METHODS, THEORIES AND MODELS TO MEASURE MARKET RISK OF THE PORTFOLIO OF SHARES

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Abstract

In terms of a portfolio of shares, market risk is caused by the price change measures under discussion and that is why it is important to consider carefully the historical evolution of prices in order to be able to determine if there is a certain cyclical trend that may affect the portfolio in the future.

Key words: *market risk, sensitivity, profitability, Value at Risk, derivative transactions*

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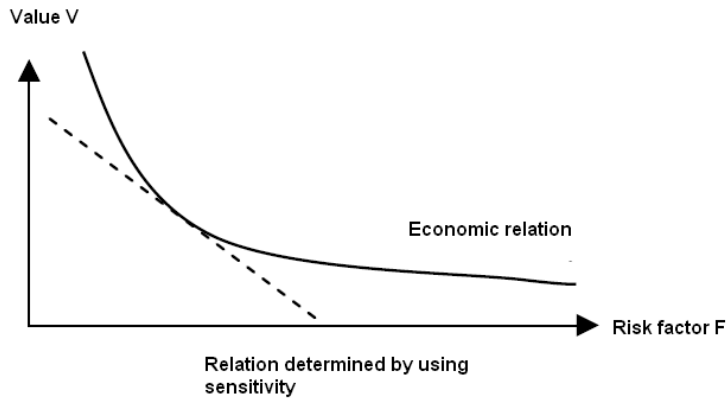
Market risk is calculated using the probability distribution of market value change over short periods of time. This is recommended because of the difficulty of estimating over long periods of time (it is difficult to change prices or volatility expected them to one year, for example) and as a result of the relevance that the calculation of the market risk on a shorter period of time.

The first step in the calculation of the market risk is the determination of the factors of interest which may affect the title held by the investor. Not all micro and macroeconomic factors affect equally the sensitivity of the price of securities owned. In the case of bonds, always market risk will occur due to the risk of changes in interest rates (both due to the risk that the bond price to drop as a result of the fact that interest in the market has increased, but also because of the risk that the reinvestment of coupons to be less profitable because the interest rate was reduced).

On the other hand, foreign exchange rate, it is important when investing in the forward or futures positions on international markets. In this case, the change in exchange rates is the most important risk factor to which it is exposed to an investor.

Once you have set the exact risk factors that may influence the positions held in the portfolio, the next step is building a model to determine the effect of the change in the portfolio's value to the factor in question.
A basic measure for measuring market risk sensitivity is. It measures the loss or the gain obtained as a result of the change of a factor of the market. For example, in the case of bonds, the bond price changes due to the modification of the rate of interest. Sensitivity it entails using the formula $\frac{\Delta V}{\Delta F}$ where ΔV represents the change in the value of the position held by the investor at the time of change of risk factor F with a unit. Graphically, this relationship is presented in the following way:

The relationship between the value of the investment and risk factor, determined using sensitivity



The effectiveness of this method of measurement depends on how big is the risk factor modification F. If modification is relatively large, then the sensitivity will lose the ability to determine market risk and become ineffective. This is because the relationship between risk factor F and V position value is convex, while sensitivity represents a linear relationship between the two elementre. As a result, at the time the change in risk factoril F is large enough, the distance between the line obtained using sensitivity and convex curve representing the economic relationship between the two factors will increase the sensitivity will not be able to be used as an estimate for the relationship.

As regards the determination of the interest risk in the case of bonds, a traditional method used is the analysis of duration. Macaulay duration for a bond is calculated as the average time to maturity of the bond weighted present value of the cash flow generated by the bond.

$$D = \frac{\sum_{i=1}^{n}(i \times PVCF_i)}{\sum_{i=1}^{n}(PVCF_i)} \qquad (1)$$

where

$PVCF_i$ represents the amount of cash flow from time t.

Using the obtained, one can determine the bond's price sensitivity to interest-rate changes:

$$\Delta\% \quad in \quad price \quad obligation \approx -D \times \Delta y / (1 + y) \qquad (2)$$

where
y is the rate of interest, and Δy is modifying or loss of interest rate during a specified period of time. As the duration is greater than the price of the bond is more sensitive to interest-rate changes.

The advantage of this method is that it is easy to calculate, simple data being collected and implemented in the model. The major disadvantage, however, is that it only takes into account price sensitivity to changes in interest rates, and not include other risks faced by the investor.

In order to determine market risk in the case of investment in securities other than bonds, over time has developed the theory of portfolio. This theory is based on the idea that investors choose to invest in securities depending on the rate of return offered and the risk to which they are exposed (standard deviation of return). Thus, they will choose portfolios with high returns and associated with minimal risks.

The portfolio theory is based on a model of Mack (1952) with regard to the principles of the selection of assets of a portfolio. According to this model, a rational investor that aims at maximizing the utility will compile a portfolio investments to depending on the average and variance of profitableness. The assumptions which formed the basis of this model were capital market efficiency and distribution of the breakevens.

Later, in 1965, Sharpe and Lintner had developed his model Mack by including in the portfolio has an asset without risk. Thus, the stock market is in equilibrium when all investors hold a combination of risk-free asset and the market portfolio (consisting of all risky securities available on the market at any given time).

Another important model portfolio theory is William Sharpe, a model developed in 1964. CAPM (Capital Asset Pricing Model) was the first model produced which highlighted the direct link that exists between the profitability of a movable and the profitability of a portfolio which includes all titles available on the market at a time. This model determines the profitability required for any mobiliar risky title.

Followed, in 1973, the publication of another important model in risk analysis, namely the Black-Scholes model, which assesses the price of an option depending on the maturity of the option, the prize received by the seller, the risk-free interest rate, the price of exercise of the option and the standard normal cumulative distribution. The formula used in this method is:

$$C = SN(d_1) - Ke^{-rt}N(d_2), \quad (3)$$

where:

C represents type call option award;

S is the price of the instrument for which it was built option;

K is the option exercise price;

r is the risk-free interest rate;

t is the period of time until the option matures;

$N(d_1)$ and $N(d_2)$ are standard normal cumulative distributions.

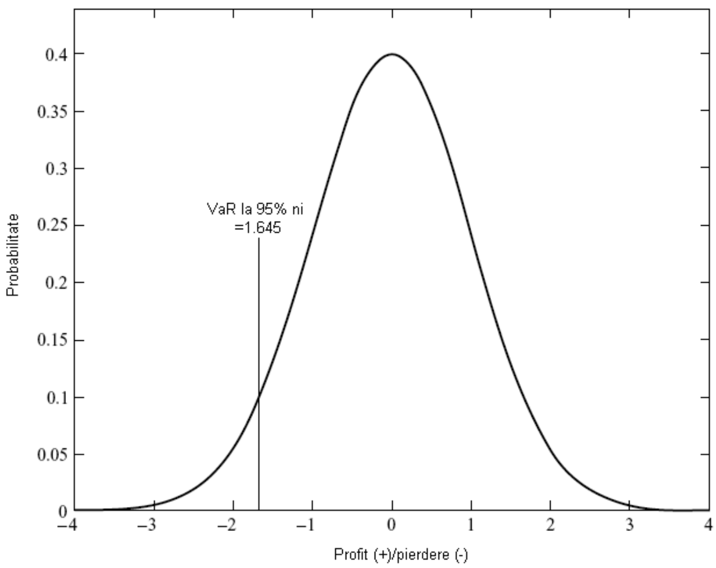
Models of portfolio theory is useful in determining the ways the risks to which they are exposed and allow investors incorporating several types of risks. These models also allow the presentation of how those risks interact with each other. The major disadvantage is that, more often than not, the data contain errors or are difficult to harvest. For example, the CAPM model, the risk-free interest rate and the profitability of the market easily, instead calculating variable beta is complicated because account must be taken both of the risk to the individual title in question but also the overall portfolio risk. Data for an extended period of time are necessary for determining the beta, which most often are not available. As the beta depends on the composition of the portfolio owned, it must be estimated whenever alterations are portfolio.

As a result of all these disadvantages, in the 1970s and 1980s have initiated studies for introducing new models for determining market risk capital market. This has led to the emergence of the risk-measurement model on the stock market using VaR (Value-at-Risk), the most popular method of measurement of market risk. VaR is considered beeing introduced by J.P. Morgan, which has become operational sometime around 1990.

Linsmeier and Pearson, the article written in 1996, have described VaR as „a brief and simple statistics to measure the possible losses of a portfolio. More specifically, VaR is a measure of the losses incurred as a result of normal market changes. Losses greater than the VaR can only occur with extremely low probabilities [...]. Once this is understood, the result obtained is easy to interpret".

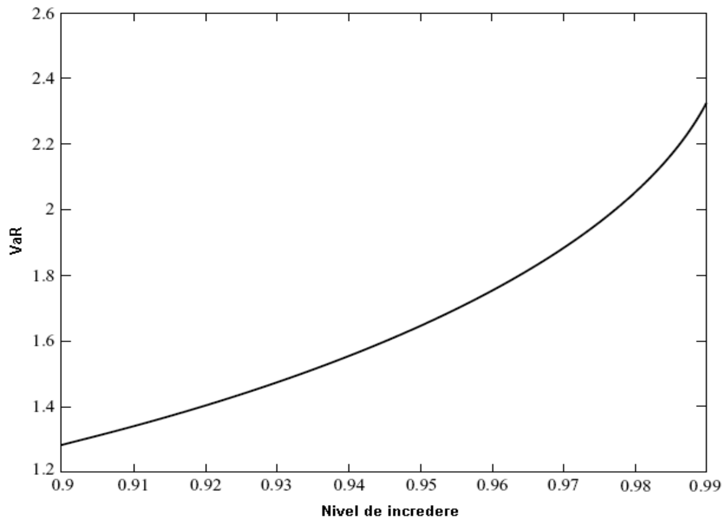
VaR is defined as the maximum loss you can encounter an investor over a specific period of time, at a certain level of probability. For example, at a probability of 95, in 95 out of 100 days, the maximum expected loss is X $. Losses greater than the VaR shall meet with a very low probability, in the example above it is only 5. Lime is shown in the figure below. This is nothing but the probability density function of gains and losses incurred by an investor for a certain period of time at a confidence level of 95. At this level of trust, the amount of interest on the Ox is-1.645, which indicates a loss of possible mazimă 1.645.

Graph VaR determined that the relationship between profit and probability



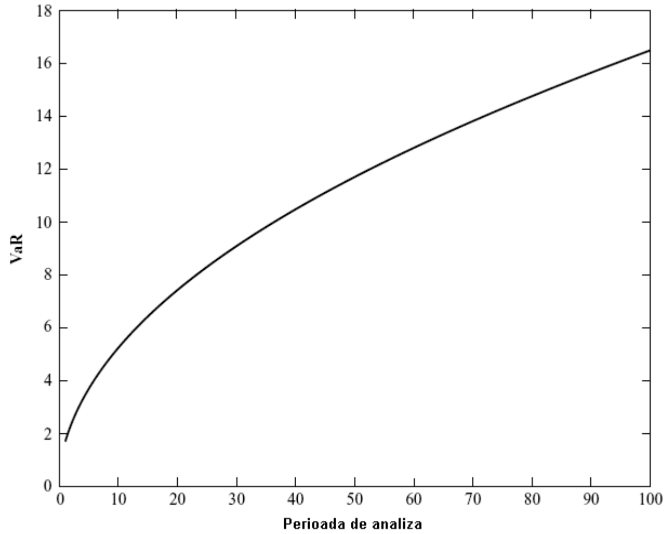
Once with changing levels of trust will change and the maximum loss determined using var. for example, at a confidence level of 99, the maximum possible loss will increase to 2.326, which leads to the idea that VaR will increase with higher levels of trust. Moreover, as noted in the chart below, lime will increase at a rate of increasing the level of trust increases.

Maximum Loss determined using the VaR method in comparison with the level of trust



Another feature of VaR is that this measure will depend on the time horizon for the determination of the maximum possible loss. For a confidence level of 95 and a review period of between one day and 100 days, lime will have a distribution similar to that of the following chart. Thus, the maximum loss will increase with the number of days of radical analysis from 1.645 for a day at 16.449 for 100 days.

Maximum Loss determined using the method, reported in the period of analysis



Between the portfolio theory and VaR models are there so some similarities and important differences.

- Portfolio theory considers the risk as defined by the standard deviation of the portfolio's profitability, while VaR interprets the maximum risk that the probability of a loss.

- VaR method is more general than the models of portfolio theory: If the portfolio theory believes that profitability is normally distributed, the VaR can be adapted and used in various ways for distribution of its profitability.

- Portfolio theory of attention only market risk, while the VaR can be applied in the determination of other types of risks, such as credit risk, operational risk and liquidity risk.

The advantages of using lime for determination of market risk to which it is exposed to an investor are numerous, including:

- VaR is a consistent measure for measuring the risk associated with multiple factors and several types of portfolios (bonds, equities, options, etc.), which means that it allows to compare the results between more investment.

- VaR allows aggregation of individual investment risks for determining risk of an entire portfolio, taking into account the manner in which risk factors are related to each other.

- VaR is a measure involving probabilities and, as a result, investor, Manager or allow to have a complex picture of the probability of loss of the portfolio.

Of course there are problems which may be encountered when using this method of determining market risk and which have been debated by researchers over time:

- Beder (1995) suggested that the estimates obtained using different methods of calculating VaR are very different and are also exposed to the risk of deployment model. If investors take decisions about their investments based on an erroneous calculation of VaR, failures can be considerably higher than those determined by the model, as well as risks.

- VaR disregards how investors react on the capital market. When determining market risk by using VaR model, investors, in particular, traders will try to adjust their positions and strategies so as to take advantage of understated or overstated titles. The VaR will no longer be calculated correctly and the maximum loss probability will be flawed.

- Taleb (1997) showed that VaR can lead to destabilization of the financial system. If we consider that most of those who are interested in the dynamic measure of VaR are traders, which have its coat according to its portfolio of assets to defend against changes in prices of shares in their portfolio, their actions can lead to the appearance of correlation between risks that are assumed to be uncorrelated. This leads again to the idea that companies are exposed to a greater risk than those determined using the method of VaR.

Over time there were found and other limitations of the method VaR. like any statistical model, and lime may be subject to errors of data collection and use the model errors or deployment.

In addition to what was presented above, it should be stressed that the VaR indicates the maximum that the investor can lose 95 of cases. However, there are 5 in cases where the losses are far bigger than those specified by the VaR but are not included in the estimates obtained using the method.

If we use the VaR method to estimate the maximum potential loss for a portfolio returns and distributed normally with the formula used will be:

$$VaR = -\alpha_{ni}\sigma_{c/p} - \mu_{c/p}, \qquad (4)$$

where

$\mu_{c/p}$ represents the average value of gains/losses;

$\sigma_{c/p}$ is the standard deviation of earnings/losses;

α_{ni} is standard normal variable corresponding to the confidence level chosen. For a trust level of 95%, α_{ni} is -1.645.

If instead we believe that there is a normal distribution of gains/ losses from securities portfolio, we believe that rentabilităţile have the normal distribution, with μ_r being the average of breakevens and σ_r being their standard deviation, then the formula used for determining the VaR would be:

$$VaR = -(\mu_r + \alpha_{ni}\sigma_r)P_{t-1}, \qquad (5)$$

where

P_{t-1} represents the value of the assets held in the prior period.

To apply VaR to portfolios composed strictly of shares owned at various companies, but we do not have information on the volatility of the stock price or profitability of the correlation between these dates, we can consider that the yield of interest (R_A) is in direct relationship with the market yield (R_M), by the following relationship:

$$R_A = \alpha_A + \beta_A R_M + \varepsilon_A, \qquad (6)$$

where

α_A represents the company's idiosyncratic risk;

β_A represents the risk coefficient that determines the relationship between the market yield and yield analysis actions;

ε_A represents the regression residual term.

Subsequently, the variance is determined using the acţunilor profitability

$$\sigma_A = \beta_A^2\sigma_M^2 + \sigma_\varepsilon^2, \qquad (7)$$

where

σ_A is the standard deviation of the action;

σ_M is the standard deviation of the market.

From this equation is the formula used for determining the VaR:

$$VaR = -\alpha_{ni}\sigma_M x_A = -\alpha_{ni} x_A \sqrt{\beta_A^2\sigma_M^2 + \sigma_\varepsilon^2},, \qquad (8)$$

where

x_A represents the amount invested in shares of the company in question.

If the investor's portfolio is made up of shares of n companies,

$$VaR = -\alpha_{ni}\sigma_M x_A - \alpha_{ni}\sigma_M x_B - ... - \alpha_{ni}\sigma_M x_N = -\alpha_{ni}\sigma_M(\beta_A x_A + \beta_B x_B + ... + \beta_N x_N) \qquad (9)$$

As seen in the previous pages, the main element that impactează market risk is liquidity instruments traded and the stock market overall. The impact of market risk liquidity is observed in terms of the cost of liquidity (the difference between the asking price and the price offered for a tool) and liquidity risk (the risk that the instrument in question may not be readily saleable).

To include in the calculation of VaR and the impact of the cost of liquidity, Kevin Dowd in his work about the measurement of market risk analyzed the first impact of this cost on the profit or loss as a result of investor holding the portfolio concerned. The cost of the transaction, as is expected, it will increase the volume traded for the instrument in question and with the difference between the asking price and the price offered.

Another impact, this time inversely proportional, it will take into account the period of time during which it is desirable to implement sale: the training period and the sale is over, so the cost is lower because the investor will wait for proper moment to act. As a result, the cost transaction (*TC*) will be defined as:

$$CT = [1 + MI / MP]^{\lambda_1} (VL \times spread / 2) \exp(-\lambda_2 pt) , \qquad (10)$$

where

MI is the size of the package tools;

MP is the size of the market;

VL is the value being wound up at the end of the period of the trading (PT);

spread is the difference between the price and the sale price offered;

λ_1 and λ_2 are parameters larger than zero; λ_1 is transaction cost elasticity in relation to position reported to the amount of the total investor positions on the market; λ_2 is the rate of increase in the cost of the transaction with the increase in the retention period of the instrument.

The first period equation presented above shows the impact of the position held by the investor himself has on transaction cost. So, the investor has less of the total volume of the instrument on the market, this time limit will be closer to 1; otherwise, the value of this term will be greater than 1. The second term represents the effect of the difference between the price and the sale price offered on total cost of the transaction, in respect of the total volume sold during the period of analysis. Last deadline is indirect connection between the preparation period up to implement and the cost sale transaction. On the basis of this formula, we can determine the market risk for any period of ownership of the portfolio, taking into account the cost transaction.

$$CT = [1 + MI / MP]^{\lambda_1} (VL \times \Delta / 2) \exp(-\lambda_2 pt)$$
$$CT = [1 + MI / MP]^{\lambda_1} [(MI - LVaR) \times \Delta / 2] \exp(-\lambda_2 pt) \qquad (11)$$

$LVaR$ it is determined as Var calculated without taking into account the cost of liquidity, in which it would add the cost transaction, such as

$$LVaR = VaR + CT \qquad (12)$$

Considering *k* being positive cost of the transaction, without taking into account *VL*, that is:

$$k = [1 + MI / MP]^{\lambda_1} (\Delta / 2) \exp(-\lambda_2 p), \qquad (13)$$

we can express $LVaR$ this way:

$$LVaR = \frac{VaR + kMI}{1 + k} \qquad (14)$$

what can be filmed in

$$\frac{LVaR}{VaR} = \frac{1 + \frac{kMI}{VaR}}{1 + k} \qquad (15)$$

In conclusion, the impact on transaction Var largely be dependent on the k and the raport MI / VaR, which will always be greater than 1.

Another model for the determination of market risk taking into account the cost of liquidity has been developed by Bangia. in the year 1991. This model assumes that the position held for analysis is very small compared with the volume whole market. In this case, stock trading stake will not affect in any way the general liquidity of the capital market, that is it will be independent from the reprisals.

In this case, Bangia took the view that the risk of liquidity take account only of the difference between the price and the sale price offered (spread) and its volatility. If we use a confidence level of 95, considering that the difference between the price and the sale price offered for the portfolio of shares held is an average over μ and volatility σ, and the data set is normally distributed, then it can be determined that the closing price of the shares will not exceed $((\mu + 1.645\sigma)/2)$ of the total portfolio owned and sold. In this case, $LVaR$ can be calculated using the formula below, with appropriate values for medium and for volatility.

$$LVaR = [1 + (\mu + 1.645\sigma) / 2]VaR \quad (16)$$

Using other confidence interval will obtain other values of liquidity impact on price transaction.

At the time of the desired positions of winding-up of a portfolio, the investor shall bear the cost of transaction which will have a direct effect on profitability a result of the operation. In the case in which we consider in the analysis and impact of this cost, the value Var will undergo some changes. The cost transaction will be all the greater the volume of assets held for sale increases.

This is also the case in principle as a result of the effect of a large volume of assets put up for sale has on potential buyers and the liquidity asset in question - in this case there is an adverse reaction to potential investors who do not have enough information and as a result They are leaking in relation to the way in which it ought to levy sale of a large number of shares, for example.

On the other hand, if the investor is willing to prepare for sale an extended period of time, waiting just for the proper moment for trading, transaction costs can be reduced considerably. As a result, transaction cost, *CT* can be defined as :

$$CT = [1 + MT / MP]^{\lambda_1} (PL \times \Delta / 2) \exp(-\lambda_2 \times PD) \, , \quad (17)$$

where

MT is the transaction size;

MP represents market size assets concerned;

PL the position size is wound up at the end of the period of analysis, which is in fact the retention period of the assets, *PD*

Δ is the difference between the price and the sale price offered;

λ_1 it is calculated as price elasticities transaction cost ;

λ_2 represents an indicator in the modification of the trading cost as the retention period of the assets increases.

Writing the equation above in terms of LVaR, this wouldn't be enough to be expressed as follows:

$$CT = [1 + MT / MP]^{\lambda_1} ((MT - LVaR) \times spread / 2) \exp(-\lambda_2 \times PD) \text{, where}$$
$$LVaR = VaR + CT \quad (18)$$

As a result, $LVaR$ can be defined as

$$LVaR = \frac{VaR + kMT}{1 + k} \text{, where}$$
$$k = [1 + MT / MP]^{\lambda_1} (spread / 2) \exp(-\lambda_2 \times PD) \quad (19)$$

In conclusion, in the case in which the cost transaction is relatively low, k it will be approximately zero, which means that the cost of the transaction will not impacta significantly the value of Var. If, on the other hand, k will have a high value, lime may be affected by up to 50 of his value calculated without taking into account the transaction costs.

If we want to analyze the way in which react market in the case of transactions with derivative, we can utilize the Krakovsky method used in the work of 1999. The method determines the impact on liquidity title for which are traded derivative in question.

The first step in using this model is building a variables of liquidity, L, as reverse partial derivation of the price title on the basis of which are traded derivative, S, in relation to the tranzactioned quantity, N. So, $L = 1/(\partial S / \partial N)$. At the same time shall be considered as derived price of action shall be determined according to the following formula:

$$\partial S = \mu dt + \sigma dx + 1/LdN \quad (20)$$

If liquidity is very high, the last term tends to zero and do not have any impact on this equation. Thus, if we are interested in determining call or put options normal, using formula Black-Scholes , we come to the following equation:

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{\sigma^2}{2\left[1 + \frac{1}{L}\frac{\partial^2 V}{\partial S^2}\right]^2}\frac{\partial^2 V}{\partial S^2} - rV = 0, \quad (21)$$

where

V is the value option;

r is the rate of interest without risk.

The difference between Black-Scholes formula and the one above is of the existence of which includes liquidity. After calculation of the value option taking into account its liquidity, for determining VaR, Krakovsky used Monte Carlo method.

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