THEORETICAL AND PRACTICAL STATISTICS*: TERRITORIAL SERIES/ OF SPACE – SYSTEM OF INDICATORS AND INDICES, SUGGESTIVE GRAPHICS REPRESENTATIONS

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Abstract

Territorial statistical series were and are used to analyze phenomena depending on space or place where they are produced, having a special importance at macroeconomic level. In our opinion, to understand how the local series are very helpful, we have to know the system of indicators, the methodology for calculating the indices as a measure of phenomena variability and possibilities / issues related to graphical representation.

Keywords: statistical variables, level indicators, gap indicators, territorial indices, individual indices, group indices, weights, graphical representation

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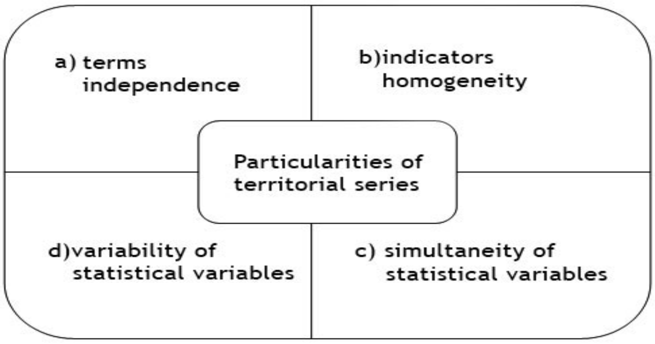
Territorial series is known as consisting of two rows of data: a row of data with characteristics values and a row of data consisting of administrative territorial units. A series of space expresses the values of statistical variables, ordered in relation to territorial divisions they belong.

The analysis in territorial profile operates with divisions of space: municipalities, districts, counties. If the analysis is done at national level or across countries, continents and other regional areas, the analysis is subject to international statistics. Territorial divisions can be studied independently as a monographic study or can be analyzed as relative measures of coordination, which compares the level of a phenomenon in different places. The type of analysis serves to international comparison of indicators.

Comparability of information and the meaning of indicators used for the investigated aspect constitutes the basic criteria for choosing indicators in order to form a territorial range.

* Suggestions for specialists, including from territorial branches of statistics for comparisons in the published papers.

In analyzing the variation of different socio-economic characteristics, there are used series of space that present some peculiarities. Example:



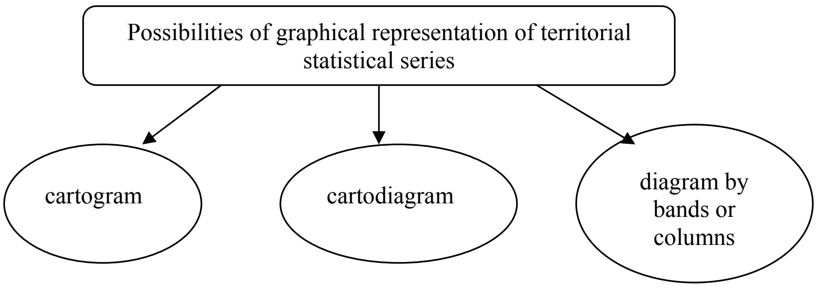
Terms independence – the specific levels for different space units are not mutually conditioned. Particularity allows separate characterization of each unit either in the comparing process either in series counting process.

Homogeneity - all indicators being compared must be based on a unified methodology: the same socio-economic content, the same definition and the same scope.

Simultaneity - all variants of different statistical variables relate to one and the same time of observation or the same recording time.

Variability - any variable to be subjected to statistical spatial analysis is the result of the combined influence of the key factors with the random ones. Differences in these characteristics occur from one unit to another area.

Concerning the content terms, **space series** are classified as follows: statistical series of space consisting of absolute values, statistical series of space consisting of derived sizes.



Cartogram is a separate hatch of the characteristics level located in the appropriate space on the map. An important role has the chart legend which helps to distinguish, based on a variation study, the qualitative types.

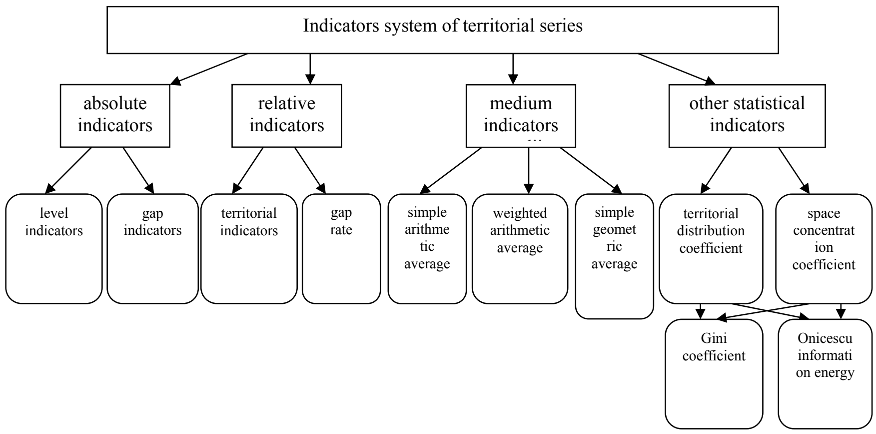
Cartodiagram involves building, within an area of the map, a statistical diagram or a symbol that reflects the aspect of territorial analysis.

Diagrams by bands or columns are used, especially, if the number of statistical units of space is reduced. The height of each column, or the length of a band, illustrates the different size of the indicator from a territorial unit to another.

Spatial investigation of phenomena is made through a **system of indicators**, built both according to the type of phenomena and also depending on the purpose of statistical analysis.

The system of indicators envisages, on the one hand the differences between units of space, and on the other hand the typical and significant phenomena, series uniformity and uneven dispersion of characteristics in space.

Like the chronological series, territorial statistical series are based on a system of territorial indicators presented below.



The analysis of statistical space series is also suitable for the usage of other methods and statistical techniques such as: dispersion analysis, parametric and non-parametric correlation.

Absolute indicators are expressed in the same measure units of the feature, being of two types: level indicators and gap indicators.

Level indicators refer to the feature itself (e.g for a series of space formed of *n* absolute terms).

Space unit	1, 2 … … n
Characteristics level	$y_1, y_2 \dots \dots y_n$

Level indicators express the volume of characteristics found in the space unit.

Gap indicators are obtained by comparing, in absolute values, the characteristic level (y) into two territorial units, and express advancement (gap) of a territorial unit, to another territorial unit established as basis for comparison.

If we study the characteristics of y into two territorial units A and B, the gap indicator is calculated by difference:

$$\Delta^{y}_{A/B} = y_A - y_B \text{ or } \Delta^{y}_{B/A} = y_B - y_A$$

Relative indicators are obtained by comparing as report, the feature level of two territorial units. It examines, first, whether the content of the comparison result is logical and has economic content.

Territorial indicators, which are obtained by dividing the values of a feature in space are most often used. Example: when studying the same variable y for two territorial units A and B, two individual indices are obtained:

$$i^{y}_{A/B} = \frac{y_A}{y_B}, \text{ or } i^{y}_{B/A} = \frac{y_B}{y_A}$$

If it is requested, the influence of units of space structure and decomposition on influence factors of a complex characteristics variation, it is used the territorial group indices (aggregate), built on a weighting system.

Rate gap is obtained as a ratio between the gap indicator and another indicator as basis for comparison (expressed usually as a percentage):

$$\Delta^{y(\%)}_{A/B} = \frac{y_A - y_B}{y_B} \cdot 100 = \left(i^{y}_{A/B} - 1\right)100$$

$$\Delta^{y(\%)}_{B/A} = \frac{y_B - y_A}{y_A} \cdot 100 = \left(i^{y}_{B/A} - 1\right)100$$

In statistical practice, there are cases when it is difficult to choose a unit area as reporting basis. We resort to the usage of average, median and means as reporting basis. **Construction of indicators**:

Average indicators are expressed by the medium level of space series $\left(\bar{y}\right)$. For the series of space consisting of absolute values, the average level builds on the simple arithmetic average, and for the series of space made up of relative measures (of structure and intensity), the medium levels are calculated as weighted arithmetic averages. If space series consist of relative measures of dynamics, the average level of characteristic is calculated as the geometric simple average.

Other statistical indicators are used to characterize the degree of uniformity of distribution in space, the most widely used indicator being the territorial distribution coefficient or coefficient of concentration in space, which applies to space series consisting of absolute values.

Uniformity of distribution in space, namely coefficient of space concentration, can be calculated in several ways.

First version, known as the **Gini coefficient (G)**, belonging to the Italian statistician Corrado Gini, consists of the square root from the sum of weights squares of "n" units of space, in the total of studied group:

$$G = \sqrt{\sum g_i^2}, \text{ i = 1, 2, … n.}$$

g_i = weight of a unit in the total.

The value of coefficient is in the interval $\left(\sqrt{\frac{1}{n}}, 1\right)$:

Interpretation:

- if $G = \sqrt{\frac{1}{n}}$, means that „n" units of space have equal weights and the territorial distribution is absolutely uniform.

- if G = 1, the studied variable concentrates in a single territorial unit.

Boundaries between coefficient "G" have some drawbacks in interpretation, when comparing communities with different territorial organization (e.g comparisons between countries, between geographical areas or continents). In international statistical practice, there have been proposed some methods to correct Gini's formula, so that limits to be comprised between $[0, 1]$, no matter how many units the researched community would have.

A second option for measuring the degree of uniformity of territorial distribution resulted from the adoption by academician Octav Onicescu of concepts specific to systems theory at the spatial analysis needs. Onicescu developed the concept of **informational energy** as a system involving community as a system and space units as states of the system. Informational energy serves as an indicator of territorial distribution, being calculated by the formula:

$E = \sum g_i^2$, with i= 1, 2, … n

Informational energy of system cohesion is adjusted according to the relation:

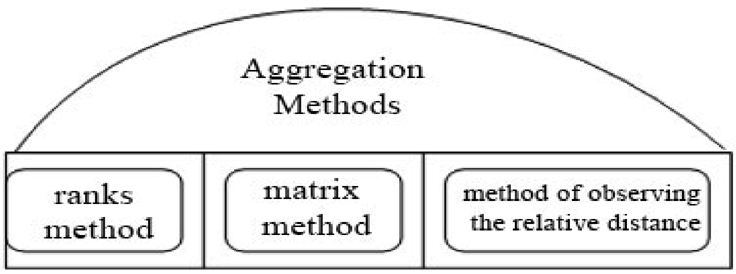
$$E' = \frac{\sum g_i^2 - \frac{1}{n}}{1 - \frac{1}{n}}$$

, being a correction process that makes the result to fit in $[0, 1]$, no matter the states presented in the studied group.

Hierarchy of territorial units according to certain criteria, has a great importance both nationally (for the harmonious development of all units of space) and internationally, to measure the differences between countries in order to develop optimal strategies for development and help.

For a judicious classification and hierarchy it is not indicated the use of a single statistical indicator, no matter how relevant it is, and it is recommended a multi-criteria analysis, using a complete system of indicators.

Any hierarchy begins with the identification and selection of indicators providing a comprehensive characterization of each territorial administrative unit. After a series of provisional rankings based on each indicator selected, it is proceed to the choice of a method of aggregation into a single indicator, on which the ranking is done properly.



Rankings method is based on a hierarchy according to the size of the characteristics giving level ranks (serial numbers) to each territorial unit. The unit with the highest level of feature gets rank one, and further on, in turn, until the unit that records the lowest level of the feature, which will rank highest.

If a space unit records more features (criteria) according to which the ranking is done, for each of these variables successive ranks are assigned.

Example, in synthesis

Unit of space	Rank assigned according to				Total score	Final rank
	Characteristics (A)	Characteristics (B)	Characteristics (C)	Characteristics (D)		
0	1	2	3	4	5	6
a	2	1	1	4	8	2
b	1	3	2	1	7	1
c	4	2	3	3	12	3
d	3	4	4	2	13	4

Ranks are assigned for all four characteristics that define territorial unit.

Ranks are summed (horizontally), resulting the “total score” (column 5). For the lowest score is given rank 1, etc.. getting the final “rank” on which units of space are ranked according to four criteria (features).

Ranking method has the advantage that its results can be exploited in analyses involving correlations between variables by nonparametric methods. The method has the disadvantage of losing some information on the two leveling of different characteristics: once the ranks are assigned for each feature and the ranks are assigned for total scores. By leveling, the distances between the two levels of characteristics, in different space units, are replaced by an arithmetic progression with ratio one.

Matrix method is distinguished by the fact that we can calculate the distances between levels of characteristics, distributed in different space units. No more distorting real distances between two different space units and provide superior results compared to rank method.

If the number of units of space and number of features is greater we resort to electronic computer.

The method of relative distance observation is a simple method that allows fully preserving of information about the actual distances between the levels of characteristics recorded in units of area. Firstly, the method consists in establishing a notional unit whose characteristics have minimum levels (or maximum) and choosing a method of measuring the distance between real and notional units (finding a process of information aggregation for each actual unit).

In all presented methods it is followed the information aggregation in a synthetic index which allows to measure the real disparities between units as well as using the results of multi-criteria ranking in the statistical results based on nonparametric methods.

Territorial indices have a wide applicability in comparing some statistical characteristics located in different space units. A territorial index is to be calculated as the ratio between the terms of a statistical series of space, expressing the level variation of characteristics in relation to space.

At territorial indicators, it is emphasised the choice of the reporting basis and weighting system. Choosing the report base is primarily based on economic or social reasons. Whenever a unit area can be appreciated in terms of economic or social indicators (degree of endowment with natural resources, technical resources, number of hospital beds per capita, the number of radio and television subscriptions etc.) to serve as a basis for reporting. It is estimated that due to the socio-economic development, a unit of space cannot be kept too long in the base. Also, it should not be chosen in the reporting basis a territorial unit of exception, but one in which the development parameters justify the comparison. Weighting system applies the same known principles. Territorial indices are made up as individual indices (i) and group indices (I).

If we note the variable that is compared with *y* and the two units of area A and B, we get two different indicators in terms of direction of comparison:

$$i^y_{A/B} = \frac{y_A}{y_B} \text{ and } i^y_{B/A} = \frac{y_B}{y_A}$$

Between the two individual territorial indices, there is a relation of space reversibility $\left(i^y_{A/B} \cdot i^y_{B/A} = 1\right)$. In statistical analyses, the two meanings of comparison are not used simultaneously.

Territorial group indices are to be drawn by comparing the complex phenomenon in space A, at the level of the complex phenomenon in space B. The complex phenomenon ($\sum y_i$), in which occurs the weighting factor, frequency. General relation for calculating the index of territorial group is based on comparison purposes:

$$I^y_{A/B} = \frac{\sum y_A}{\sum y_B} \text{ or } I^y_{B/A} = \frac{\sum y_B}{\sum y_A}$$

$\sum y_A$ = total level in a space unit A;

$\sum y_B$ = total level of the phenomenon from a unit of space B.

Reversibility principle is not always respected at group indices. If quantitative factor is directly added, the quality factor is manifested at the level of elements investigated as an average. There are the variables directly expressed as relative measures of intensity.

The index of territorial group, appears as a relation between two environments:

$$I_{A/B}^{\bar{x}} = \frac{\bar{x}_A}{\bar{x}_B} = \frac{\sum y_A}{\sum f_A} : \frac{\sum y_B}{\sum f_B} = I_{A/B}^{y} \cdot I_{A/B}^{f}$$

In space profile it is shown the link between the complex variable and its factors of influence:

$$I_{A/B}^{y(\bar{x},f)} = I_{A/B}^{\bar{x}} \cdot I_{A/B}^{f}$$

If the quantitative factor is not directly totalised, for comparisons in space it is chosen the appropriate weight. For example, the quantitative factor can be weighted with x_A or x_B, and the group index which expresses the variation of quantitative factor in different areas will be:

$$I_{A/B}^{f} = \frac{\sum x_A f_A}{\sum x_A f_B} \text{ and } I_{A/B}^{f} = \frac{\sum x_B f_A}{\sum x_B f_B}$$

In the case of non totalised qualitative factor, the weight is made with f_A or f_B and group index that expresses the variation of qualitative factor in different spaces becomes:

$$I_{A/B}^{x} = \frac{\sum x_A f_A}{\sum x_B f_A} \text{ and } I_{A/B}^{x} = \frac{\sum x_A f_B}{\sum x_B f_B}$$

The index factors are weighted with frequencies specific for territorial units. In these conditions it is not ensured the reversibility of factors. To establish a relationship between complex phenomenon and the corresponding factors, it is used a weighting system proposed by Fischer, which is based on the geometric average of the two weighting variables.

Therefore group indices are:

$$I_{A/B}^{f} = \sqrt{\frac{\sum x_A f_A}{\sum x_A f_B} \cdot \frac{\sum x_B f_A}{\sum x_B f_B}} \text{, for quantitative factor.}$$

$$I_{A/B}^{x} = \sqrt{\frac{\sum x_A f_A}{\sum x_B f_A} \cdot \frac{\sum x_A f_B}{\sum x_B f_B}} \text{, for qualitative factor.}$$

Both indices as the ratio of two averages and territorial aggregated indicators are subject to research in space, especially in international comparisons.

Note that, based on differential dynamic indicators in regional analyzes, it is given a special interest in territorial forecasts. To develop techniques for forecasting by extrapolation, a few questions arise: How long the indicator level can double or triple? What will be the time necessary for a territorial unit to catch up with another territorial unit or even to overtake? What pace should be required to reach a certain level?

To answer some of these questions brought forward, the advancing coefficients are calculated, either for two territorial units or for two variables that make up a community to track their evolution over time.

If we note by I_A the chronological index set for territorial unit, and I_B, the chronological index determined for territorial unit B, the advancing coefficient becomes:

$$I^{y}_{A/B} = \frac{I_A}{I_B}$$

Reporting two dynamics of the same phenomenon (a dynamic in space A and a dynamic in space B), the advancing coefficient shows the number of times the unit reported grows faster than the evolution of the reporting unit as a reporting basis.

To some questions, answers the territorial extrapolation technique. When you know the average annual growth of territorial units, the question to calculate after how many years (period of time), the phenomenon would change a number of times, or whenever the phenomenon will change after a while, in both cases maintaining the same growth conditions. If we assume a geometric relationship between the statistical series terms of space and if we known the average index of growth index, we resort to the relationship:

$$\underline{\bar{I}}^{\,t} = K$$

$\bar{I}$ - the average index of growth in time of the phenomenon;

T – the number of years when the change will be produced of K times;

K – coefficient of phenomenon change after passing "t" years [e.g the phenomenon will double (K=2), will triple (K=3) etc.]

It is known that territorial units evolve with different development levels and rhythms. In this case the question arises to find out when to catch up with a unit of space or another, considering that their development will be in geometric progression, with ratio equal to the average growth rate.

There are absolute levels y_A and y_B for the calculating time ($y_A < y_B$), and average indices of growth for the two territorial unit I_A and I_B ($I_A > I_B$).

At the same time, y'_A and y'_B, the phenomenon level in the final moment t. Therefore, the two absolute levels y'_A and y'_B will have the same dimension in the final moment t, $y'_A = y'_B$.

Between y and y' there are the relations:

$y'_A = y_A \cdot \bar{I}_A^t$

and $y'_B = y_B \cdot \bar{I}_B^t$ / $y_A \bar{I}_A^t = y_B \bar{I}_B^t$

(the two territorial units will have the same absolute level).

Applying the logarithm in relation, it results:

$$\log y_A + t \log \bar{I}_A = \log y_B + t \log \bar{I}_B$$

$$t = \frac{\log y_A - \log y_B}{\log \bar{I}_A - \log \bar{I}_B}$$

t represents the time horizon when the two absolute levels of territorial units will be equal.

If *t* is too high, therefore time horizon is far, it is resorted to the estimation of the average rhythm necessary for that space unit remained behind concerning the development.

If unit A proposes to catch up with the gap in comparison with B unit, it results:

$$\log \bar{I}_A = \log \bar{I}_B + \frac{1}{t}\left(\log y_B - \log y_A\right)$$

Estimating the average rate, for the unit space remained behind, is correlated with other stakeholders of the territorial unit, where they will appreciate how fast the growth must be, given the economic effort required for such an increase.

Conclusion

Regarding territorial indices we tried to highlight some features specific to comparative analyses in territorial benchmarking. General principles of composition of indices remain valid for the important area of territorial / space comparisons.

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