DYNAMIC / CHRONOLOGICAL (TIME) SERIES

- theoretical presentation, structure, relationships between indices

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Abstract

Time series, which emphasize the temporal characteristic of phenomena, are an important in the context of macro-economical analyses. Specific to the dynamic series is the fact that they are defined for complex entities, characterized by a high level of variation for indicators, including structural temporal variations.

Key words: *series, terms, methodology, time interval, average*

The chronological series composed of two rows of parallel data, the first rows show variation of the time periods variation and the second rows show the variation of the phenomenon or characteristics studied over time. The chronological series has the following features: variability, homogeneity, interdependence to its terms. The variability is because each term is obtained by centralizing of individual data different as level of development. These individual data exist because, that in social phenomena acts besides the essential causes, determinants causes, and a sufficient number of nonessential causes. When analyzing time series, we have to take into account the fact that they are prepared for the complex units. For them, the degree of variation of indicators includes structural variations from one time unit to another.

Interdependent of terms: the indicators are successive values of the some phenomena recorded in the some territorial or administrative units. In time series need to know the trend curve specific to each stage of development, and statistical expressing is the action of the low even causing them. Because of the interdependence among terms, in the case of chronological series it's necessary to know the trend line (curve) which is specific to each development stage and which expresses statistical and in quantitative manner the action of law that determine them. Taking into account all these particularities, the statistical analysis of chronological series shall be based on a system of indicators that characterize many quantitative relations of the inside of series and at the period to which data is referring.

How to analyze the dynamic series of streams

Following the processing of time series are obtained absolute, relative and average indicators which together will characterize from statistical point of view the development of phenomena studied by the interpretation of the objective trend of them development in each stage.

The absolute indicators, that characterize a time series, are:

y_i –absolute levels of terms of series;

$\Delta_{i/0}$ -absolute change(increase or decrease),calculated with fixed base;

$\Delta_{i/i-1}$ – absolute change(increase or decrease) calculated with the base chained.

The relative indicators, that characterize a time series, are:

I_i – dynamic index calculated with fixed base;

$I_{i/i-1}$ – dynamic index calculated with the base chained;

$R_{i/0}$ – rate of increase (decrease) calculated with fixed base;

$R_{i/i-1}$ – dynamic index calculated with the base chained;

$A_{i/0}$ – absolute value of a percentage of increase (decrease) with fixed base;

$A_{i/i-1}$ – absolute value of a percentage of increase (decrease) with fixed base which the base chained.

Average indicators are:

$\bar{y}$ – the average level of the time series;

$\bar{\Delta}$ – the average level of the absolute change(increase or decrease),

$\bar{I}$ – the average index of dynamics;

$\bar{R}$ – the average rate growth rate.

Methodology of calculating the time series indicators is elaborated on the example of time series interval:

$y_i = f(t_i)$,

where:

y_i = the values of the studied variable ;

t_i = the numerical values of the studied variable.

The absolute indicators of the time series are expressed in the units of measurement of the phenomenon under study. If we note with t the time variable and with y the studied variable, series will take the following values: from y_0 / y_1(initial period) to y_n (last term of the series):

For the variable „y" the specialized literature uses the following notations:

$y_1, y_2 \dots y_i \dots y_n, \quad i = \overline{1 \cdot n}$

or:

$y_1, y_2 \dots y_t \dots y_T, \quad t = \overline{1, T}$.

In this paper we use:

$y_0, y_1 \dots y_i \dots y_n, \quad i = \overline{0 \cdot n}$.

In this case, the series is comprised by n+1 terms.

The first absolute indicator is the indicator of series volume, which can be calculated only for the interval series because its terms are summed.

Absolute increase (or decrease) is calculated from the level of a single period considered as base of reference or from a period of time to another. In the first case, we obtain absolute increase (or decrease) calculated with fixed base, in the second case we obtain absolute increase (or decrease) calculated with the chained base known as absolute increase (or decrease) with variable base [1].

Absolute change with fixed base is noted $\Delta_{i/0}$ and obtain as difference between each level of period y_i and the level of reference period y_0 :

$\Delta_{i/0} = y_i - y_0.$

In the time series with y_i terms $(i = \overline{0 \cdot n})$ we obtain n absolute increase.

Between absolute increase with fixed base and absolute increase with variable base it is the following relation:

$$\sum_{i=1}^{n} \Delta_{i/i-1} = \Delta_{n/0} ,$$

That is:

$(y_1 - y_0) + (y_2 - y_1) + \dots + (y_n - y_{n-1}) = y_n - y_0$.

Using this relation we can calculate:

$\Delta_{i/0} - \Delta_{i-1/0} = \Delta_{i/i-1}$

$(y_i - y_0) - (y_{i-1} - y_0) = y_i - y_{i-1},$

Absolute increase may also take the negative values, frequently using the expressions „zero growth" and „ negative growth".

The relative indicators of the time series are very important for analyzing the dynamic of the socio-economic phenomena.

These indicators are used frequently to determine the proportions and correlations between the various branches and sectors of the national economy. Relative size showing how many times a phenomenon over time has changed is called dynamic index and it can be calculated with fixed base and variable base.

Dynamic index with fixed base is denoted $I_{i/0}$ and calculated by the following formula, expressed in percentage:

$$I_{i/0} = \frac{y_i}{y_0} \cdot 100,$$

Dynamic index with variable base ($I_{i/i-1}$) is calculated upon the formula:

$$I_{i/i} = \frac{y_i}{y_{i-1}} \cdot 100,$$

In the time series we can obtain dynamic indices with fixed base equal to the number of the series terms and with a less term for the dynamic index with variable base. Between these relative indicators there are relations that permit the transition from one shape to another.

$$\prod_{i=1}^{n} I_{i/i} = I_{n/0},$$

so:

$I_{1/0} \cdot I_{2/1} \cdot I_{3/2} \ldots I_{n/n-1} = I_{n/0}$;

Appropriately it can passes from the indices with fixed base to the indices with variable base, too:

$$\frac{I_{i/0}}{I_{i-1/0}} = \frac{y_i}{y_0} : \frac{y_{i-1}}{y_0} = \frac{y_i}{y_{i-1}}.$$

So:

$$\frac{I_{i/0}}{I_{i-1/0}} = I_{i/i-1}.$$

But, in practice is important to analyze the increased current level as towards the benchmark (fixed or variable) of the phenomenon studied.

The growth rate with fixed base is calculated as the ratio of absolute change with of each period and the level corresponding to the year taken as reference

This indicator in noted $R_{i/0}$ and usually is used as a percentage:

$$R_{i/0} = \frac{y_i - y_0}{y_0} \cdot 100 .$$

This indicators can be written:

$$R_{i/0} = \frac{y_i - y_0}{y_0} = \frac{y_i}{y_0} - \frac{y_0}{y_0} = \frac{y_i}{y_0} - 1.$$

But, $\frac{y_i}{y_0}$ is dynamic index with fixed base, so, the relation is:

$R_{i/0} = I_{/0} - 1.$

In percentages we have:

$R_{i/0}\ \% = I_{i/0}\ \% - 100$

So, if we know dynamic index with fixed based, the growth rate with fixed base is easy to obtain.

The growth rate with variable base ($R_{i/i-1}$) is calculated as ratio between absolute change with variable base for each year and the level that corresponding previous year, usually expressing themselves in percentages:

$$R_{i/i} = \frac{y_i - y_{i-1}}{y_{i-1}} \cdot 100.$$

Likewise:

$$R_{i/i-1} = \frac{y_i - y_{i-1}}{y_{i-1}} = \frac{y_i}{y_{i-1}} - \frac{y_{i-1}}{y_{i-1}} = \frac{y_i}{y_{i-1}} - 1,$$

So:

$R_{i/i-1} = I_{i/i-1} - 1$

If we use percentages, then the following relationship applies:

$R_{i/i-1} = (I_{i/i-1} \cdot 100) - 100.$

To be noted that the transition from the growth rate with fixed base to the growth rate with variable base to be made only after it transforms into dynamic indices:

$I_{i/0} = (R_{i/0} + 1) \cdot 100$

and

$I_{i/i-1} = (R_{i/i-1} + 1) \cdot 100,$

because multiplying the growth rate with variable base is denote as equally with the growth rate with fixed base for the entire period:

$$\prod_{i=1}^{n} R_{i/i-1} \neq R_{n/0} \,.$$

Using relations by moving from a base to another and from indices to rhythms can lead to restoration of indicators that not known.

Calculation of indices with variable base:

Ii/i-1(%) = Ri/i-1 (%) + 100

Calculation of indices with fixed base:

$$\prod_{i=1}^{n} I_{i/i-1} = I_{n/0}$$

And the base year multiplies by 100.

For the interpretation of annual growth rates should consider the following concerns:

One aspect is referred to the indices with the fixed base, can be compared between them because of the same denominator, but indices with the variable base can not be compared directly, because they have different bases.

The absolute value of the percentage of growth rate with fixed base, expresses how many units of growth recorded a year back every percentage of growth, it is the entire period, because level that was considered is equal to 100% corresponding the base year (y_0).

$$A_{i/0} = \frac{\Delta_{i/0}}{R_{i/0}\%} = \frac{\cancel{y_i - y_0}}{\frac{\cancel{y_i - y_0}}{y_0} \cdot 100} = \frac{y_0}{100}$$

The absolute value of the percentage of growth rate with variable base is noted with $A_{i/i-1}$ and is based on the same reasoning, that is:

$$A_{i/i-1} = \frac{\Delta_{i/i-1}}{(R_{i/i-1})100};$$

$$A_{i/i-1} = \frac{\Delta_{i/i-1}}{R_{i/i-1}\%} = \frac{y_i - \cancel{y_{i-1}}}{\frac{y_i - \cancel{y_{i-1}}}{y_{i-1}} \cdot 100} = \frac{y_{i-1}}{100}$$

$$A_{i/i-1} = \frac{y_{i-1}}{100};$$

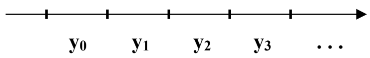
This indicator makes the connection between absolute indicators and relative indicators helping their correct interpretation.

Calculation of absolute and relative indicators leads to the characterization of individual relationships between the terms of a series, taken by two. These indicators show the variability of the terms of a time series as a result of the influence of all causes and conditions that determine the evolution of this phenomenon.

When analyzing dynamics of phenomena, we can calculate the average level and average rate.

The average of the time series.

If we represent on an axis the terms of a chronological series of intervals, they will appear as:



The average of the time series of interval be calculated using simple arithmetic average:

$$\overline{y} = \frac{\sum_{i=0}^{n} y_i}{n+1}.$$

If the terms of the time series is denoted by $y_i, \ (i = \overline{1,n})$, then, the average of the time series is:

$$\overline{y} = \frac{\sum_{i=1}^{n} y_i}{n}$$

If the terms of the series are noted y_1, y_2 ... y_i ... y_n, then we have n terms and n-1 chain-based growths. The calculation formulas must be adapted ant:

$$\overline{\Delta} = \frac{\sum_{i=2}^{n} \Delta_{i/i-1}}{n-1} = \frac{\Delta_{n/1}}{n-1}.$$

Likewise, the average annual change is:

$$\overline{\Delta} = \frac{\sum_{i=1}^{n} \Delta_{i/i-1}}{n},$$

where:

n = the number of absolute increase with variable base, that is the number of terms less one.

Average annual absolute changes/ increase is depends on the difference between the first and last term, because, based on the relation that exists between absolute changes with fixed base and variable base, the amount of absolute changes with fixed and variable base is equal with absolute change for entire period:

$$\overline{\Delta} = \underset{(1)}{\frac{\sum_{i=1}^{n} \Delta_{i/i-1}}{n}} = \underset{(2)}{\frac{\Delta_{n/0}}{n}},$$

whence:

$$\overline{\Delta} = \frac{y_n - y_0}{n},$$

It results that in this case, we can use two formulas for calculating the average annual absolute change.

If in the inside of the same series can be found opposite trends, that on the graph, corresponding to a change in the form of a second degree parabola with a maximum or minimum point, then the series should be divided into two parts, calculating the respective average indicators.

If, y_i, $i = \overline{1,n}$, then we have:

$$\overline{\Delta} = \frac{\sum_{i=2}^{n} \Delta_{i/i-1}}{n-1} = \frac{\Delta_{n/1}}{n-1}$$

Average growth index looks how many times increase the average of the phenomenon level studied during a year.

$I_{1/0} \cdot I_{2/1} \cdot I_{3/2} \cdot I_{4/3}, \ldots I_{n/n-1} = I_{n/0}$;

$$\bar{I} \cdot \bar{I} \cdot \bar{I} \ldots \bar{I} = I_{n/0},$$

$$I_{1/0} \cdot I_{2/1} \cdot I_{3/2} \cdot I_{4/3} \ldots I_{n/n-1} = \underbrace{\bar{I} \cdot \bar{I} \cdot \bar{I} \cdot \ldots \bar{I}}_{de\ n\ ori}.$$

$\bar{I}$ being a constant size, the product of the second member of equal may be replaced by its value raised to a power equal with the number of terms $(\bar{I}^n)$.

$I_{1/0} \cdot I_{2/1} \cdot I_{3/2} \cdot I_{4/3}$, ... $I_{n/n-1} = \bar{I}^{n}$

Or:

$$\prod_{i=1}^{n} I_{i/i-1} = \bar{I}^{n},$$

$$\bar{I} = \sqrt[n]{\prod_{i=1}^{n} I_{i/i-1}} .$$

It is important that index (sg)/indices (pl.) be taken as coefficients and not as percentages.

We see that using the geometric average formula, where:

$\bar{x}_g$ = average index;

x_i = indices for average.

The product of the indices with variable base is equal with the index with fixed base for the entire period:

$$\bar{I} = \sqrt[n]{\prod_{i=1}^{n} I_{i/i-1}} = \sqrt[n]{I_{n/0}} = \sqrt[n]{\frac{y_n}{y_0}} .$$

(1) (2)

They are used depending on the available data. For longer series, the second formula is more suited.

For the time series denoted from 1 to n we have:

$$I = \sqrt[n-1]{\prod_{i=2}^{n} I_{i/i-1}} = \sqrt[n-1]{\frac{y_n}{y_1}} .$$

(1) (2)

If we have more successive sub periods, general average index can be calculated as the weighted geometric mean of average growth indices.

$$\bar{\bar{I}} = \sqrt[\sum_{i=1}^{k} n_i]{\bar{I}_1^{n_1} \cdot \bar{I}_2^{n_2} \cdot \bar{I}_3^{n_3} \ldots \bar{I}_k^{n_k}}$$

where:

$\bar{\bar{I}}$ = general average index;

$\bar{I}$ = partial average index;

n_i = variable-based index number of each sub period ;

k = number of sub periods – number of partial average index.

The average growth rata shows how much this phenomenon has increased in relative size, in the analyzed period, on average from one unit to another of interval.

The average growth rata is calculated as the difference between dynamic environment index, expressed as a percentage.

$$\overline{R} = (\bar{I} \times 100) - 100;$$

Conclusion

The indicators obtained by processing a chronological time series can be organized in a system, where each indicator may emphasize one aspect of the development of the phenomena studied.

The of these indicators is determined by the manner in which the time series, by the significance of the period chosen for the evolution of the phenomenon studied, by the homogeneity of empirical data used and the length of the time series. The number of terms must be large enough to satisfy the law of large numbers, they interpret statistical regularities evolutionary phenomena.

When working with heterogeneous time series, with various development trends it is necessary to calculate these indicators on each step as partial indicators, or indicators obtained from processing is not real and practical and theoretical conclusions have not made proper basis and can not be calculations used for forecast.

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