The Bayesian Modelling Of Inflation Rate In Romania

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ABSTRACT

Bayesian econometrics knew a considerable increase in popularity in the last years, joining the interests of various groups of researchers in economic sciences and additional ones as specialists in econometrics, commerce, industry, marketing, finance, micro-economy, macro-economy and other domains. The purpose of this research is to achieve an introduction in Bayesian approach applied in economics, starting with Bayes theorem. For the Bayesian linear regression models the methodology of estimation was presented, realizing two empirical studies for data taken from the Romanian economy. Thus, an autoregressive model of order 2 and a multiple regression model were built for the index of consumer prices. The Gibbs sampling algorithm was used for estimation in R software, computing the posterior means and the standard deviations. The parameters' stability proved to be greater than in the case of estimations based on the methods of classical Econometrics.

Keywords: *Bayesian econometrics, Bayesian regression, Bayes' theorem, Gibbs sampling algorithm, posterior mean*
JEL Classification: *C11, C13, C51*

INTRODUCTION

Bayesian econometrics is a later developed branch of econometrics that applies Bayes' principle in economic modelling. An introduction in the Bayesian inference from econometrics is made by Zellner (1996) in 1971, but in last years a real informational explosion of researches that use Bayesian methods was observed. This phenomenon could be explained using solid arguments, the first one being the real deficiencies of the classical econometrics in estimating the regression models based in major cases on unrealistic assumptions compared to the empirical data. On the other hand, the rapid progress of the computational techniques made possible the application on a larger scale of the specific methods of the Bayesian econometrics. Moreover, the real evolution of the economic phenomena is better explained by the Bayesian approach that works out with data from samples, but also with prior information, previously determined. A presentation of the Bayesian methods for econometrical estimations, testing and prediction of the economic indicators is realised by Zellner (1985), who showed the superiority of the Bayesian methods that includes prior information compared to the classical econometrics methods. Initially, a huge part of the literature dedicated to Bayesian econometrics referred to factorial models with an unknown number of latent factors.

An introduction in modern Bayesian econometrics, highlighting the utilisation of informatics programs for estimating the Bayesian models and presenting for this context the S programming and the use of Bugs software was made by Lancaster (2004).

Most of the researchers use R and Matlab programs for Bayesian estimations; in this research I used some codes written in R for estimating the linear and Bayesian regression models using Gibbs sampling method. The Matlab program is used in Bayesian inferences and estimations by Koop, Porrier and Tobias (2009) that offered many empirical examples for initiation in Bayesian econometrics secrets.

A literature retrospection in this domain is made by Geweke, Koop andvan Dijk (2011), discussing the posterior simulations, the " Markov chain Monte Carlo" (MCMC) methods, the state space models , non-parametrical techniques and filtering in Bayesian approach. The authors highlight the applicability of Bayesian econometrics techniques in numerous domains of the economical sciences, among them being micro-economics, macro-economics, marketing, management, finance, commerce. For applications of Bayesian approach not only in economical sciences, but also in medicine, natural sciences, engineering, ecology, politics, industry and sociology you can see more at O' Hagan and West (2010).

The necessity of the application of Bayesian econometrics methods is required by the objective finality: taking decisions at different levels and various domains. Approaches in both theoretical and practical ways are described by Heij, de Boer, Franses, Kloek and van Dijk (2004): data choice econometrics (truncated data or census data, logit and probit models, multinomial and ordered choices) and the time series econometrics (univariate time series, vector-autoregressive models, simultaneous equations models, SUR models, panel data, trend, volatility). For different Monte Carlo approaches for Bayesian analysis of the simultaneous equations models you can see more at Van Dick H.K. (2011). Bayesian inferences for clusters based on Markov chains are made by Fruhwirth-Schnatter, Pamminger, Weber and Winter-Ebmer (2011). Recently, an efficient inference for ARMA models with switching regime was proposed by Kim J. and Kim C. (2013). The main critiques brought to Bayesian approach, according to Gamerman and Lopes (2006), are related to the difficulty of computing the marginal likelihood and the normalization of Bayesian factor. The advantages of Bayesian approach in economy, compared to classical modelling are numbered by Müller and Mitra (2013). Moreover, the Bayesian methods allow the study of properties of the non-optimal estimators and statistics.

The disadvantages of the ordinary least squares method of estimation in the classical linear regression were mentioned by Lindley and Smith (1972). The method could not be applied for dimensions greater than 2. Transformations of the variable are made in order to achieve this condition and the hypothesis of normality is fixed to error, these assumptions making difficult the estimation process. On the other hand, the lack of these conditions for Bayesian approach brings in many situations better results.

The regression models used in forecasting do not fulfil the necessity of development and frequent update. The classical regression model does not succeed in achieving these objectives. Therefore, the Bayesian regression is a very good solution to extend the low volume data series. It is actually an adaptation of famous Bayes theorem, taking into account two types of information:

1. A prior information;
2. Experimental data.

The purpose of this research is related to the following aspects: the explanation of the application of Bayes' theorem in economics, the presentation of the linear regression model that is estimated using the Bayesian approach and the realization of two empirical examples using Gibbs sampling algorithm, specific to Bayesian econometrics, for explaining the evolution of some macroeconomic indicators in Romania. This research is a novelty for the Romanian literature, Bayesian estimations of linear regression models never being made before. The Bayesian approach proved to be better than the classical one, the probability distribution being determined for the coefficients. Thus, the decisional process at macroeconomic level is improved by considering not a punctual value of estimators, but a probability distribution, even for variance. The knowledge of errors' variance in Bayesian approach offers the possibility of assessing and even reducing the uncertainty that affects the econometric model and the forecasts based on it.

BAYES' THEOREM APPLIED IN ECONOMICS

If A and B are two events, the first one being unknown and the second one being known, in Bayesian approach, B is associated to known data and A to model coefficients. The following notations are used:

y- set of data

y*- set of unobserved data

Mi- set of models, where i=1,2,...,m

θ^i - parameters that Mi depend on

$p\left(\frac{\theta^i}{M_i}, y\right)$ - posterior density

p(Mi/y)- posterior probability of the model (the model is based on this model)

p(y*/y)- predictive density that the forecast is based on

The conditional probability of A, when B is known, represents the probability that A takes place, when B has already taken place: pr(A/B)=pr(A,B)/pr(B).

According to Bayes' theorem: $pr(A/B) = \frac{pr\left(\frac{B}{A}\right) \cdot pr(A)}{pr(B)}$

We consider only one regression model that depends on parameters θ^i. according to Bayesian theory, $pr(B/A) = \frac{pr\left(\frac{A}{B}\right) \cdot pr(B)}{pr(A)}$, but if A is replaced by y and B with θ, then: $pr(\theta/y) = \frac{pr\left(\frac{y}{\theta}\right) \cdot pr(\theta)}{pr(y)}$ (y data being known, what can we know about θ). There is a controversy between econometricians, more of them considering that θ is not a random variable. p(y) not depending on θ, it can be ignored and the following approximation could be done for the posterior density being computed after the knowledge of the data: pr(θ/y) $\approx pr\left(\frac{y}{\theta}\right) * pr(\theta)$.

$pr\left(\frac{y}{\theta}\right)$- likelihood function

$pr(\theta)$- posterior density, that does not depend on data

Marginal density is based on integration: $\left(\frac{y^*}{y}\right) = \int [p(y^{\dagger} *, \theta^{\dagger} *]/ y) d\theta$, being equivalent with $p\left(\frac{y^*}{y}\right) = \int [p(y^*, y, \theta]) p\left(\frac{\theta}{y}\right) d\theta$ (1)

THE LINEAR REGRESSION MODEL IN THE BAYESIAN APPROACH. THE ESTIMATION ALGORITHM GIBBS SAMPLING

The following regression model is considered in matrix form:

$Y_t = AX_t + u_t$, where $u_i \to N(0, \sigma^2)$ (2)

Y matrix for the dependent variable has the dimension nx1, while X is the matrix of independent variables with the size nxk, where k is the number of independent variables and n is the number of observations of each data series

The objective is the determination of the estimators matrix, the errors variance being: σ^2.

The classic econometrics solves this problem by estimating and maximising a likelihood function, resulting in the end the estimator for matrix A and the estimated variance of the errors. So, the classic econometrics is based on the utilisation of all data.

The Bayesian econometrics offers the following solution that supposes the successive demarche of the next steps:

1. The researcher intuits the values of parameters' estimators, using the information regarding the A matrix and the errors' variance, but the information is not related to the values of the data series for X and Y. The intuitions are called prior belief, being related to the researcher's experience and to previous studies for similar models but for other data sets. It is important to notice that these beliefs are expressed as probability distributions. For example, the prior on the matrix A coefficients follows a normal distribution with average and a variance-covariance matrix denoted by $\sum_0$. More sure the researcher is about the appreciations regarding the coefficients, lower the variance is.

2. The second phase is also met in the classic econometrics and it supposes the collecting of the data for X and Y and the estimation of the likelihood function:

$$F\left(\frac{Y_t}{A}, \sigma^2\right) = (2\pi\sigma^2)^{-\frac{T}{2}} \cdot \exp\left(-\frac{(Y_t - AX_t)^T (Y_t - AX_t)}{2\sigma^2}\right) \quad (3)$$

The researcher updates the expectations regarding the model parameters using the data for X and Y and the estimated likelihood function. Practically, the prior probability distribution is combined with the likelihood function in order to get the posterior repartition. This distribution is defined in the terms of Bayesian theorem: $H\left(A,\frac{\sigma^2}{Y_t}\right)=\frac{F\left(\frac{Y_t}{A},\sigma^2\right)\times P\left(A,\sigma^2\right)}{F(Y)}$. (4)

In other words, the prior distribution is gotten by dividing the product between the likelihood function and the prior probability by the marginal likelihood (the marginal density of data, which is a scalar). So, the prior distribution is proportional with the likelihood function by a prior number of times. Therefore, in estimating the coefficients of the simple regression model the following relationship is used: $H\left(A,\frac{\sigma^2}{Y_t}\right)\alpha F\left(\frac{Y_t}{A},\sigma^2\right)\times P(A,\sigma^2)$.(5)

The joint density (product between the marginal density of Y and the conditional density of the parameters or the product between the marginal density of the parameters and the conditional density of data) can be computed in two ways, being denoted with:

$$G(Y_t,A,\sigma^2)=F(Y_t)\times H\left(A,\frac{\sigma^2}{Y_t}\right)=F\left(\frac{Y_t}{A},\sigma^2\right)\times P(A,\sigma^2) \quad (6)$$

Gibbs sampling is a numerical method used for estimation and it is applied in 3 possible cases:

- Estimation of prior distribution of A under the hypothesis of known variance of errors;
- Estimation of prior distribution of variance under the hypothesis of known A matrix;
- Estimation when both parameters are unknown.

a. Estimation of prior distribution of A under the hypothesis of known variance of errors;

The next steps are followed in this case:

a1. In practice a prior normal distribution is chosen for A, which is a conjugate distribution. This is combined with a likelihood function and a posterior distribution results, the repartition being the same (the normal one). The form of prior distribution of average A_0 and a variance-covariance matrix denoted by $\sum_0$ is:

$$(2\pi)^{-\frac{K}{2}}\left|\sum_{0}\right|^{-\frac{1}{2}} \exp\left[-0.5(A-A_0)^T \sum_{0}^{-1}(A-A_0)\right] \alpha \exp\left[-0.5(A-A_0)^T \sum_{0}^{-1}(A-A_0)\right] \quad (7)$$

a2. The likelihood function is defined as :

$$F\left(\frac{Y_t}{A},\sigma^2\right) = (2\pi\sigma^2)^{-\frac{T}{2}} \cdot \exp\left(-\frac{(Y_t-AX_t)^T(Y_t-AX_t)}{2\sigma^2}\right) \alpha \exp\left(-\frac{(Y_t-AX_t)^T(Y_t-AX_t)}{2\sigma^2}\right) \quad (8)$$

a3. The computation of posterior distribution as:

$$H\left(\frac{Y_t}{A},\sigma^2\right) \alpha \exp\left[-0.5(A-A_0)^T \sum_{0}^{-1}(A-A_0)\right] \text{ x } \exp\left(-\frac{(Y_t-AX_t)^T(Y_t-AX_t)}{2\sigma^2}\right)$$

For the normal distribution, Hamilton (1994) and Koop (2003) used the following formulae for normal distribution average and variance:

$$M* = \left(\sum_{0}^{-1}+\frac{1}{\sigma^2}X_t^T X_t\right)^{-1}\left(\sum_{0}^{-1}A_0+\frac{1}{\sigma^2}X_t^T Y_t\right) = \left(\sum_{0}^{-1}+\frac{1}{\sigma^2}X_t^T X_t\right)^{-1}\left(\sum_{0}^{-1}A_0+\frac{1}{\sigma^2}X_t^T A_{ols}\right) \quad (9)$$

$$V* = \left(\sum_{0}^{-1}+\frac{1}{\sigma^2}X_t^T X_t\right)^{-1}$$

b. The estimation of posterior variance under the hypothesis of a known matrix A

b1. The normal distribution admits negative values, fact that justifies the choice of an inverse Gamma distribution or a Gamma distribution with the parameter $\frac{1}{\sigma^2}$.

Let's consider a variable denoted by "v" with normal distribution and T numbers that are identic and independently distributed : $v_t \sim N\left(\frac{0,1}{\theta}\right)$

The sum of squares for this variable follows a Gamma distribution with the parameters T (number of degrees of freedom) and the scale parameter θ.

The probability density function corresponding to Gamma distribution is: $g(W)\ \alpha W^{\frac{T}{2}-1}\exp\left(\frac{-W\theta}{2}\right)$. (10)

The mean of Gamma distribution is given by: $g(W)=\frac{T}{\theta}$ (11)

The form of prior density is: $\frac{1}{\sigma^2}^{\frac{T}{2}-1} \exp\left(\frac{-W\,\theta}{2}\right)$ (12)

b2. The likelihood function is defined as:

$$F\left(\frac{Y_t}{A},\sigma^2\right) = (2\pi\sigma^2)^{-\frac{T}{2}} \cdot \exp\left(-\frac{(Y_t - AX_t)^T(Y_t - AX_t)}{2\sigma^2}\right) \alpha (\sigma^2)^{-\frac{T}{2}} \exp\left(-\frac{(Y_t - AX_t)^T(Y_t - AX_t)}{2\sigma^2}\right) \quad (13)$$

b3. The computation of posterior distribution (a Gamma distribution with degrees of freedom

$T_1 1 = (T_1 0 + T)/2$ and $\theta_1 1 = ([\theta_1 0 + (Y_1 t - AX_1 t)] ^T (Y_1 t - AX_1 t))/2)$ as:

$$-\frac{1}{2\sigma^2}\left((Y_t - AX_t)^T(Y_t - AX_t)\right) \cdot \frac{1}{\sigma^2}^{\frac{T_0}{2}-1-\frac{T}{2}} \exp\left(-\frac{1}{2\sigma^2}\left([\theta_0 + (Y_t - AX_t)^T(Y_t - AX_t))]\right] \cdot \frac{1}{\sigma^2}\right)$$

The mean of conditional posterior distribution is: $\frac{T_0 + T}{\theta_0 + (Y_t - AX_t)^T(Y_t - AX_t)}$. (14)

c. We will consider the case when both parameters are unkonwn. The three steps are the following, according to Blake and Mumtaz (2012):

c1. The computation of the joint prior distribution

c2. The likelihood function

c3. The computation of posterior distribution

The joint posterior distribution for A and variance is: $\frac{1}{\sigma^2}$.

$$H\left(\frac{1}{\sigma^2},\frac{A}{Y_t}\right) \alpha\, p\left(A,\frac{1}{\sigma^2}\right) x\, F\left(\frac{Y_t}{A},\sigma^2\right) \quad (15)$$

The inference about parameters is based on the computation of conditional prior distributions, defined by Koop (2003) as: $H\left(\frac{\frac{1}{\sigma^2}}{Y_t}\right) = \int_0^\infty H\left(\frac{1}{\sigma^2},\frac{B}{Y_t}\right) dB$. (16)

Gibbs sampling is a numeric method for estimating the coefficients of the linear regression model that uses the conditional distributions to approximate the joint and the marginal repartitions. A general presentation of the method is followed by a description in the context of the linear regression model.

A joint distribution of k variables is considered: $f(x_1, x_2, \ldots, x_k)$.

Our objective is the determination of the marginal distributions: $f(x_i), i = \overline{1,k}$

Starting from the conditional distributions $f\left(\frac{x_i}{x_j}\right), i \neq j$, the Gibbs sampling algorithm approximates the marginal repartition by following the next steps, an easier demarche than the integration of the joint distribution:

Step 1: The starting values are considered: $x_1^0, x_2^0, \dots, x_k^0$

Step 2: Selection of sample x_1^1 from the distribution of x_1 conditional on current values of $x_2, \dots, x_k$

$$f\left(\frac{x_1^1}{x_2^0}, \dots, x_k^0\right)$$

Step 3: Selection of sample x_2^1 from the distribution of x_2 conditional on current values of $x_1, x_3, \dots, x_k$

$$f\left(\frac{x_2^1}{x_1^1}, x_3^0, \dots, x_k^0\right)$$

Step k: Selection of sample x_k^1 from the distribution of x_k conditional on current values of $x_1, x_3, \dots, x_{k-1}$

$$f\left(\frac{x_k^1}{x_1^1}, x_2^1, \dots, x_{k-1}^1\right)$$

According to Casella and Edward (1992), if the number of iterations converges to infinite, the draws from the conditional distributions (the samples) converges to the marginal or joint repartitions of x_i at an exponential rate. So, if we arrive to a large number of steps, we can easily approximate the marginal distribution to the empirical repartition of x_i. If the Gibbs algorithm is applied P times and only the last M draws of x_i are retained (M values for $x_1, x_2, \dots, x_k$), the histogram for $x_1, x_2, \dots, x_k$ is an approximation for the marginal density of $x_1, x_2, \dots, x_k$. Therefore, the estimator for the average of the marginal posterior repartition for x_i is: $\frac{\sum_{b=1}^{M} x_i^b}{M}$, where b- number of Gibbs iterations. The variance of the marginal distribution is the number of Gibbs iterations that are necessary for convergence.

The form of the conditional distributions should be a prior known by the researcher $f\left(\frac{x_i}{x_j}\right)$. Moreover, random draws could be taken from these distributions.

Let us to consider the following regression model (AR(2) model) for (monthly index of consumer prices for Romania), used in computing the inflation rate, in the period 1991: January – 2013: April:

$$Y_t = \alpha + A_1 Y_{t-1} + A_2 Y_{t-2} + u_t, u_t \sim N\left(0, \sigma^2\right) \tag{17}$$

The RHS variables are: 1, Y_{t-1}, Y_{t-2}

A= $\{\alpha, A_1, A_2\}$- vector of coefficients

Objective: the approximation of the marginal distribution for coefficients (α, A_1, A_2) and variance (σ^2)

The priors and initial values are set. A normal distribution is set for the coefficients. This implies that the prior averages are specified for each coefficient (A0)- a 3x1 vector and the prior variance ($\sum_0 - a\ 3x3\ matrix$).

$$p(A) \to N\left(\begin{pmatrix}\alpha^0 \\ A_1^0 \\ A_2^0\end{pmatrix} \cdot \begin{pmatrix}\sum_\alpha & 0 & 0 \\ 0 & \sum_{A_1} & 0 \\ 0 & 0 & \sum_{A_2}\end{pmatrix}\right) \tag{18}$$

An inverse Gamma distribution with prior for σ^2 and the prior degrees of freedom T_0 and the prior scale matrix as θ_0. We will work with inverse Gamma distribution.

$$p(\sigma^2) \to \Gamma^{-1}\left(\frac{T_0}{2}, \frac{\theta_0}{2}\right) \tag{19}$$

The OLS estimator for σ^2 is set as starting value. The large number of Gibbs iterations will determine an insignificant influence of the starting value on the results for linear regressions.

Then, we sample from the conditional posterior repartition of A, having a starting value for σ^2 $(normal\ distribution)$.

$\frac{A}{\sigma^2}, Y_{\downarrow}t)$ -> N(M*,V*)

$$M* = \left(\sum_{0}^{-1} [+ \frac{1}{\sigma^2} X_t^T X_t\right)^{-1} \left(\sum_{0}^{-1} A_0 + \frac{1}{\sigma^2} X_t^T Y_t\right)] \tag{20}$$

$$V* = \left(\sum_{0}^{-1} [+ \frac{1}{\sigma^2} X_t^T X_t\right)^{-1}]$$

The following algorithm is used to compute the draw for A.

Algorithm a: Let z be a k*1 vector that is sampled from a normal distribution of average m and variance v. Let z_0 be the first k*1 numbers from the standard normal distribution, numbers that could be transformed in order to have the mean m and the variance v: $z = m + z_0 * v^{0.5}$. In our case, we have the following relationship:

$$A^1 = M^* + [\bar{A} + (V^*)^{0.5}]$$

The draw for is computed using the previous formula, where A^1 *is the first Gibbs iteration*.

$\bar{A}$ - a vector from the standard normal distribution

The variance σ^2 is drawn from the conditional posterior distribution, being given, the repartition being an inverse Gamma one:

$$H(\sigma^2 \backslash A, Y_t) \rightarrow \Gamma^{-1}\left(\frac{T_0}{2}, \frac{\theta_0}{2}\right) \tag{21}$$

$$T = T_0 + T \tag{22}$$

$$\theta_1 = \theta_0 + (Y_t - A^1 X_t)^T (Y_t - A^1 X_t)$$

The following algorithm is used in order to sample a scalar denoted by z from the Inverse Gamma distribution (T/2 degrees of freedom and scale parameter D/2).

Algorithm b. We generate T numbers from a standard normal distribution (z^0). Then, z is computed as: $z = \frac{D}{(z^0)^T z^0}$, being drawn from an Inverse Gamma repartition.

After getting A^1, …, A^p, the last M values of A are used to form the empirical repartition of the parameters (an approximate of the marginal posterior distribution). The first iterations (P-M iteration) that are not taken into consideration are considered burn-in iteration.

For estimate the linear regression in R we could use the function proposed by Professor Doug Schroeder which is available on http://fisher.osu.edu/~schroeder.9/AMIS900/GibbsLinRegr.R . The prior average for beta is represented by ”prior”, prior variance for beta, nu0=df for sigmasq. For sigmasq we have prior scale.

The output from Gibbs sampler is used to make inference. A sequence of draws from the approximate marginal distribution of the parameters is obtained. The draws average is an approximation of the posterior mean, providing a point estimate for parameters. The computed percentiles using these draws are used to get the posterior density intervals. The 5th and the 95th percentiles approximate the 10% highest posterior density intervals.

The marginal likelihood (defined like $F(Y) = \int [F(Y/A, \sigma^{\uparrow}2]$) p(A $\sigma^{\uparrow}2)d\beth, where\ \beth = A, \sigma^{\uparrow}(2\ ))$, is the posterior distribution with the parameters integrated out.

A model M1 is preferred to M2 if $F_{M1}(Y) > F_{M2}(Y)$ or the Bayesian factor $\frac{F_{M1}(Y)}{F_{M2}(Y)} > 1.$

The prior mean for the coefficients is: $\begin{pmatrix} \alpha^0 \\ A_1^0 \\ A_2^0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

while the prior variance is represented by the identity matrix: $\begin{pmatrix} \sum_{\alpha} & 0 & 0 \\ 0 & \sum_{A_1} & 0 \\ 0 & 0 & \sum_{A_2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. 10 000 replications were saved for this application, the total number of iterations being 50 000. Each coefficient has a posterior mean and the standard deviation (table no. 1).

The coefficients, the posterior means and the standard deviations

Table no. 1

Coefficient	Posterior mean	Standard deviation
	-0.0391	0.099883
	0.0063	0.010486
	0.0611	0.09542

Source: own computations

The posterior means are rather low, while the standard deviations are less than 0.1, fact that suggests the parameters' stability, the model being better than the classical autoregressive one.

CONCLUSIONS

The Bayesian approach, which knew a major development in the last 20 years, finds its applicability in many domains of economics, being the support for taking decisions in various conditions. The Bayesian Econometrics has many different applications, the Bayesian linear regression model being another perspective of modelling the variables' dependences. The inclusion of prior information determines better estimations for the parameters, the situation being also reflected by the low values of the coefficients compared to the models from classical econometrics. The future research directions should take into account the selection of the best prior values of the parameters based on the results provided by classical econometrics.

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