system itself can be considered as deterministic (when the evolution in the future is precisely determined from its present state by some evolution law), or stochastic or random (when the evolution on the future cannot be precisely derived from its present state but only probabilistically).

THE CONCEPT OF SIMULATION AND MODEL DEVELOPMENT

The quality of the chosen model

The validity of the simulation of the economic process depends on the accuracy of the chosen model, a high quality model yields simulations that provide precise, clear, quantifiable results that are close to reality.

The procedure (Heche et al, 2003) of identifying the most appropriate model can be schematically presented as follows:

Primary − −→ Original − −→ Experiment − −→ Primary Quantifiable
rule − − − −→ process − −→ monitoring − − − −→ results
Comparative Procedures ***Statistical Tests***
Secondary − −→ Chosen − −→ Process − −→ Secondary Quantifiable
rule − − − −→ model − −→ simulation − − − −→ results

In the process of choosing the most adequate model for simulations, we identify the following concepts, and situations:

-both the rules that make the initial (original) process operational and the rules that drive the model are known, this is possibly an ideal situation, when the rules can be compared, by interchanging the variables or other entities, possibly leading to high complexity optimization problems.

-only one set of rules in known, in which case one has to make assumptions on the other set of rules and on the validity of the experimental results obtained by observing the economic phenomena, leading to non-linear optimization problems.

-both sets of rules are unknown, however it is possible to quantify and compare the outcomes of the original process with the results of the simulations of the chosen model.

In this case, an essential role is played by statistical tests, which are used to decide whether the chosen model needs to be rejected in the light of the data. Hypothesis testing are a powerful instrument in such approaches.

It is possible to obtain results from simulations of the chosen model, however data on the original process is not available. In this case accepting the results of the simulations, without a comparison with the original process, assumes major risks in the subsequent decision making.

The simulation of processes with continuous times and states, these processes are subordinated to a mathematical formalism (Heche et al, 2003) based on differential equations, for which we adopt the following notation convention:

$x \in R^n$	*Vector representing the state of the szstem at a given time*
$u \in R^m$	*vector representing the external actions or commands on the systems*
$y \in R^k$	*vector of th outcomes of the system*
$A: n \times n$	*the transition matrix of the system*
$B: n \times m$	*the matrix of actions an commands on the evolution of the system*
$C: k \times n$	the transition matrix, $\frac{input}{output}$ of the system

The law that describes the dynamics of the economic system is given by the formula:

$$x^{\cdot} = Ax + Bu$$
$$y = Cx \quad (1)$$

If a feed-back loop is present, with the matrix R representing the transformation of a state of the system into a command, with $u = Rx$, we have the following differential equations:

$$x^{\cdot} = (A + BR)x$$
$$y = Cx \quad (2)$$

The system is described as follows:

Figure 1

$$-u \to (B) \to (A) - x \to (C) - y \to$$
$$- - (R) < -$$

The symbol *A* in the above diagram corresponds to the linear system $x^{\cdot} = Ax + w$, obtained under the assumption that w represents an input of the system.

As scientifically validated and operationally applicable software solutions, (Cormen et al, 2009) it is recommended to use the computer algebra systems $Maple, Matlab, Mathematica$, which offer general modules for system integration, concept adapted from the engineering sciences, these software packages present exceptional facilities, due to their friendly graphic interface, and the ability to take in an and executing algebraic operations with an arbitrary number of vectors, each vector consisting of n components that are time dependent.

Starting with an initial condition represented by a vector $x(0)$ of n components, the summation or integration operator computes the sum or integral, from 0 to t, of the previous vectors, we identify a module that takes in as input vectors of the form $x(t) = (x_1(t), \dots, x_n(t))$, ale whose components are functions of time, and yields as outputs functions of time obtained by multiplying the input vector x by the matrix $A: n \times n$.

The general scheme is to translate concepts from advanced technology (from the world of electronics) to economic processes (managerial processes, capital price dynamics), this simulations are based on linear differential systems with constant coefficients.

The time dependent vectors represent the interaction between the system and the environment, and are generated via some simple circuits, which makes this type of simulation models adequate for economic problems from a wide spectrum of business structures.

To analyze the process following this scheme, if we put $u \equiv 0,$ and suppressing the feedback loop, that is, for $R \equiv 0,$ we obtain the homogeneous system $x^{\cdot} = Ax.$

Independently of initial condition $x(0),$ the system is stable if all roots of the characteristic polynomial $\det[A - \lambda I],$ have negative real parts; these roots are the eigenvalues of the system.

Considering the feedback loop described by $R,$ one obtains stability for the system from figure graphics 1, independently on the initial conditions, in the case when all roots of the characteristic polynomial $\det[A + BR - \lambda I],$ have negative real part.

We point out that the simulation of non-linear system of the type $y^{\cdot} = \phi(y) + w,$ present a higher degree of complexity than in the linear case, as the qualitative analysis of the solution is much more intricate.

THE SIMULATION OF DISCRETE SYSTEMS AND EVENTS: METHODOLOGY AND MODELS

In simulation of economic processes of stochastic type, the most common paradigm s that of modeling via discrete events, via markovian processes (Cormen et al., 2009) in continuous time, where the state space and the transition law are explicit, it is important to point out that the simulation of discrete events has generated the creation of some specialized computer programming languages, of expert type, which are adapted to the requirements of the scientific models as well as to the business-operational type paradigms, such as *SIMULA, GPSS, SIMSCRIPT*, in subsequent development, object oriented programming languages concepts, and discrete events simulation models have been developed, which influenced in a significant way the development of modern client systems.

In what follows, we present a simulation model based on a sequence of type G/G/1, specific to economic phenomena in the areas of logistics and distribution of goods.

ARRIVALS	**WAITING**	**SERVER**	**DEPARTURES**
Arrival processes	**Service disciplines**	**Service processes**	**Departure processes**

We introduce the following notation that is utilized in probability laws:

"M", is associated with exponential law, "*MARKOVIAN*","*MEMORYLESS*, the arrival process is of *POISSON* tyoe, and the time intervals between two successive arrivals of clients are random variables.

"D, is associated with a "*Degenerate law*", the arrival of clients occurs at regular time intervals.

E_k", process associate to a process when the time intervals between two successive arrivals is a random process following d'ERLANG law of order "k".

"G" no assumption is made on the arrival process

In waiting-time theory (Filipowicz and Kwiecien, 2008) , which is part of probability theory, we denote by *G/G/*1 a sequence (queue) of waiting times, in a single-server system, when the arrival and the services are distributed within some arbitrary interval, as described below:

$G - general\ type\ of\ distribution\ of\ arrival\ times, no\ hypothsis$
$G - general\ distribution\ of\ waiting\ times, S^- = 1/\mu$
$1 - single\ service, load\ \rho = \lambda S^-,\ for\ a\ stable\ sequence, \rho < 1$

The model the waiting sequence, *G/G/*1 is a complex system for which utilizes general types of approximations, one of the simplest being described by the $Pollaczek - Khinchin$ formula

$$W^-_{G/G/1} \cong (\frac{C_A^2 + C_S^2}{2}) W^-_{M/M/1} \tag{3}$$

In waiting-time theory, the previous formula describes the relation between the length of the queue and the distribution of service times via $LAPLACE\ transform$, and it also relates the average length of the waiting queue and the average service time, in this expression, C_A^2 is the square of the variation of the random variable that describes the time intervals between two successive arrivals of clients, distribution following a probabilistic law under the assumption that the events are identically distributed and independent, and C_S^2 is the random variable that models the service ich are also assumed to be identically distributed and independent, the value $W^-_{M/M/1}$ corresponds to the average waiting time in a queue $M/M/1$, with the same rate of arrival and service as in the system under consideration.

Lindley Equation

We propose the simulation of the behavior of such a queue with a model of the type FIFO, which studies the waiting times via Lindey equation, we consider a sequence of arrivals of clients indexed by n, using the formalism from below:

C_n	$the\ n - th\ client\ attiving\ to\ the\ system$
a_n	$the\ time\ of\ the\ arrival\ of\ C_n$
d_n	$the\ time\ of\ departure\ of\ C_n$
t_n	$time\ interval\ between\ C_{n-1}\ and\ C_n$
s_n	$service\ time\ of\ C_n$
w_n	$waiting\ times\ for\ C_n$

The process of interest is defined by the succession $\{w_n, n = 0, 1, \dots\}$ of waiting times, before establishing the fact that the process is markovian, the time of departure of C_{n+1} is

$$d_{n+1} = \begin{cases} d_n + s_{n+1}, & if\ d_n \geq a_{n+1} \\ a_{n+1} + s_{n+1}, & if\ d_n \leq a_{n+1} \end{cases} \quad (4)$$

The first restriction corresponds describes the situation that C_{n+1} needs to wait for the departure of C_n in order to access the server, the second restriction describes the situation when C_{n+1} arrives to an empty system, by definition, the waiting time of C_{n+1} is given by

$$w_{n+1} = d_{n+1} - a_{n+1} - s_{n+1}$$

subtracting $(a_{n+1} + s_{n+1})$ from (4), it follows

$$w_{n+1} = \begin{cases} d_n - a_{n+1}, if\ d_n - a_{n+1} \geq 0 \\ 0,\ if\ d_n - a_{n+1} \leq 0 \end{cases}$$

Since $w_n = d_n - a_n - s_n$, it follows:

$$w_{n+1} = \begin{cases} w_n + s_n - t_{n+1}, if\ w_n + s_n - t_{n+1} \geq 0 \\ 0, if\ w_n + s_n - t_{n+1} \leq 0 \end{cases}$$

Introducing the variable $u_n = s_n - t_{n+1}$, we obtain the fundamental relation:

$$w_{n+1} = \begin{cases} w_n + u_n, if\ w_n + u_n \geq 0 \\ 0, if\ w_n + u_n \leq 0 \end{cases} \quad (5)$$

The sequences $\{t_n, n \geq 0\}, \{s_n, n \geq 0\}$ are mutually independent, and the form independent random variable, the sequence $\{u_n, n \geq 0\}$ is also form by mutually independent random variables, and, due to (5), the sequence $\{w_n, n \geq 0\}$, defines a markovian process, analyzing the distribution function $U_n(z)$ and u_n in terms of the time intervals between successive arrivals of the clients, denoted by $A(t)$, and service times denoted by $B(s)$, for $z \in R$,

$$U_n(z) = P[u_n = s_n - t_{n+1} \leq z],$$

$$U_n(z) = P[u_n = s_n - t_{n+1} \leq z] = \int_{t=0}^{\infty} P[s_n \leq z + t | t_{n+1} = t] dA(t) = \int_{t=0}^{\infty} B(z+t) dA(t) \quad (6)$$

The previous expression shows that the distribution function $U_n(z)$ is independent of n, hence it can be written as $U(z)$, thus, the waiting queue is stable and the single variable u_n has negative expectation if

$$E[u_n] = E[s_n - t_{n+1}] = E[s_n] - E[t_{n+1}] = E[S] - E[A] = e[A](\rho - 1),$$

where $E[S]$, $E[A]$ represent the expectation of the service time and of the time intervals between successive arrivals of clients, the expectation u_n is negative if and only if $\rho = E[S]/E[A] < 1$, denoting $W_n(y)$ the distribution function of the waiting times w_n for C_n, we obtain for $y \geq 0$ according to (6),

$$W_{n+1}(y) = P[w_n + u_n] = \int_0^{\infty} P[u_n \leq y - w | w_n = w] dW_n(w) = \int_0^{\infty} U(y-w) dW_n(w) \quad (7)$$

The waiting time is assumed to be stable, and the processes $\{w_n, n \geq 0\}$ are of ergodic type, a process is said to be "ergodic" if its statistical properties can be derived from a randomly chosen sample, and any collection of random samples extracted from the process exhibit the typical statistical properties of the process in its entirety, non ergodic processes have a chaotic rate of change, the process under consideration admits an invariant distribution $W(y)$, which verifies the functional equation:

$$W(y) = \int_0^{\infty} U(y-w) dW(w), y \geq 0$$

The waiting times have non-negative dimension, hence $W(y) = 0$, for $y < 0$, for a queue of the type *G/G/*1 that is stable, (Kleinrock, 1975) the stationary distribution of the waiting times satisfies Lindley's integral equation:

$$W(y) = \begin{cases} \int_0^{\infty} U(y-w) dW(w), y \geq 0 \\ 0, y < 0 \end{cases} \quad (8)$$

This is an equation of Wiener-Hopf type, which does not admit an closed form solution, except for very simple, particular cases, however it can be solve with numerical methods that are easy to implement via software applications, the approach to this type of problems is an important method in mathematics, (Harb et al, 2016) on solving system of integral equations, which has many important applications, there are also applications in the field of partial differential equations in two dimensions, with mixed boundary conditions; the method can be applied with the aid of the standard Fourier transform, other possible transforms can be used, such as the Mellin transform.

The transforms are applied to the equations underlying the respective processes and to the boundary conditions, we define a dual space of complex functions "+", " − ", which can be identified with analytic functions on the upper and on the lower half-plane of complex numbers, on this domain the two functions are endowed with a "low dimension strip structure" that has as a component the real axis, the two functions are integrated and define a single analytic function on the complex plane, applying Liouville's Theorem, it follows that the resulting function is a polynomial, with its degree determined by the boundary conditions, which turns out to be a constant or zero in many cases.

We should mention here the "Liouville's Theorem", in the context of statistical mechanics, is a fundamental tool to describe the evolution of the dynamics of a system consisting of a large number of particles, which in our case are represented by modern economic organizations, assimilated to points, forming altogether a system of point masses.

In the case of simulation based on discrete model, the system underlying the economic process is described at any moment by a well-defined state, which does not change until a new event occurs.

To study the dynamics of a system portraying a specific economic situation, one initializes the simulation with an event that triggers a sequence of subsequent events that take place within a certain time horizon in the future, or modifies the evolution of posterior events that were already scheduled to take place in the future, these events only occur at the discrete times, as suggested by the name of the model. On the modeling aspect, we remark that for a process structured on specific activities, for example the successive operations on an assembly line, decisional tree specific to production management, we are not concerned with the specific activities that mark the beginning and the final of the process, but only the resulting final effect.

As long as the activity takes place, there is no visible effect on the system besides that locking of the resources, so the state of the system does not change for this type of activity throughout its evolution, this is in contrast with models based on continuous time, described previously, for which the state of the system is continuously updated, in applications to economic operations, for case studies with specific requirements, sometimes one uses "mixed" simulations, which comprise both a continuous part and a discrete part.

Simulation elements, discrete events

We identify the following main elements in the process of discrete event simulation, relevant for our discussion:

- descriptor of status elements of the systems that underlie the demand for the determination of simulation running;
- indicators (counters, quantifiers) of positions (places) where the results obtained are stored;
- a chronology for future events, together with upgraded algorithms and procedures, allowing the management of an event insertion in the correct position, searching the next event and its exclusion from this chronology;
- various types of events, each with its own description, those inducing actions of events on the system status, conditional triggers, the chronology position of the future events, upgrading and updating the various recordkeeping and statistics indicators.

OPERATIONALIZATION OF SIMULATION: ALGORITHMS AND IMPLEMENTATIONS

The operationalization of simulation activity (Bratley et al., 1983) presupposes the elaboration of three fundamental phases, (Heche at al., 2003), (Cormen et al, 2009) as follows:

$\boldsymbol{START}$: At the beginning, the system is positioned in its initial status, a first event, $START$, initiating the process onset time, from the point $t = 0$, the record of subsequent events in the time spreadsheet and forecasts especially the event $STOP$, marking the end of simulation;

$\boldsymbol{PROCESS}$: Successive events are processed in their chronological order of occurrence within the time agenda: the movement takes place from the initial moment of the first temporary event, this event being then removed from the time schedule (spreadsheet), after it is executed (operationalized), the requested status change takes place, we record the subsequent event in the appropriate position of the time spreadsheet, we upgrade the corresponding indicators and statistics, the time the next event produces being mentioned;

$\boldsymbol{STOP}$: Reaching the time when the significant event takes place $STOP$, marks the end of simulation, this can be triggered by other events, or it is a temporary scheduled event from the beginning of the simulation, to indicate that the time granted to this process type expired, the algorithms and procedures cease their operationality and the results are presented.

For the transposition in operationality for the simulation of managerial decision, high-level simulation software solutions are accessed, but which request a high number of parameters to induce the particular simulation model.

Software facilities specialised and dedicated to simulations offer various graphical interfaces leading to a better understanding of phenomena, graphical animation representing a communication platform between the actors involved in the modelled and simulated systems, advanced-level simulation digital solutions are suitable for the rapid elaboration of simplified prototypes, the systems that must be simulated in managerial operationality have a higher and higher complexity degree, with the evolution and development of business models and the quantity of information that must be identified, selected and submitted to the simulation process.

Compared to much more rigorous and relatively more stable engineering processes, economic processes have a higher dynamism and a much more complex uncertainty degree, due to the involvement of human decision-maker, as an uncertainty and perturbation factor.

Generic simulation of discrete events is advantaged by the increasing capacity of the computing power of the new computer generations, but also by the data stocks under the form of databases and even data deposits.

Application, simulation of a waiting line, *G/G*/1

We have briefly presented (Kleinrock, 1976) the waiting line of the type $G/G/1$, the total customers arriving into a system of this type are taken over under the responsibility of the one serving, if it is free, it is composed of an infinite capacity queue, where each person waits that the precedent person be served in his/her arriving order, before its turn, this representing the operation rule $FIFO$ (acronyme for "First In First Out",method for row management, of the waiting queues), (Filipowicz and Kwiecien, 2008).

The time intervals between two successive customer arrivals form a row $\{\alpha_i\}$ of independent random variables, submitted in the same distribution functions $F_\alpha(t)$, with $E[\alpha] = \frac{1}{\lambda}$.

Within an analogous approach, the serving times for successive customers form a row $\{\beta_i\}$ of independent random variables, all of them distributed according to a distribution function $F_\beta(t)$, with $E[\beta] = \frac{1}{\mu}$, the row of serving times is composed of independent entities of the customer arriving processes.

The system is stable and reaches its stationary regime if and only if $\rho = \frac{\lambda}{\mu} < 1$, as we assume in the continuation of the developed reasoning.

The final target of simulation is the estimation of an average number N^- of present customers, so that the average standing time T^- of a customer within the system when it evolves in stationary regime.

In specialty literature, it has an increased difficulty degree for lines with the structure *G/G*/1 and it is much easier for particular cases such as waiting lines *M/M*/1. This particularities offer good opportunities of testing the simulation software, comparing the generated results with the values resulted by mathematical calculations, we define the variable *n* indicating at each moment *t* the value of the function n_t , the number of customers that are present in the system, and the one in progress of being served.

We note that if *n* defines the status of a Markov process in the case of a waiting line of the type *M/M*/1, this is not also valid for the line *G/G*/1 being the argument for which this more general waiting line has a higher complexity degree to be analytically treated as Markov type and, therefore (Kleinrock, 1976).

It is preferably to proceed to simulations to obtain quantitative indications on the behaviour of this type of systems, the value $n_{cum} := \int_0^t n_\tau d\tau,$ represents the total number of customers X the time units being served by the system in the timeframe $[0, t]$.

We calculate the average number of customers present in the system at some moment in the interval $[0, t]: N^- = \frac{n_{cum}}{t}$.

The simulation process (Heche et al, 2003) imposes the definition of four structural events, ***START, STOP***.

ARRIVAL, DEPARTURE being thus four algorithm structures changing the system status, triggering in their turn events and managing statistical scheduling.

$\boldsymbol{THE\ ALGORITHM\ OF\ SIMULATED\ EVENTS}$

$\boldsymbol{START} \coloneqq Simulation\ initialization\ and\ launching$

$The\ number\ of\ customers\ extisting\ in\ the\ system, n \coloneqq 0.$

$The\ cumulation\ of\ system\ existence\ times, n_{cum} = 0.$

$Current\ times\ t \coloneqq 0.$

$Total\ duration\ of\ the\ simulation\ process\ D_{tot}.$

$Forecasting\ the\ occurrence\ of\ the\ STOP\ event\ , at\ the\ moment\ t_{fin} \coloneqq D_{tot}.$

$The\ operationalization\ of\ the\ law\ F_\alpha(t), the\ arrival\ of\ the\ first\ customer, achievement\ of\ \alpha.$

$Forecasting\ a\ first\ event\ of\ ARRIVAL\ type, at\ the\ moment\ t_{arr} \coloneqq \alpha, within\ the\ scheduling.$

$\boldsymbol{ARRIVAL}$: *the process of arrival of a new customer takes place at the moment* t_{arr}

The event is removed from the time spreadsheet.

Be $\Delta \coloneqq t_{arr} - t; t :=$
t_{arr}, *the update of the clock for the times for the occurrence of the*

current ARRIVAL event, after which the time Δ *is calculated between this one*

and the previous one.

We generate a radom variable α , *which is submitted to the distribution function* $F_{\alpha,}$

up to a new ARRIVAL.

We schedule a new ARRIVAL event for the time moment $t_{arr} := t + \alpha$

within the temporary forecast.

If $n = 0$, *the line is free, then*

We generate a random variable β, *for the distribution function* $F_\beta(x)$,

representing the serving time for the new customer.

We forecast the DEPARTURE event at the time moment $t_{dep} \coloneqq t + \beta$.

Be $n_{cum} \coloneqq n_{cum} + n \times \Delta$, *the statistics update.*

Be $n \coloneqq n + 1$, *the system status update.*

There follows the search of the next event within the temporary forecast.

$\boldsymbol{DEPARTURE}$: *A customer leaves the system at a moment* t_{dep}

The event is removed from the calendar schedule.

Be $\Delta \coloneqq t_{dep} - t; t \coloneqq t_{dep}$, *the clock update at the time* t_{dep},

for the current DEPARTURE event, after the calculation of the time Δ, *between*

an eveniment and the previous one.

If $n > 1$, *at least one customer is waiting within the line*, then

We generate a random variable β, *according to the law* $F_\beta(t)$.

We forecast (schedule)the DEPARTURE event, at the moment $t + \beta$.

Be $n_{cum} \coloneqq n_{cum} + n \times \Delta$

Be $n \coloneqq n - 1$, *the update of the number of customers in the system.*

There follows the search of the next event in the forecast.

$\boldsymbol{STOP}$: $we\ present\ the\ results\ and\ the\ simulation\ end\ process\ at\ the\ moment\ t_{fin}$.

$Be\ N^{-} := \frac{n_{cum}}{T_{fin}}$, $the\ average\ number\ of\ customers\ present\ in\ the\ system.$

$Be\ T^{-} :=$
$\frac{N^{-}}{\lambda}$, $the\ average\ waiting\ times\ for\ the\ customers\ in\ the\ system, that\ can\ be\ calculated\ Little.$

$\boldsymbol{STOP}$

This presentation of the simulation algorithm, (Heche et al, 2003) offers the possibility that two or more simultaneous events be distanced, for the simulation of the previous algorithm, the occurrence of simultaneous events is null, being thus required to elaborate a way of tie breaking of simultaneous events and the coherence of the process must be preserved.

Table, scheme of discrete event simulation running, row G/G/1, we highlight four events, START, ARRIVAL, DEPARTURE, STOP

$\boldsymbol{FORECAST - CALENDAR}$		$\boldsymbol{EVENTS}$
$Times$	$Eevents$	
0	$START \rightarrow$	$START$
t_1	$ARRIVALS < -$	
…	$\rightarrow$	
…		
t_m	$ARRIVAL < -$	
t_{m+1}	$DEPARTURE < -$	$ARRIVAL$
…	$\rightarrow$	$DEPARTURE$
t_q	$ARRIVAL$	
…		
t_s	$DEPARTURE$	
…		
…		
t_{STOP}	$STOP$	$STOP$

DISCUSSION OF MOST REMARKABLE METHODS USED IN THE SIMULATION PROCESSES

$MONTE\ CARLO\ method$ is at the origin of simulation of stochastic processes (Fishman, 1996), the hazard within these processes being minimized, and the method is recommended for the resolution of various problems, involving a low computing effort compared to the difficulty of the problem.

The simulation of managerial decision-making problems (Bratley et al., 1983) involving economic organizations is applied to all classes of challenges that include operational rules, algorithms and procedures, involving the adaptation of decisions, their control, price policies, the resolution of problems that are specific to economic organizations, by simulation technologies, impose the use of some interactive algorithms and the existence of some rigorous phases, aiming at reaching a final presupposed objective, using entry data randomly generated.

The $MONTE\ CARLO\ method$ is successfully applicable in the multiple integral calculation (Rubinstein, 1981), within the resolution of differential equations with partial derivates and combination optimization, modelling of an economic system presupposes the modelling of time variables for the treatment of an item within the operationalized processes and the two arrivals by probability distributions.

Within the theory of waiting lines and those of modelling/simulation of processes that are specific to economic organizations, the arrivals admit a dual-type perception, either by random numbers of items throughout a reference period (defined time units), or by random time intervals separating two successive arrivals within the system.

Two distributions are usually used in the random arrival processes of the customers in an activity specific to business models:

- $POISSON\ distribution$ corresponds to the distribution of a number of X events that occur within a space-time frame given (for example, spatial framework - the access to a retail point, organizational sale, temporary framework - time frame mentioned, event - the arrival of a customer), $POISSON\ probability\ distribution$ of parameter λ' is $P(X = x) = e^{-\lambda'} \lambda'^x / x!$, this distribution corresponds, dually, to a time frame T separating two successive events following the exponential law of the parameter $\lambda = \frac{1}{\lambda'}$, under the formalism $P(T > t)$ $\int_t^\infty \lambda \cdot e^{-\lambda t} = e^{-\lambda t}$, this distribution formulation being preferred to the $POISSON$ one the modelling, simulation of economic processes, the probability that an event occur in the time frame t, $t + \varepsilon, \varepsilon \to 0$ is the same, regardless the time run =since the previous event, this conditioned probability corresponds to what we call $hazard\ function,$ specialised software $EXCEL$ offers the possibility of quantification with the spreadsheet $Arrival\ POISSON\ Introduction\_Simulation.xls,$ the $POISSON$ -type process being a process "$without\ memory$".

-the second distribution used for modelling the random entries of items in an economic system is $d'ERLANG\ distribution,$ this being characterized by a parameter of the form $\alpha,$ positive whole number and a parameter of the scale $\frac{1}{\lambda'},$ as follows $P(T > t) = e^{-t/\lambda} \cdot \sum_{i=1}^{\alpha-1} (\lambda \cdot t)^i / i!,$ and $E(T) = \alpha \cdot \lambda, V(T) = \alpha \cdot \lambda^2,$ when $\alpha = 1,$ we find $the\ exponential\ distribution$ for superior values, the probability density function being unimodal and approximated with a $normal\ law,$ for values of α exceeding the positive whole number 5,6, $d'ERLANGlaw$ is defined as exponential, the probability of occurrence of an event in the time frame $t, t + \varepsilon, \varepsilon \to 0$ depends on the time run since the occurrence of the previous event and on the value of the parameters α and $\lambda,$ the approach within the specialised software $EXCEL$ offers the possibility of quantification with the spreadsheet $d'ERLANG\ distribution$

$Introduction\_Simulation.xls.$

Another method specific to the simulation processes is $METROPLOIS\ method$, applied on a large scale of combination optimization processes, within this method being generated a row $\{X_1, X_2, \ldots\}$ of random variables that can be interpreted as a trajectory of $Markov\ chain,$ with n statuses with a stationary distribution $\pi = (\pi_1 \ldots \pi_n),$ with $\pi_i \geq 0,$ whatever i and $\sum_i \pi_i = 1.$

$METROPOLIS\ method$ (Heche et al., 2003) is specific to the applications of fundamental sciences (physics), engineering sciences, physical systems in thermodynamic balance, to the acquisition of random points in high complexity degree areas, characteristic which imposed its extrapolation to economic organizations, this last application representing the continuous case of the method, and, integrated with $MONTE\ CARLO\ method,$ (Fishman, 1996)offers significant results where other simulations fail, in statistics, the simulation method presented is known under the acronym $Markov\ Chain\ Monte\ Carlo$, having a high applicability degree.

THE ANALYSIS AND DISCUSSION OF SIMULATED PROCESS CONVERGENCE

The purpose of simulating the stochastic behaviour of economic system is to assess its real behaviour and performances, (Aarts and Korst, 1989), (Bratley et al, 1983) generated by an added value and profit margins, managerial performance outputs, a number of unknown parameters, susceptible to provide the searched information, must be estimated, eventually internally, within an optimization loop, and the time allotted to the performance of simulation is also an essential element for this type of approach.

One must determine „the number of replications" of simulation experiences, sufficient for the identification of simulated trajectory, after a possible elimination of the initial transitory phase, which could influence the results, and the precision degree must be high enough.

It is possible to estimate the precision of a simulation process starting from the results generated by it, namely sampling, planned experiments, probabilistic estimate of the confidence intervals, described in statistical surveys, without the certainty of a clear answer, only for certain punctual, particular situations.

The simulation results are formally represented by a row (vector) of values

$$X_1, X_2, \ldots, X_i, \ldots, X_{n-1}, X_n \quad (9)$$

they can be assimilated to the reproduction of a certain trend or specific trajectory of a stochastic process, from which we want the estimation of some parameters, (Heche et al., 2003).

The parameters can be of hope time $E[\psi(X_i)],$ whatever i, or $\psi(.)$ is a given function, a correct choice of this function allows the estimation of the dimensions of hope functions $X, \psi(X) = X;$ hope function of $X^2, \psi(X) = X^2,$ which in its turn allows the estimation of the variation of $X,$ as being $Var[X] = E[X^2] - E[X]^2$; we insert the distribution function $P[X \leq x]$ of $X,$ we also use the indicator function $\psi(X) = 1_{\{X \leq x\}}$.

If the differences X_i have all the same distribution, then it is obvious that $E[\psi(X_i)] = \gamma,$ for any *i,* the type of basic stochastic process is the decision-making factor for the chosen simulation method, we extremely identify two cases:

-the row (9) represents especially the achievement of independent random variables, identically distributed;

-the row (9) represents a fraction of the trajectory of non-stationary stochastic processes;

The first one represents the classical analysis of the sampling, one can identify *n* independent replications for the same experiment performed more times in identical conditions, the size of the sampling must be great enough to allow the application of *"great number law" and "central limit theorem"* allowing the estimation of a confidence interval for the parameter $\mu = E[X_i]$; the second case is identifiable as being non-stationary, the processes evolving in time, generating a behaviour trend of the average value $E[X_i] = \mu_i$.

The processes can have an evolution with a certain variability degree, variant or covariant between the variables shifted by a same time interval, and there is the possibility that the two effects overlap,when there is no information on the process non-stationary nature, the estimation of the parameters μ_i is possibly intuitive with a number *K* of independent replications of all experiments, obtaining the result rows (vectors), presented in a matrix structure, picture, as follows

$$\begin{pmatrix} X_1^{(1)}, \ldots, X_i^{(1)}, \ldots, X_n^{(1)} \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ X_1^{(k)}, \ldots, X_i^{(k)}, \ldots, X_n^{(k)} \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ X_1^{(K)}, \ldots, X_i^{(K)}, \ldots, X_n^{(K)} \end{pmatrix} \quad (10)$$

For each value of the index *i*, the "transversal" row of values within the previously defined matrix

$$X_i^{(1)}, \ldots, X_i^{(k)}, \ldots, X_i^{(K)}$$

for the estimation of the unknown parameter, $E[X_i] = \mu_i,$ the situation is limited to the previous case of independent observation sampling, this simple modelling being achieved with high costs and it does not use complementary information being available in the basic structures.

The analysis of the series occurred in chronological order and the elaboration of correct estimates within the non-stationary processes are major importance challenges for economic organizations and applications of superior added value generation (Heche et al., 2003).

One can identify, in addition to the row of independent random and identically distributed variables, the non-stationary stochastic processes, “stationary stochastic processes”, and we create the hypothesis that „the statistical balance” of these processes lead to an analysis that is similar to independent sampling; in the stationary case, the simulation results are identified within a row of random variables identically distributed but independent one from another, this dependency remaining invariant in relation to the time shifts (Bratley et al., 1983).

Considering these hypotheses, we identify an ergodic process, (a stochastic process is ergodic if the statistical properties are deducted from a single random sample, representative for enough long temporary sequences), confidence estimates are generated for $\mu = E[X_i]$, starting from an unique row.

The temporary average of the values of a particular process achievement tends to the hope X_j, therefore existing for a particular moment j, the probability 1, it results:

$$lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} X_i = E[X_j] = \mu \quad (11)$$

If the simulations are replications of that in non-stationary case, the occurrence average for any replication tends to the same limit as any average, transversally with the probability 1, it results

$$lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} X_i^{(k)} = lim_{K\to\infty} \frac{1}{K} \sum_{k=1}^{K} X_i^{(k)} = E[X_j] = \mu, \ a.s. \quad (12)$$

the phrase is practically a representation of the great number law, the annotation *a.s.* comes from *the theory of probabilities* meaning almost sure.

Following the developed reasoning, we can conclude the convergence of the simulation processes, even if the economic problems aim at disjunctive phenomena and the elaboration methods for the simulation processes are different as vision and approach.

CONCLUSIONS

The concepts induced by the simulation process within engineering sciences sees new dimensions and development trends for the economic processes, with a special focus on the processes involving managerial decision.

Within the latter, the development is structured on two hubs:

(1) The simulation obliges the elaboration of structured models, described by mathematical formalisms with a high rigour degree, the analyses of economic processes being ample and profound;

(2) The simulation is that process implicitly offering an easy and “friendly” communication relationship between humans and “machines”, allowing the use of data storage power and the computing one, offered by modern IT solutions, to elaborate the optimal answers, or a row of answers tending asymptotically towards the optimal operational strategic decision, for

complex and dynamic challenges generated from the modern business environment;

Within the scientific debate developed, the simulation of economic events, to be modelled by continuous and discrete events, is also taken into account.

Two modern modelling methods have been briefly presented and discussed, namely the *MONTE CARLO method*, and the *METROPOLIS method*, translated from the applications that are specific to technical, engineering sciences to economic phenomenology, (Heche et al., 2003).

We can notice the explosive development of operational strategic software products that amplify a lot the complex managerial process of decision elaboration, both operational and strategic, in temporary well-delimited horizons.

The vulnerability identified in simulation processes is accentuated simplification, economic problems having in general, in addition to the certain, quantifiable factors, a multitude of other uncertain and unpredictable factors, coming both from the interior of economic organization, decision-making organisational entities or task-forces, as well as from external ones, the business environment, legislative, environmental, competitive factors (Knuth, 1981).

With a greatly diminished mathematical formalism, following only the strictly necessary things in the comprehension and construction of models suitable for the representation of modern economic problems, the article offers an introduction to the principles and utilities offered by modelling, in the search for the uniquely optimal managerial decision, or represented by a sequence of intermediary decisions tending asymptotically towards it.

It is recommended that the results generated by the simulation processes be perceived and analysed with a certain suspicion dose, the decision-making manager having in them a considerable theoretical and practical support, but he/she has to manifest a critical, innovative and creative spirit, sometimes being close to the spirit of art creators, based on the data generated scientifically and rigorously spiritually.

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