
CALCULATION METHODS AND TECHNIQUES USED FOR STATISTICAL ANALYSIS OF RESULTS INDICATORS

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Abstract

The objective of this work is to present the role of statistical methods in facilitating managerial decisions on the combination of production factors, taking account of the limited character of them, but the two aspects of economic activity: one technical and one economic. The analysis was performed on the example of a company to which the data are processed for a period of four years.

Keywords: complementarity, interchangeability, substitution elasticity, production factors, function of the Cobb-Douglas production.

Production activity is a set of input processing operations so as to achieve outputs required by the market, respecting the objective function and tapping of the manufacturer: maximizing profit in conditions to minimize efforts. Thus, appears imperious need highlighting the methodological principles of combination and substitution of production factors, which constitute a guide for managers of firms. Economic decisions taken will be the variance quantity inputs use, increased use of either one or another of the factors or combination of factors of production change.

Depending on the nature of economic activity, managers will opt for a combination of production factors taking account of the limited character of them, but the two aspects of economic activity: one technical and one economic. From the technical point of view, specific to each production process is getting any good by uniting resources work with elements of technical capital characteristic of that field, and from the economic point of view, the combination of production factors means minimizing production costs in maximising profits. The combination of production factors implies the existence of complementarity and their interchangeability. Combine alternative from which they expect the greatest efficiency, choose following some economic calculations. For example, if a trader seeks technical and economic efficiency of production through labour substitution with capital, then he will opt for more machinery and fewer workers. Appreciation of the choice made can be realized only through the calculation of some indicators

such as: the marginal productivity of labour and capital (comparing them), the elasticity of substitution and the marginal rate of substitution (labour or capital with work). Substitution elasticities expresses the extent to which it can be maintained when a production factor is replaced by another. Normal combinations of the factors of production causes a positive elasticity of substitution, which ranges from zero to infinite depending on the ease with which one of the factors may be replaced by another, the production remains constant.

Elasticity of substitution shall be measured using substitution elasticity coefficient (e_s) a factor with a factor B. This coefficient shows how many percentage must increase the value of the ratio between the level of the factor A and the factor B (X_A/X_B), when the ratio of the marginal productivity of factor B (W_{mB}) and the factor A of (W_{mA}) increases by a percentage, so that production to stay constant.

The relationship is:

$$e_s = \frac{\frac{\Delta R}{R}}{\frac{\Delta r}{r}}$$

$$R = \frac{X_A}{X_B}, \quad r = \frac{W_{mB}}{W_{mA}}, \quad \Delta R = R_1 - R_0, \quad \Delta r = r_1 - r_0$$

If $e_s = 0$ the factors are perfectly complementary and their substitution is not possible, the factors which are used in equal proportions.

If $e_s \rightarrow \infty$ fluid replacement is perfect.

If $e_s = 1$ then the relative variation of the factors is proportional to the relative variation of the marginal productions.

As such, the elasticity is greater, so the degree of interchangeability of the factors is higher. For example, we considered the case of trading company "CONSTRUCT" S.A., which presents the situation manufactured production, tangible fixed assets and staff employed on the last four years divided in quarters.

Tangible situation, the production of goods and the number of employees in S.C. CONSTRUCT S.A.

- Quarterly data

ANII	IMOB. CORPORALE (K)	PROD. MARFĂ FABRICATĂ (Q)	Nr mediu de salariați (L)
2008 trim I	4927725	63035836	726
trim II	5055231	70352750	748
trim III	4984980	68002107	752
trim IV	3468724	48921401	742
2009 trim I	11515037	109930479	742
trim II	11593557	115427309	778
trim III	11589300	112402736	763
trim IV	11219526	100969666	733
2010 trim I	19762541	110305514	735
trim II	20221273	122435970	755
trim III	19963133	110131620	760
trim IV	17961362	74764548	738
2011 trim I	19689030	123315043	694
trim II	19910031	126156348	708
trim III	19655749	125983638	712
trim IV	19270745	117263740	690

We have calculated the data used for the following indicators: changing the absolute mobile base, tangible fixed assets and production staff, the marginal productivity of the production factor capital (W_{mK}), i.e. production factor (W_{mL}), the ratio of the production factor capital and labor production factor (R), the ratio of the two marginal productivity (r), and finally the value substitution elasticity coefficient (e_s).

Quarterly data on processing of production factors indicators

Anii	$\Delta K = k_i - k_{i-1}$	$\Delta Q = Q_i - Q_{i-1}$	$\Delta L = L_i - L_{i-1}$	$WmK = \Delta Q / \Delta K$	$WmL = \Delta Q / \Delta L$	$R = K/L$	$r = WmL / WmK$	$\Delta R = R_i - R$	$\Delta r = r_i - K_{i-1}$	$\Delta R/R$	$\Delta r/r$	e_s
2008 trim.I	-	-	-	-	-	6787.5	-	-	-	-	-	-
trim.II	127506	7316914	22	57.385	332587	6758.3	5795.72	-29.17	5795,727	-0.004	1	-0.004
trim.III	-70251	-2350643	4	33.461	-587661	6629	-17562.8	-129.37	-23358.5	-0.02	1.33	-0.015
trim.IV	-1516256	-19080706	-10	12.584	1908071	4674.8	151625	-1954.1	169188.4	-0.418	1.116	-0.375
2009 trim.I	8046313	61009078	0	7.5822	-	15519	0	10844.1	-151626	0.699	-	-
trim.II	78520	5496830	36	70.005	152690	14902	2181.11	-617.17	2181.111	-0.041	1	-0.041
trim.III	-4257	-3024573	-15	710.49	201638	15189	283.8	287.378	-1897.31	0.019	-6.69	-0.003
trim.IV	-369774	-11433070	-30	30.919	381102	15306	12325.8	117.189	12042	0.008	0.977	0.008
2010 trim. I	8543015	9335848	2	1.0928	4667924	26888	4271508	11581.5	4259182	0.431	0.997	0.432
trim. II	458732	12130456	20	26.443	606523	26783	22936.6	-104.67	-4248571	-0.004	-185	2E-05
trim. III	-258140	-12304350	5	47.665	-2460870	26267	-51628	-515.86	-74564.6	-0.02	1.444	0.014
trim. IV	-2001771	-35367072	-22	17.668	1607594	24338	90989.5	-1929.4	142617.6	-0.079	1.567	-0.051
2011 trim.I	1727668	48550495	-44	28.102	-1103420	28370	-39265.2	4032.47	-130255	0.142	3.317	0.043
trim.II	221001	2841305	14	12.857	202950	28122	15785.7	-248.85	55050.97	-0.009	3.487	-0.003
trim.III	-254282	-172710	4	0.6792	-43177.5	27606	-63570.5	-515.12	-79356.3	-0.019	1.248	-0.015
trim.IV	-385004	-8719898	-22	22.649	396359	27929	17500.2	322.227	81070.68	0.012	4.633	0.002

$$Wm_K = \frac{Qf_i - Qf_{i-1}}{k_i - k_{i-1}}, \quad Wm_L = \frac{Qf_i - Qf_{i-1}}{L_i - L_{i-1}}, \quad i = \overline{1; 16}$$

From the observation data shows that substitution elasticity coefficient value of capital (tangible) with work factor (e_s) is greater than zero, but very close to this value, which means that factors tend to be complementary. Not being possible to their substitution, factors to be used in roughly equal proportions.

Company managers are constantly faced with the problem of choosing an optimal hash variants of production factors, to ensure a certain level of production and to enable them to maximize profits. For manufacturer's behavior analysis, in terms of compatibility between this behavior and the consumer, using production functions. They describe the relationship between

entries (inputs) and outputs, i.e. the relationship between production expected to get from a good, satisfying market requirements and the quantities of the various inputs required for obtaining it.

The relationship of the production function is:

$$Q = f(a, b, c, \dots),$$

where a, b, c represent inputs.

Production function expresses dependence between the value of Q, the amount of work and capital K used is the function of Cobb-Douglas type ($Q = f(K, L)$), with the parameters, α and β , which is a constant specific to each national savings, and α and β coefficients of elasticity of production in relation to fixed assets (e_K), i.e. in relation to human capital (e_L).

$$e_K = \alpha, e_L = \beta$$

Cobb-Douglas function has the following expression:

$$Q = A K^\alpha L^\beta$$

and the coefficients α and β , is determined by its logarithm area.

Thus, the relationship becomes:

$$\ln Q = \ln A + \alpha \ln K + \beta \ln L$$

Noting $\ln Q = y$; $\ln A = a$; $\ln K = x_1$; $\ln L = x_2$, the relationship you can write:

$$Y = a + \alpha x_1 + \beta x_2$$

Of statistically, an econometric, production function Cobb-Douglas is the best known non-linear multiple regression model, which is logarithm area linearizează, becoming a model of linear regression ($Y = a + \alpha x_1 + \beta x_2$). By applying the method of least squares to estimate the three regression model parameters, a, α and β .

In the analysis of a production process, an element that shows the importance is the scale elasticity, which is determined by the relationship:

$$e = e_L + e_K = \alpha + \beta.$$

According to this relationship, identify three situations that characterize the production process:

- production process of descending scale efficiency, $\alpha + \beta < 1$. In this case, a spore of the factors generating an increase in output, but in a lesser proportion.

- production process scale yield constant, $\alpha + \beta = 1$. According to this situation, if the two entries and exits, then grow in the same proportion.

- production process of ascending scale efficiency, $\alpha + \beta > 1$. This case expresses the situation in which the two growth factors in a specified ratio generates an increase in a higher proportion of exits.

For the situation presented at S.C. CONSTRUCT S.A., analysis of production, capital and the number of employees by using Cobb-Douglas is performed by performing the following steps:

1) Logarithmic surgery is made of production, capital and number of employees for the 16 quarters.

2) By applying the method of least squares and cancelling the partial derivatives of the function f with respect to a , α and β is obtained on the system:

$$\begin{aligned} \sum a + \alpha \sum x_1 + \beta \sum x_2 &= \sum y \\ a \sum x_1 + \alpha \sum x_1^2 + \beta \sum x_1 x_2 &= \sum x_1 y \\ a \sum x_2 + \alpha \sum x_1 x_2 + \beta \sum x_2^2 &= \sum x_2 y \end{aligned}$$

In solving the system of equations, I get the following regression equation:

$$\begin{aligned} \hat{y} &= \hat{a} + \hat{\alpha} x_1 + \hat{\beta} x_2 \\ \hat{a} &= 6,6139 \\ \hat{\alpha} &= 0,4418 \\ \hat{\beta} &= 0,6925 \end{aligned}$$

The coefficient of in the expression of the Cobb-Douglas is calculated by the relationship: $A = e^a$

$$A = 745.38436$$

In the last quarter, the substitution of one unit of labor can be achieved by using additional equipment in the amount of:

$$S_{L/K} = \frac{\hat{\beta}}{\hat{\alpha}} * \frac{K_{2011 \text{ TRIM } V}}{L_{2011 \text{ TRIM } V}}$$

$$S_{L/K} = 0,692/0,441 * 19270745/690 = 43.824.494 \text{ LEI}$$

Where the L/K is the rate of substitution of one unit of labor with fixed assets having a value, in this case 43824,492 THOUSAND LEI.

As $\alpha + \beta > 1$, it can be concluded that the production process presents an efficiency of ascending scale, this means that the growth of entries in a given ratio generates an increase in output in a higher proportion.

To test the significance, the quality of the linear regression model obtained by logarithmic Cobb-Douglas function applies the procedure ANOVA.

Data necessary for the analysis – by quarters

ANII	\hat{Y}_i	$\hat{Y}_i - \bar{Y}$	$\left(\hat{Y}_i - \bar{Y}\right)^2$	$Y_i - \hat{Y}_i$	$\left(Y_i - \hat{Y}_i\right)^2$	Y_i^2	$Y_i - \bar{Y}$	$\left(Y_i - \bar{Y}\right)^2$
2008 trim I	17.984	-0.398	0.158745	-0.025	0.0006	322.5	-0.423	0.17918
trim II	18.016	-0.366	0.134299	0.053	0.0028	326.5	-0.313	0.09827
trim III	18.014	-0.369	0.13613	0.021	0.0005	325.3	-0.347	0.12073
trim IV	17.844	-0.538	0.289922	-0.138	0.0191	313.5	-0.677	0.45804
2009 trim I	18.374	-0.008	6.96E-05	0.141	0.0199	342.8	0.133	0.01764
trim II	18.41	0.027	0.000755	0.154	0.0238	344.6	0.182	0.03299
trim III	18.396	0.014	0.000191	0.141	0.02	343.6	0.155	0.02405
trim IV	18.354	-0.028	0.0008	0.076	0.0058	339.7	0.048	0.00228
2010 trim I	18.625	0.242	0.058717	-0.106	0.0113	342.9	0.136	0.01856
trim II	18.64	0.257	0.066062	-0.016	0.0003	346.8	0.241	0.0578
trim III	18.614	0.231	0.053364	-0.096	0.0093	342.9	0.135	0.01813
trim IV	18.567	0.184	0.033976	-0.437	0.191	328.7	-0.253	0.06383
2011 trim I	18.565	0.182	0.033244	0.065	0.0043	347.1	0.248	0.06137
trim II	18.584	0.201	0.040438	0.069	0.0048	347.9	0.271	0.07317
trim III	18.582	0.199	0.039726	0.07	0.0049	347.9	0.269	0.0724
trim IV	18.551	0.169	0.028507	0.029	0.0008	345.2	0.197	0.03897
TOTAL	194.91	176.5	1.074945	-	0.319	5408	-	1.33759

Data regarding the analysis carrying out

Variations source	in	The sum of square ranges deviations	No degrees of freedom	Dispersion	The calculated value F
Explained by the model SPE		$\sum \left(\hat{y}_i - \bar{y}\right)^2 = 1.074945$	k-1=1	$\sigma^2_{expl} = 1.074945$	$F_c = \frac{\sigma^2_{expl}}{\sigma^2_{rez}} = 47.17293$
Residual SPR	$= \sum_{i=1}^n (e_i)^2$	$\sum \left(y_i - \hat{y}_i\right)^2 = 0.319$	n-2=14	$\sigma^2_{rez} = 0.0228$	-
Total SPT		$\sum \left(y_i - \bar{y}\right)^2 = 1.393945$	n-1=15	-	-

Testing the significance of the regression model is done by checking the test statistic F.

If $F_{\text{calculated}} > F_{\text{tabelat}}$ confirms the validity of the regression model.
 $F_{\text{calc}}=47,17$; $F_{0,05;1;14}=4,6$ the model is valid.

The validity of the regression model can be established and by calculating the coefficient of determination, which indicates that part of the variance of the variable Y that is explained by the model adopted.

Determination coefficient is

$$R^2 = 1 - \frac{0.319}{1.393945}$$

$$R^2 = 0,7614$$

This means that the regression model adopted explains 76,14% of the variance of the variable Y. Testing the significance of the result obtained shall be made to the size F calculated by the relationship:

$$F_{R^2} = \frac{(n-2)R^2}{1-R^2} = \frac{14 * R^2}{1-R^2} = 44,699$$

In terms of a threshold of significance of 5% from the F distribution determines, for a probability of 95% and $n_1 = 1$, $n_2 = 14$ degrees of freedom $F_{\text{tabelat}} = F_{0,05; 1; 14} = 4.6$.

$F_{R^2} > F_{\text{tab}} \Rightarrow$ determination coefficient expresses a real dependency.

It has thus been checked through this test that linear regression model is correctly estimated. In terms of a threshold of significance of 1% and a probability of 99%, $F_{\text{tab}_{\alpha;k-1;n-k}} = F_{0,01;1;14} = 8,86$

$F_{\text{calc}} > F_{\text{tab}} \Rightarrow$ confirmed the validity of the regression model adopted.

Conclusions

The main ideas stemming from the treatment of this subject are:

-manufacturer's behavior relies on the use of several methods, comparing the results obtained by their use and choosing optimal variant taking into account consumer behaviour;

-optimal combination of production factors and the price depends on the factors;

-economic analyses involve the use of a series of chronological data as long periods so that, through their processing, allowing getting information, relevant conclusions.

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