ECONOMETRIC MODELS FOR DETERMING THE EXCHANGE RATE

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Abstract

The simple econometric models for the exchange rate, according to recent researches, generates the forecasts with the highest degree of accuracy. This type of models (Simultaneous Equations Model, MA(1) Procedure, Model with lagged variables) is used to describe the evolution of the average exchange rate in Romanian in January 1991-March 2012 and to predict it on short run. The best forecasts, in terms of accuracy, on the forecasting horizon April-May 2012 were those based on a Simultaneous Equations Model that takes into account the Granger causality. An almost high degree of accuracy was gotten by combining the predictions based on MA(1) model with those based on the simultaneous equations model, when INV weighting scheme was applied (the forecasts are inversely weighted to their relative mean squared forecast error). The lagged variables Model provided the highest prediction errors. The importance of knowing the best exchange rate forecasts is related to the improvement of decision-making and the building of the monetary policy.

***Key words*: exchange rate, forecasts, forecasts accuracy, Granger causality**

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The determination and prediction of the exchange rate are key issues at the macroeconomic level, especially for central banks interested in the montary policy establisment. Although several methodologies have been developed in order to determine the exchange rate, the recent researches have invalidated the ideas set out in literature. Engle [1] showed that simple econometric models generate predictions with high accuracy. The same author points out that the exchange rate had the role of assets price.

Other authors showed that the ‘70s of last century models generated forecasts with a higher degree of accuracy than those based on random walk. According to Popescu [2], many authors have used panel data set or high volume data series and they concluded that the reduced econometric models provide a good estimate of the exchange rate.

Making a retrospective in literature a multitude of proposed models stands. However, some authors, like Rogoff [3], consider the determination of the exchange rate a difficult demarche. The same observation was formulated by Williamson [4], who identified the limits of standard model of Rogoff and proposed a behavioral model as alternative.

Currently the emphasis is on the assumption of expectations that is taken into account in building the model for the exchange rate. Analyzing the models recently proposed in literature, many of them explain the dependent variable (the exchange rate) as a weighted sum of variables that make up this variable, variables called "fundamentals". Predictions of fundamentals are made starting from the exchange rate, under their variations to the exchange rate values. If the fundamentals are integrated of first-order and the discount factor is close to 1, Engel and West [5] showed that the exchange rate followed a random walk process.

In literature, there are several traditional ways of accuracy measurement, which can be ranked according to the dependence or independence of measurement scale. A complete classification is made by Hyndman and Koehler [6] in a study in the field:

- Scale-dependent measures;
- Measures based on percentage errors;
- Measures based on relative errors;
- Relative measures;
- Scaled errors.

If we consider, $\hat{X}_t(k)$ the predicted value after k periods from the origin time t, then the error at future time (t+k) is: $e_t(t+k)$. Hyndman and Koehler introduce in this class of errors "Mean Absolute Scaled Error " (MASE) in order to compare the accuracy of forecasts of more time series. Scale error is defined as: $es_t = \dfrac{e_t}{\dfrac{1}{n-1}\sum_{i=2}^{n}|X_i - X_{i-1}|}$ and mean absolute scale error as: MASE= mean | es_t |.

In practice, the most used measures of forecast error:

- Root Mean Squared Error ($RMSE = \sqrt{\frac{1}{n}\sum_{j=1}^{n} e_X^2(T_0 + j,k)}$);
- Mean error $(ME = \frac{1}{n}\sum_{j=1}^{n} e_X(T_0 + j,k))$;
- Mean absolute error ($MAE = \frac{1}{n}\sum_{j=1}^{n}| \ e_X(T_0 + j,k) \ |$).

The sign of indicator value provides important information: if it has a positive value, then the current value of the variable was underestimated, which means expected average values too small. A negative value of the indicator shows expected values too high on average.

A common practice is to compare the forecast errors with those based on a random-walk. "Naïve model" method assumes that the variable value in the next period is equal to the one recorded at actual moment. Henri Theil, a famous expert in economic forecasting and econometric theory, proposed the calculation of U Statistics, that takes into account both changes in the negative and the positive sense of an indicator: $U = \sqrt{\frac{\sum (X_{t+k} - \hat{X}_t(k))^2}{\sum X_{t+k}^2}}$.

U Theil's statistics is calculated in two variants by the Australia Treasury:

$$U_1 = \frac{\sqrt{\sum_{t=1}^{n}(a_t - p_t)^2}}{\sqrt{\sum_{t=1}^{n} a_t^2} + \sqrt{\sum_{t=1}^{n} p_t^2}} \text{ and } U_2 = \sqrt{\frac{\sum_{t=1}^{n-1}\left(\frac{p_{t+1} - a_{t+1}}{a_t}\right)^2}{\sum_{t=1}^{n-1}\left(\frac{a_{t+1} - a_t}{a_t}\right)^2}}.$$

The used notations:
a- effective results;
p- predicted values;
t- time;
e- error (e=a-p);
n- number of time periods.

A value close to zero for U_1 implies a high accuracy. If $U_2 = 1$=> there are not differencies in terms of accuracy between the two predictions. If $U_2 < 1$=> the prediction is more accurate than the naive one. If $U_2 > 1$=> the prediction is less accurate than the naive one.

Purchasing power parity theory in its relative form, after Pecican [7], it is established that in case of two coins initially in equilibrium the exchange rate evolves to those values that are obtained by the variations of the relative prices of the two selected states. In Romania a frequent cause of prices increase is the variations of the exchange rate.

The estimation and testing parameters of regressions models:

$$CIP_{t/0} = \alpha_0 + \alpha_1 ER_{t-1} + \alpha_2 CIP_{t-1/0} + \varepsilon_1$$
$$ER_t = \beta_0 + \beta_1 ER_{t-1} + \beta_2 CIP_{t-1/0} + \varepsilon_2$$

where CIP_t – fixed based consumer price index
ER_t – exchange rate.

The data series for consumer index of prices (CIP) in constant prices (October 1990=100) and monthly exchange rate are presented and they cover the period January 1991-March 2012, being published by National Institute of Statistics and National Bank of Romania. Due to the high volume of data for the mentioned period, in the following table, the first and the last three values for each data series were presented.

The exchange rate and CIP in constant prices (October 1990) in January 1991- March 2012

Month	Exchange rate	CIP in constant prices (Oct. 1990=100)
1991:01	0.0047	158.1
1991:02	0.0048	169.2
1991:03	0.0047	180.4
2012:01	4.3428	379764.02
2012:02	4.3506	382196.09
2012:03	4.3652	383786.38

Source: www.bnr.ro, www.insse.ro

Using Granger causality methodology we checked that in specified period the changes in exhange rate determined prices variations.

Granger causality test

Pairwise Granger Causality Tests			
Date: 06/13/12 Time: 11:03			
Sample: 1991:01 2012:03			
Lags: 2			
Null Hypothesis:	Obs	F-Statistic	Probability
IPC does not Granger Cause CS	253	0.45736	0.63349
CS does not Granger Cause IPC		5.94316	0.00301

A value less than 0,05 for the probability displayed by Eviews implies the rejection of null hypothesis. For a level of significance less than 5%, the result was that the variation of the exchange rate is a cause of CIP changes in January 1991-March 2012.

The data series for exchange rate is stationary, being necessary the elimination of the seasonal factors. The data modeling was done in EViews.

- **Simultaneous equations model (model A)**

$$ER_t = 0,4 + 1,16 \cdot 10^{-5} \cdot CIP_t \quad (1)$$

$$CIP_t = 12254,48 \cdot ER_{t-1} + 0,8408 \cdot CIP_{t-1} \quad (2)$$

The forecasted values for the exchange rate are obtained introducing the predicted values from equation (2) in equation (1).

- **Model with lagged variables (model B)**

$$ER_t = 0,4159 + 6,12 \cdot 10^{-6} \cdot CIP_{t-1} + 5,57 \cdot 10^{-6} \cdot CIP_{t-2}$$

- **Moving average model (MA(1)) (model C)**

The data series for the exchange rate being stationary, the elimination of the seasonal factors was necessary, resulting the model: $ER_t = 2,262 + 0,978 * \varepsilon_{t-1} + \varepsilon_t$. Another model was built, taking into account the registered value in April 2012 to predict the value from May 2012: $ER_t = 2,269 + 0,98 * \varepsilon_{t-1} + \varepsilon_t$.

Predictions for 2 months for average exchange rate ROL/EUR using the mentioned models

	Simultaneous Equations Model (model A)	Model with lagged variables (model B)	MA(1) Model (model C)
April 2012	4,418	4,893	4,429
May 2012	4,424	4,903	4,432

From the analysis of the accuracy indicators of forecasts, a low variability of errors was gotten for predictions based on A and C models. The prognosis for April and May 2012 based on these models are better than those based on random walk. The positive values for ME show the tendency of underestimating the values of exchange rate for all predictions.

Measures of forecasts accuracy for the three econometric models

	RMSM	ME	MAE	MASE	U1	U2
Model A	0,031327	0,0139	0,02805	0,924681	0,003549	0,704385
Model B	0,491641	0,4909	0,49095	17,40957	0,052835	11,04494
Model C	0,037724	0,0235	0,02955	0,947872	0,004269	0,848397

A generalization of Diebold-Mariano test (DM), established by two experts interested in comparisons of predictive accuracy, is used to determine whether the MSFE matrix trace of the model A is significantly lower than that of the model of B. If the MSFE determinant is used, the DM test can not be used in this version, because the difference between the two models MSFE determinants, Athanasopoulos and Vahid [8] show that it can not be written as an average. In this case, a test that uses a bootstrap method is recommended. The DM statistic is calculated as:

$$DM_t = \frac{\sqrt{T} \cdot [tr(MSFE_A)_h - tr(MSFE_B)_h]}{s} =$$

$$\frac{1}{s} \cdot \sqrt{T} \cdot [\frac{1}{T} \sum_{t=1}^{T} (em_{1,h,t}^2 + em_{2,h,t}^2 - er_{1,h,t}^2)]$$

T-number of years for which forecasts are developed, $em_{i,h,t}$ - the h-steps-ahead forecast error of variable i at time t for model A, $er_{i,h,t}$ - the h-steps-ahead forecast error of variable i at time t for model B, s- the square root of a consistent estimator of the limiting variance of the numerator.

The value of DM statistics (1,24) is higher than the critical one, so, if we use model A we have a better forecasts accuracy than using model C.

We refer to the most used combination approaches:

- optimal combination (OPT), with weak results according to Timmermann [9];
- equal-weights-scheme (EW);
- inverse MSE weighting scheme (INV).

Bates and Granger [10] considered two predictions p1;t and p2;t, for the same variable Xt, derived h periods ago. If the forecasts are unbiased, the error is calculated as: $e_{i,t} = X_{i,t} - p_{i,t}$. The errors follow a normal distribution of parameters 0 and σ_i^2. If ρ is the correlation between the errors, then their covariance is $\sigma_{12} = \rho \cdot \sigma_1 \cdot \sigma_2$. The linear combination of the two predictions is a weighted average: $c_t = m \cdot p_{1t} + (1-m) \cdot p_{2t}$ .The error of the combined forecast is: $e_{c,t} = m \cdot e_{1t} + (1-m) \cdot e_{2t}$ .The mean of the combined forecast is zero and the variance is: $\sigma_c^2 = m^2 \cdot \sigma_1^2 + (1-m)^2 \cdot \sigma_{2t}^2 + 2 \cdot m \cdot (1-m) \cdot \sigma_{12}$. By minimizing the error variance, the optimal value for m is determined (m_{opt}): $m_{opt} = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2 \cdot \sigma_{12}}$. The individual forecasts are inversely weighted to their relative mean squared forecast error (MSE) resulting INV. In this case, the inverse weight (m_{inv}) is: $m_{inv} = \frac{\sigma_2^2}{\sigma_1^2 + \sigma_2^2}$. Equally weighted combined

forecasts (EW) are gotten when the same weights are given to all models.

Combined forecasts based on econometric models on the forecasting horizon April-May 2012

Month Models A+B	Combined forecasts (OPT scheme)	Combined forecasts (INV scheme)	Combined forecasts (EW scheme)
April 2012	3,7055	4,789739	4,892992
May 2012	3,7055	4,79887	4,902992
Month Models A+C	Combined forecasts (OPT scheme)	Combined forecasts (INV scheme)	Combined forecasts (EW scheme)
April 2012	4,425071	**4,426609**	4,429
May 2012	4,429143	**4,430261**	4,432
Month Models B+C	Combined forecasts (OPT scheme)	Combined forecasts (INV scheme)	Combined forecasts (EW scheme)
April 2012	4,627452	4,661	4,42902
May 2012	4,633446	4,6675	4,432021

Applying INV scheme, the combined forecasts based on A and C models have a rather high degree of accuracy, the U1 being 0,004099 (high accuracy), and U2 the value 0,814386, which indicates a better prediction than the naive one. Taking into account the U1 values, the highest degree of accuracy was achieved by predictions based on the simultaneous equations model, then by the combined forecast starting from MA(1) model and the simultaneous equations model with INV weighting scheme, followed by those got using MA(1) procedure and finally the model with lagged variables.

Conclusions

Econometric Models for the determination of the exchange rate were developed in order to analyze the evolution and to make predictions. Given that theory provides several possible models to explain the same variables, it is important to choose the model that generates best predictions in terms of accuracy.

Three possible models to explain the evolution of the exchange rate in Romania were proposed: **a Simultaneous Equations Model**, a **MA (1) Procedure** and a **Model with lagged variables**. **The Simultaneous Equations Model** that takes into account the Granger causality generated the most accurate predictions on the horizon April-May 2012. The INV weighting scheme determined a high accuracy, close to that of the simultaneous equations model. Knowing of best estimates of the exchange rate is necessary in order to build the monetary policy.

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