THE STRUCTURE OF ECONOMETRIC MODELS USED BY MACRO MODELLING CENTERS

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Abstract

When modelling a national economy, complex economic and social processes must be taken into account, as well as the relationships between economic agents entering into economic systems. Systems are composed of individual economic agents or their interconnected bodies, such as their organizations. Based on the major functions played in economic processes, the following groups of economic agents can be distinguished: households, enterprises, public institutions (including financial ones) and foreign agents. Different types of markets are also distinguished - goods and services markets, labor markets and money markets. Sellers and buyers in markets also participate in barter transactions, which are followed by flows of goods and services, price adjustments, and financial flows. The criterion that is most widely used emphasizes the predominant type of economic activity, leading to the distinction of sections and industries, such as agriculture or manufacturing. By investigating the activities of the bodies of economic agents, knowledge of the mechanisms underlying their operation and growth can be expanded. This is an important area of empirical analyzes based on macroeconometric models. Macroeconometric models were initially built using mainly classifications that distinguished the type of economic activity (SCN). A more recent trend is the application of the type of economic agent as the primary criterion, other criteria being given a secondary role.

Key words: *economic processes, economic agents, market, supply and demand, econometric models.*

JEL classification: *C10, E10*

Introduction

In this article the authors set out to describe how the equations of macroeconomic models are specified, including consumer demand and investment demand functions, production functions, and equations explaining

prices, wages, and financial flows. In each case the underlying economic theories will be summarized, detailed presentations of which are available in mathematical economics monographs.

This description will be accompanied by a presentation of empirical research results based on macroeconomic models. The results, most frequently in the form of estimates of the respective elasticities, will be presented for the major countries of the world to show the degree to which the macro-characteristics of the behavior of economic agents have been approximated and stable. This explains why we refer to country-specific macromodels to describe the specifications of the equations, but in characterizing the estimation results we use the results obtained from the multinational world models.

The construction and use of macroeconometric models are related to many other important aspects such as estimation methods of equation parameters, especially those involving cointegration analysis, or numerous tests that are widely used by authors of contemporary macroeconomics (metric models). Also, descriptions of the many model applications, including the results of multiplier analyses, policy simulations, and forecasts, will not be provided. These issues require special discussion and more space, which can be found in many monographs and contributions to world literature.

It is not feasible to define what a typical structure of a macroeconometric model is, because several types of models have been developed in the past, having different theoretical bases and various arrangements of the specified equations. They were predominantly demand-driven Keynesian models, called the mainstream. There are advanced studies in theoretical and applied econometrics that shed light on CGE models, which are static and partially deterministic.

This class of models has been generalized, taking advantage of modern business cycle theory, which has led to the development of dynamic stochastic general equilibrium (DSGE) models. DSGE models distinguish within the manufacturing sector between producers of intermediate goods (including domestic producers) and producers of final goods, which also include suppliers of domestic and imported goods.

The specification of the equations that explain the profit maximization (or cost minimization) behavior of enterprises can be discussed. Except for the equations explaining changes in inventories, the most important role is played by production functions. The functions are used in generating production potential and production stocks and in constructing functions that explain the demand for factors of production. Investment demand functions are determined, as are job demand functions.

Literature review

Boonman H. et al (2023) quantify the macroeconomic and environmental impacts of an innovation-driven shift to a circular economy, with particular focus on distributional impacts across regions and sectors. Gorenflo R. et al. (2014) are concerned with a complete and self-contained description of all aspects of the theory and application of Mittag-Leffler functions. Thus, the introduced functions are very useful in solving differential and integral equations of fractional order. Huifeng C. (2023) approaches a model that links standard measures of bond illiquidity, such as bid-ask spreads and parameters that drive bond illiquidity at the microeconomic level, with macroeconomic variables. Tarasov V.E., Tarasova V.V. (2017) made in their study a discrete-time self-consistent description of economic accelerators, which is based on exact finite differences. Thus, for the discrete-time approach, the exact difference model equations have the same solutions as the corresponding continuous-time models, and these discrete and continuous models describe the same behavior of the economy. Tarasov V.E., Tarasova V.V. (2016) highlighted the fact that economic processes with long and short memory in continuous time, which are characterized by the power law, must be described by fractional differential equations. Pokrovskii V.N. (2012) review the theory of commodity production, thereby providing the tools both for consistent interpretation of empirical data and for sketching development scenarios. Vinante C. et al. (2021) consider in their paper the evaluation of the circular economy at the company level, thus highlighting the fact that the available literature reviews do not clarify what and how to actually evaluate, while many evaluation methods they do not take into account the latest developments in the field.

Data, Results and Discussion

Quantitative and price adjustment mechanisms of market clearing are considered in this paper. Quantitative adjustments include changes in inventories and changes in the rate of capacity utilization, exports and imports. Also, through specific models, aspects related to the modeling of balances and imbalances on the markets of goods and services and on the labor market are treated.

The theoretical descriptions are supplemented by the introduction of equations that explain the demand of public institutions for consumer goods, the demand for changes in stocks and foreign trade. Also considered are changes in the labor market, which determine unemployment rates, along with the characteristics of the NAIRU and NAWRU.

We have presented in this study how prices of final goods, prices of factors of production and average wages are determined in the public sector,

thus concluding the description of the price and wage system. Finally, the major feedbacks (multipliers) from the household and business sectors are demonstrated.

Financial flow modelling presents equations that explain income, expenditure and balances (savings), which have been constructed for institutional agents, for example households, businesses, public institutions (mainly the state budget) and foreign agents (using the balance of payments). The specifications of the equations that explain direct and indirect taxes are also addressed. The linkages between institutional agencies and linkages with real sectors are also demonstrated.

The equations that explain money and credit markets are distinguished and discussed separately. Loans for certain groups of agents and their bank deposits are explained. The money request function is specified. The equations that explain interest rates are shown. Particular attention is paid to decisions by central banks to adjust interest rates to allow the inflation target to be reached. The specification of the exchange rate is discussed at length, and the analysis concludes with examples showing interrelationships between inflationary processes and specific interest and exchange rates, analyzed by the cointegration method.

• Classifications of the national economy in macro models

Economic agents are usually classified into macroeconomic models based on international statistical criteria and rules. They derive from the system of national accounts (SNA), which was developed at the UN Statistical Office as a result of the pioneering efforts of R. Stone and has been used worldwide since the 1990s.

We remind you that until the end of the 1980s the former centrally planned economies (CPIs) operated a special system of balances of the national economy, called the Material Production System.

In the above systems, private economic agents are classified according to their predominant economic activity. In the enterprise sector, agents are usually enterprises (often carrying out various activities) rather than factories, which are technologically homogeneous units of a uniform profile. The SCN has recently been extended to account for socio-demographic processes. Since processes have been commonly presented as a flow matrix, the extended system is called a social account matrix (SAM).

Early macroeconomic models used activity classifications close to the SNA and emphasized macroeconomic orientation. Accordingly, mainstream models have typically distinguished between real and financial processes (flows), linked by (broadly understood) price adjustments.

Actual processes typically include:

• generating gross domestic production (GDP),
• production of goods and services,
• employment and the workforce,
• fixed capital and its replacement,
• The distribution and use of GDP, followed by the generation of final demand,
• export and import of goods and services,
• home consumption,
• the consumption of public institutions,
• investments in fixed capital and changes in inventories.

Factors of production can be further classified by prevailing technology, raw materials used, etc., as well as broken down into protected and open sectors. A more detailed breakdown of consumption, investment and foreign trade is also possible using common commodity classifications.

Financial processes include:

• current transactions of economic agents:
  - income (income),
  - expenses (costs with the user),
  - surplus or deficit,
• capital transactions of economic agents:
  - assets and their changes,
  - liabilities and their changes,
  - balances of capital transactions,
• wages and prices, including exchange rates and interest rates.

Certain types of financial flows are generally specific to different types of economic agents, so they are modelled separately for different institutional sectors.

In some analyses, economic agents are assigned to sectors based on the theoretical support accepted by the authors of the models, which postulate the optimizing behavior of economic agents rather than institutional criteria. This type of classification can be illustrated using a grouping of economic activities.

Households:

• consumption
• residential investments
• labor supply
• financial assets (partially distinguished in the financial sector)

Enterprises:
- producer prices
- production
- fixed capital and investments in fixed capital
- employment, number of employees and hours worked
- wages in the business sector.

Further disaggregation is possible and then criteria based on the type of economic activity are mostly used. For example, consumption can be broken down into consumption of durable goods, non-durable goods and services, production can be divided into agriculture, manufacturing, etc.

Markets are classified by:
- goods and services
- the work
- money.

A deeper breakdown of markets is usually achieved through special classifications such as SITC in foreign trade. Models that characterize the behavior of markets are usually composed of systems of equations, which explain the demand from potential buyers, the supply from potential sellers, and the clearing prices of the markets. If the interconnections in a market subsystem overlap the entire national economy, then computable equilibrium (CGE) models can be constructed. Developed at the World Bank since the 1980s, CGE models have spread throughout the world.

• **Macroeconometric models**

Econometric models of national economies belong to the class of econometric models that have reached the highest degree of excellence. Certain countries operated in a state of relative isolation, implementing their own national economic policies and using uniform socio-economic statistics. Macroeconometric models from various countries have been used not only to conduct macroeconomic analysis, but also to conduct sector studies, analysis of particular markets, analysis of financial processes, etc.

The limited availability of statistical data has prevented regional models from being widely used in some countries. On the other hand, however, multi-country models have gained importance in recent years. This applies in particular to world economy models, which contain separate models for the main industrialized countries, as well as models of distinct regions in the rest of the world.

A special class of macro-models are input-output models, which largely represent intra- and/or cross-sectoral commodity flows and related

price systems. They can be linked to econometric sub-models that explain, on the one hand, the generation of income and the resulting final demand and, on the other hand, the generation of output and the factors of production. This provides the basis for building integrated models of the national economy.

In the construction of macroeconometric models, alternative assumptions can be used regarding the major economic mechanisms that define specific economic regimes. The first group of models consists of a system in which demand and supply of goods and factors of production effectively adjust to respective price and wage changes. These are the models of the national economy in equilibrium. CGE models belong to this class of models. The class can be coherently represented by the following system of equations:

• a demand equation y_t^c:

$$y_t^d = c(p_t \dots) \tag{1}$$

• a supply equation y_t^o:

$$y_t^o = o(p_t \dots) \tag{2}$$

We believe that demand equals supply ($y_t^d = y_t^o = y_t$).

The solution is given by the market clearing price (p_t^*), mean $p_t = p_t^*$.

The above models are rarely built in their pure form because the assumption that prices are the only instruments of market adjustment is rarely met and the adjustments are not necessarily efficient. There are rigidities in price and wage behavior as well as cost adjustments. This neo-Keynesian view has been accepted for dynamic, stochastic general equilibrium (DSGE) models.

Most macro models assume, however, that the prevailing economic mechanisms involve quantitative adjustments, such as changes in inventories, changes in levels of capacity utilization and employee working time, and in exports and imports. Price adjustments, if they occur, are either late or minor (this does not mean that prices are constant, but that they change mainly due to changes in unit costs).

In the real market, the above adjustments are rarely effective. There are several markets where the equilibrium condition is not met. Macro models that assume that such markets exist are called broad disequilibrium models. Imbalances can take different forms and occur in different markets.

The first to be distinguished are the macro models that assume an excess supply of factors of production, that is, with fully unused production capacities and unemployment, and also with a likely excess supply of goods and services. If these conditions are met, it is reasonable to assume that supply

follows demand. For this reason, the models are called demand-driven or demand-driven. Thus, we can write the relation:

$y_t^o > y_t^c \Rightarrow y_t^o \rightarrow y_t^c$ and $y_t^c = y_t$ (3)

where: $y_t^o - y_t^c = y_t^o - y_t$ is oversupply.

If the supply of factors of production or of goods and services (or foreign currency reserves) is tight, then market transactions will represent supply realizations. These models most commonly contain not only supply equations, but also estimates of excess demand and their respective imbalances. They are called supply-constrained models or supply-driven models. They can be presented as:

$$y_t^o < y_t^c \Rightarrow y_t^o = y_t \quad (4)$$

where: $y_t^c - y_t^o = y_t^c - y_t$ is excess demand.

Disequilibrium conditions may actually differ between sections and markets. The simplest approach is to assume that the minimum condition holds:

$$y_t = \min (y_t^c, y_t^o) \quad (5)$$

In the real market, quantity and price adjustments occur simultaneously, but their intensities are different. The efficiency of these adjustments, especially in the short term, is market specific. Adjustments are frequently assumed to be efficient in goods and services markets, that is, where demand equals supply.

This follows from the assumption that, due to underutilized production capacities, unemployment and sufficient foreign exchange reserves, changes in demand will cause short-term adjustments in supply. The adjustments will not involve significant changes in prices (excluding commodities characterized by rigid supply, such as agricultural crops).

The above conditions, marked by idle production capacity and long-term unemployment, are described by the following class of models representing the Keynesian unemployment regime:

$$X_t^o \geq X_t^c = X_t \quad N_t^o > N_t^c = N_t \text{ and } K_t^o > K_t^c = K_t \quad (6)$$

where:

X_t = output (GDP),

N_t = labor force,

K_t = fixed capital.

In conditions characterized by excess demand for goods and services, two economic regimes are generally distinguished. In the classical

unemployment regime, it is assumed that entrepreneurs will not decide to produce enough products to satisfy demand, unless they consider it profitable. Then, restrictions in satisfying demand are followed by employment restrictions. Thus:

$X_t^o < X_t^c \Rightarrow X_t^o = X_t$ and $N_t^o > N_t^c = N_t$ and the most common $K_t^o > K_t^c = K_t$ (7)

According to frequent developments, excess demand is caused by insufficient supply resulting from the lack of one of the production factors. This economic regime is called a suppressed inflation regime to emphasize the unsatisfactory efficiency of price adjustments. If the supply of labor is restricted, then excess demand affects both the goods and labor markets, described by the relationship:

$X_t^o < X_t^c \Rightarrow X_t^o = X_t$ and $N_t^o < N_t^c \Rightarrow N_t^o = N_t$ (8)

Within the above regime, other sources of deficit (of insufficient supply) can be distinguished, such as restrictions in the supply of energy or raw materials which on a macro scale can be attributed to limited foreign exchange reserves. These conditions were typical of former centrally planned countries.

In practice, pure forms of developing country regimes are quite rare, so modelers construct mixed models assuming the coexistence of different regimes.

$X_t = C_t + G_t + J_t + (E_t - M_t)$ (9)

The basic behavioral and technological relationships were as follows:

- the consumption function $C_t = c(Y_t, r_t, C_{t-1})$ (10)
- the investment function $J_t = j(X_t, r_t, K_{t-1})$ (11)
- export function $E_t = e(WT_t, p_t^w/p_t, E_{t-1})$ (12)
- the imports function $M_t = m(X_t, p_t/p_t^m, M_{t-1})$ (13)
- employment function $N_t = n(X_t, N_{t-1})$ (14)
- producer price equation $p_t = p(w_t, N_t/X_t, p_t^m)$ (15)
- average wage equation $w_t = w(u_t, p_t)$ (16)
- population activity equation $N_t^o / L_t = n(u_t, w_t/p_t)$ (17)
- the money demand equation $M_t^c = m(Y_t, p_t, r_t)$ (18)

where:

- the endogenous variables are:

C_t = household consumption (constant prices),
E_t = exports (constant prices),
J_t = gross investment (constant prices),
K_t = fixed capital (end of period, constant prices),

M_t = imports (constant prices),
N_t = labor force,
N_t^o = labor supply,
p_t = producer prices,
p_t^m = import prices,
u_t = unemployment rate,
w_t = nominal average salary,
Y_t = real disposable household income,

• the exogenous variables are:

G_t = the real expenses of public institutions,
L_t = population size,
WT_t = global world exports,
p_t^w = world prices.

The consumption function represents a Keynesian orientation.

Consumption is mainly determined by real household disposable income. The lagged consumption introduced due to inertia (the Brown effect) can also be interpreted as a summary characteristic of the lagged distribution of real incomes. The interest rate explains changes in consumption caused by changes in savings.

The investment function is a version of the flexible acceleration function. It is assumed that the desired level of fixed capital K_t^* is determined by the level of output and the interest rate

$$K_t^* = k_t(X_t, r_t) \quad (19)$$

and that fixed capital expands in proportion to the difference between the desired and actual stocks of fixed capital:

$$\Delta K_t = \lambda(K_t^* - K_{t-1}) \quad (20)$$

Thus, the investment will be equal to the amount of the fixed capital increase and its depreciation $D_t=d_tK_{t-1}$:

$$J_t = \Delta K_t + D_t = i(X_t, r_t, K_{t-1}) \quad (21)$$

In the stylized model, it is assumed that the demand for public investment is exogenous.

Foreign trade equations are more or less standard. Exports depend on world demand and imports on total domestic demand. In both cases demand is adjusted accordingly by relative prices.

The employment function is most commonly generated from the inversion of the production function. Gaps play an important role in employment adjustments.

Financial moments are an equation that explains money in terms of real income, prices, and the interest rate. The remaining components of financial flows, including revenues and expenditures of the state budget, are explained by the relevant identities.

• **Microeconomic foundations of model specification**

At the beginning, attention was paid to the fact that the macroeconomic relationships presented in the models must be specified respecting the microeconomic fundamentals. Following neoclassical theory, households and firms were assumed to optimize their activities. Households maximized utility and firms maximized profits (or minimized costs) under conditions of imperfect competition.

Households' consumption, investment (residential) and labor supply functions were derived from utility maximization. By solving the profit maximization problem of enterprises, the equations explaining the demand for factors of production, the producer price equations and the wage equations can be constructed, assuming predetermined production functions.

The specification of the equations differs, because in addition to flows, stocks available to economic agents have been introduced. In particular, according to the permanent income hypothesis (Friedman), the consumer demand function was expanded by introducing a variable that represented the stock of personal wealth of the household Vt (containing initially financial assets, but later also physical assets, mainly apartments and residential buildings) . Later, real income from labor in the future, called human wealth, was introduced, following the long-standing Modigliani hypothesis.

The demand for factors of production was specified taking into account the impact of replacing labor with machinery and equipment. In the investment function, the costs of using investments were introduced as an additional variable (their major component being the interest rate). This made it possible to determine the expected returns on investments. In recent years, special attention has been paid to the need to allow for the costs of installing new equipment, which have involved corresponding delays in the investment process. Attempts have been made to use Tobin's Q concept.

In specifying the occupation function, real wages or wage-to-profit ratios were entered.

• **Modelling of the offer**

The original specification of the structural equations generally assumed that, as a result of market transactions, the demand for goods and services and the demand for factors of production were satisfied. It was therefore assumed

that the supply of goods and services and the supply of factors of production followed demand. For this reason, the supply functions were not explicitly specified in the macromodels. An exception was the labor market where the labor supply was generated from a separate equation, which made it possible to estimate the unemployment rate, which characterizes the imbalance in this market.

The possibility of frictional imbalances in goods and labor markets attracted attention quite early. Imbalances could be removed by stock adjustments, so some models were fitted with separate equations explaining changes in final goods stocks or by adjustments to exports or imports. In the latter case, the characteristics of potential demand gaps had to be calculated, which were also used to determine producer prices.

These features can be built in different ways. The most commonly used are capacity utilization ratios WX_t, which can be obtained from appropriate surveys or calculated from production trend deviations.

$$WX_t = X_t / X_t^p \quad (22)$$

where:

X_t = effective outputs

X_t^p = potential outputs

Production functions were explicitly specified in only a few annual models, and the impacts of technological progress were exogenous (largely represented by an exponential trend). The prerequisite for introducing new specifications with endogenized total factor productivity (TFP) was the development of endogenous growth theory. TFP growth was assumed to be dependent on an increase in knowledge capital represented by human capital and cumulative R&D expenditures, both domestic and foreign. The above functions were most commonly used in disequilibrium models and long-run models.

The allocation of goods, including imports, was described by supply functions defined for certain groups of intermediate and end users. In most cases, export supply followed external demand, while domestic consumption demand and, in particular, investment demand were streamlined.

Chronic unemployment present in industrialized countries has been the subject of numerous studies involving the use of disequilibrium models.

Extended disequilibrium models are used to describe the functioning of centrally planned economies, especially during the period when chronic imbalances prevail in goods and services markets.

In the supply sectors of the early demand-driven models, the decisive role is played by the wage and price equations. The specification of wage

equations has been modified to account for real wage levels as functions of the short-run unemployment rate and long-run labor productivity. Taking into account the fact that wages play a special role in the formation of prices in imperfect markets, the concept of the non-accelerating rate of unemployment (NAIRU) was formulated as an alternative to the concept of the natural rate of unemployment.

With the relaxation of international financial flows and the associated abandonment of control over exchange rates, research attempts have been made to endogenize their development. The most significant has become the theory that assumes that the exchange rate based on purchasing power parity (PPP) varies according to changes in the ratios of interest rates in the compared countries (uncovered interest parity UIP) followed by the situation of capital flows and changes in the interest rate premium risk. Subsequently, attention was paid to the formation rigidities of wages, prices and costs related to the institutional environment. This neo-Keynesian perspective is generally adopted in DSGE models.

Conclusions

From the study done and presented in this article, a series of theoretical and practical conclusions emerge. First, the classifications of national economies in macro models were extended into models that introduced dynamic and stochastic relationships, i.e. into dynamic, stochastic general equilibrium (DSGE) models. In particular, domestic and imported goods were clearly distinguished between the commodity flows of the models. Domestic production was carried out in the intermediate goods production sector, and domestic and imported goods were combined in the final goods production sector, being in fact a representation of the domestic trade sector.

Another conclusion is that early developing country models assumed that external demand was met, while supplies to domestic markets were residual, due to deficiencies affecting domestic production potential.

Another conclusion is that the system of equations presented in this paper contains all the major and characteristic feedbacks of macroeconometric models. The expansion of real disposable income is followed by a corresponding increase in GDP and real incomes, accounting for employment lags and predetermined wages.

Last but not least, the specification of equations that explain the supply of certain groups of goods and services could be found in general equilibrium computational models, where equilibrium prices and supply-driven models are to be determined, assuming the realization of supply and the presence of excess demand in the markets. Also, supply-driven models

have usually been built in countries with centrally planned economies and in developing countries that are in the early stages of their development. The key role in these models was played by the production functions of industry and agriculture, generating the supply of goods.

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