STATISTICAL MODELS USEFUL FOR MEASURING MARKET PERFORMANCE

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Abstract

In this article the authors aim to highlight, based on a proper study, how the performance of a portfolio or set of portfolios should be measured when placing them on the market. Starting from the fact that the capital market has a sufficient degree of volatility, the investor may consider as important a few elements of which we mention first of all the risk reduction for efficient portfolios. In this regard, any investor aims to invest effectively in the capital market. He must make a study on volatility, that is, the standard deviation of the rate of return, in order to know whether out of all the portfolios that seem to be effective, the one that responds best to the reward /volatility ratio must be chosen. It is based on the fact that inadequately diversified portfolios always have a low rate of return. In order to assess the performance of a single security or portfolio that constitutes only a part of an investor's holdings, it is necessary that volatility is only to some extent the actual way of determining the risk actually incurred. Combining securities or portfolio with loans can be reached any point along a line that we consider drawn by the points of incidence presented graphically. The relationship between the characteristic line of a security or portfolio in relation to reward/ volatility is the key element that can underlie the decision that the investor makes. In determining these aspects that are very important when choosing the portfolio or a line that it has to follow is made, we have used an appropriate methodology on the use of statistical-mathematical equations, which shows how this portfolio return evolves when it is placed on the capital market. It is also important to establish and, in this regard, we use the statistical variables, how the usefulness of placing the portfolio according to the results and the interest rates for risk-free commitments. We used graphical representations to more clearly illustrate how measuring portfolio performance is appropriate and to suggest how investors should act for the anticipated study.

Keywords: *portfolios, performance, variables, investor, capital market*.

JEL classification: *C10, G10*

Introduction

The measurement of portfolio performance starts from the fact that in the process of placing portfolios of assets on the capital market, successive measurements are made, illustrating the efficiency with which these portfolios are placed on the capita market.

Of course, in many cases the risk is ignored or treated inappropriately, which in the end manifests itself in a low yield or non-compliant with what the investor has enshrined. The question arises of establishing the best portfolio context in which it starts from the average yield of a portfolio previously considered good in terms of variability with another portfolio that was better aiming to establish what is the ratio between the two and choose the steep line that has indicates the performance of the portfolio with the best return.

Portfolios will be located along a line, but they will all have the same ratio to consider, i.e. reward/variability. Portfolios may appear ineffective, but after the pronounced and deep study we are talking about can have an efficiency, and sometimes even a high one.

The reward/variability ratios of portfolios may more or less coincidentally vary around an associated value determined by the capital market. The reward/variability ratio is designed to measure the performance of a portfolio. In order to evolve the performance of a security or portfolio, it is necessary to use a different measure. In theory, variability does not really represent the risk that the investor can bear. Given the definition of safe market line, slope can be considered the price of the share reduction. For any such case the security or the portfolio that complements the portfolio line we use an equation for measuring it.

The real interest rate also has a particular influence on the purpose of the portfolio or assets placed on the capital market. In this sense we are talking about a line associated with a value of a portfolio resulting from calculations and then the graphic representation that the steeper the reward / volatility ratio, the more this indicates a high return for the portfolio or the assets placed on the capital market.

If we take into account the relationship between the characteristic line of a security or portfolio in relation to the reward / volatility, we will be able to find that three special situations arise as graphically presented. The line shall pass through the point where both yields are at least equal to their average values and the slope determines by effective covariance the actual change in profitability of the market portfolio.

The ability to borrow is considered by investors to yield results only if it emerges from the south carried out that it is going through a point where variants intersect. That is the point that we can raise.

We can appreciate the fact that the reward/volatility ratio can be viewed as a constant that places itself at a reference point through which all portfolios under analysis pass. Of course, a positive differential yield shows that the performance will be superior to that of a market-based portfolio with considered validity. Differential yield is closely related to this reward/volatility.

Summing up, we find that the reward/volatility ratio can be used to compare securities or portfolios with each other in order to choose the optimal option that brings the best return that the investor anticipates.

Literature review

The issue of optimal placement of asset portfolios on the capital market is a topical one and has been studied by many researchers. Thus, Amuer H.B. and Prigent J.L. (2010) were concerned with the study of structural portfolio management. Armeanu D. (2008) was concerned about the profitability and risk of the portfolio consisting of two securities. Baule R. (2010) presented a paper on the selection of the optimal portfolio for a small investor, given the risk and cost of the transaction. Buraschi A., Porchia P. and Trojani F. (2006) were concerned about the correlation of risk with the optimal portfolio chosen. Cox J. and Huang C.F. (1989) analyze optimal consumption and the conditions for placing portfolios according to the share price. In 2011 Geromichalos A., Simonovska I. analyzes a number of aspects regarding the liquidity of assets in the formation of international portfolios. Harvey, C. R. and others (2010) turned their attention to analyzing Portfolio Selection with high moments. Li J. and Smetters K. (2011), analyzed a number of aspects related to the choice of the optimal portfolio in the context of ensuring the indexation of social security. In 2012 West K.D. makes a presentation on the econometric analysis of the use of a model when the reduction factor is close to one.

Methodology, data, results and discussions

Many measurements of previous performance have been proposed. Most either ignore the risk entirely or treat it inappropriately.

Let's return to the definition of the capital market line. Its slope can be considered the price of risk reduction for efficient portfolios:

$$r_e = \frac{E_M - p}{\sigma_M}$$

where r_e= risk reduction price for efficient portfolios
E_M = expected rate of return of the market portfolio
p = pure interest rate
σ_M = standard deviation of the rate of return of the market portfolio

All of these values are about predictions. To measure the performance of the market portfolio, actual values must be used. An analogous relationship is:

$$\frac{A_M - p'}{\sigma_M'}$$

Where: A_M = average rate of return of the market portfolio
p' = the actual pure interest rate
σ_M' = variability (standard deviation of the effective rate of return) of the market portfolio

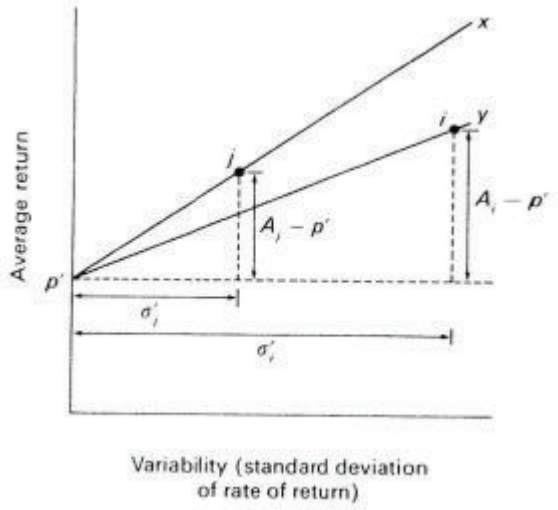
Figure 1 illustrates variability. The point indicates the actual interest rate for risk-free commitments. Points $p'i$ and *j* represent the performance of portfolios *i* and **j**.

The question arises of establishing the best portfolio.

In terms of average return, *portfolio i* was better, but in terms of variability, the *j* portfolio was better. By combining loans or loans with investing in the j-portfolio, an investor could have reached any point along the *p'ix* line. By combining loans or loans with investments in *portfolio i*, he could have reached any point along the *p'iy* line. But the *p'jx* line dominates the *p'iy* line. Given the ability to lend or borrow at the *p'* rate, the performance of portfolio *j* was significantly superior to that of portfolio *i*. A natural measure of performance is thus the slope of the line associated with the portfolio. But the slope of the right is the reward-variability ratio, as in Figure 1. The higher the ratio, the steeper the line and the better the performance of the portfolio.

Variability (standard deviation of the rate of return)

Figure 1



Now we consider a world in which $A_p = E_p$, $\sigma'_p = \sigma_p$ and $p' = p$. Under the effect of these conditions, all portfolios that appear to be effective, including the market portfolio, will prove to be effective.

In a chart of the type shown in Figure 1, all such portfolios will be located along the same line. They will all have the same reward-variability ratio. Inadequately diversified portfolios will all have lower rates.

Portfolios that appear to be ineffective may prove to be effective. Even within highly diversified portfolios, reward/variability ratios may differ considerably.

These conditions, the reward-variability ratios of highly diversified portfolios will vary more or less randomly around the value associated with the capital market line. Reports for other portfolios will vary more or less randomly around lower values. Persistent differences between reward-variability ratios will only occur in cases involving inappropriately diversified portfolios.

The reward-variability ratio is designed to measure the performance of a portfolio. It is assumed that the investor has placed a substantial part of his wealth in the portfolio in question. Variability is thus the relevant measure of the level of risk actually incurred.

In order to assess the performance of a single security or portfolio that constitutes only part of an investor's holdings, a different measure is

required. Variability will not actually represent the risk actually incurred. A more appropriate choice is volatility.

We start from the definition of the safe market line. Its slope can be considered the price of reducing the risk for the securities. For any security or portfolio that is plotted along the line.

$$r_s = \frac{E_i - p}{b_i}$$

Where: r_s = the price of the risk reduction for securities

E_i = expected rate of return of the title or portfolio *i*

p = pure interest rate

b_i = volatility of the title or portfolio *i*

The appropriate measure of past performance is the reward-volatility ratio, namely:

$$\left(\frac{r}{b}\right)_i = \frac{A_i - p'}{b_i'}$$

Where: = $\left(\frac{r}{b}\right)_i$ the reward-volatility ratio of security or portfolio *i*

A_i = average rate of return of the security or portfolio *i*

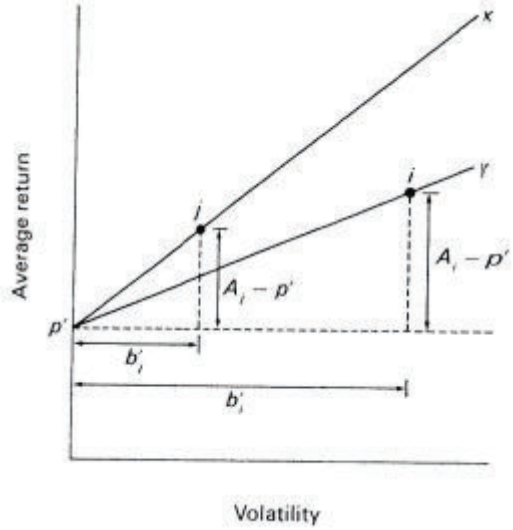
p' = the actual pure interest rate

b_i' = actual volatility of the title or portfolio *i*

Figure 2 illustrates the usefulness of this approach. Point *p'* represents the real interest rate and points *i* and *j* the performance of two securities or portfolios. By combining securities or portfolio *i* with loans, any point along the *p'iy* line can be reached. But such points are clearly dominated by points along the *p'jx* line, which can be achieved through loans combined with securities or j portfolio. The steeper the line associated with a value or portfolio, the better it is. As shown in Figure 2, the reward-volatility ratio is the slope of such a line.

Rreward-volatility contribution

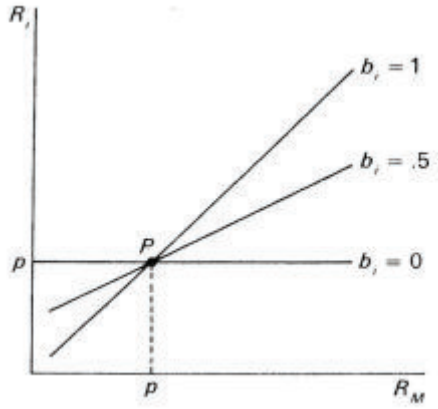
Figure 2



In a world where $A_i = E_i$, and $b_i' = b_i$ and $p' = p$ all securities and portfolios would have identical rates between reward and volatility. If the forecasts are random, the reward-volatility ratios will vary randomly around the value associated with the asset market line. There will be no particular differences.

Characteristic prediction-based lessons

Figure 3

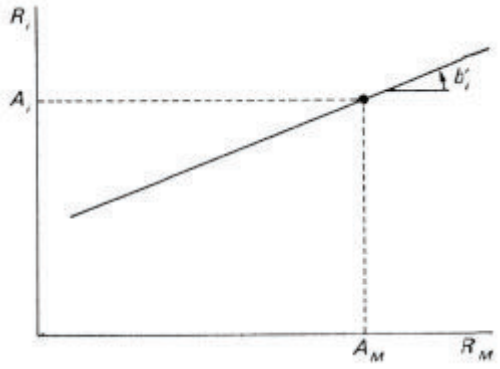


It is important to consider the relationship between the characteristic line of a security or portfolio and the reward-volatility ratio. Starting from the balancing condition for characteristic lines based on predictions, each must go through the point where both rates of return are equal to the pure interest rate. Three lines that meet the requirements are shown in Figure 3.

The actual relationship between the rate of return of a security or portfolio and that of the market portfolio can be represented by a real characteristic line. The line shall pass through the point where both returns are equal to their average values and the slope (volatility) is determined by dividing the actual covariance by the actual variance of the rate of return of the market portfolio. Figure 4 shows such a case.

Rate of return of a security or portfolio and that of the market portfolio

Figure 4

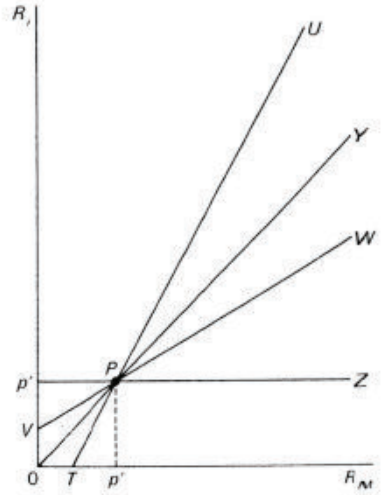


We will consider the performance of the market portfolio, which can be described by a characteristic line that makes an angle of 45^0 with the origin, such as the *OPY* line in Figure 5. The performance of a risk-free title can be described by a perfectly horizontal characteristic line, such as *p'PZ* in Figure 5 (*p'* represents the pure real interest rate). Any combination of lending plus investments in the market portfolio can be represented by a line between these two, for example, *vpw*. Any combination of loan plus investment in the market portfolio can be represented by a line such as *TPU*.

Given the ability to borrow at *the p'* rate, an investor could have achieved results along any desired characteristic line passing through point *P*, at which both rates of return are equal to *p'*.

Performance of the market portfolio

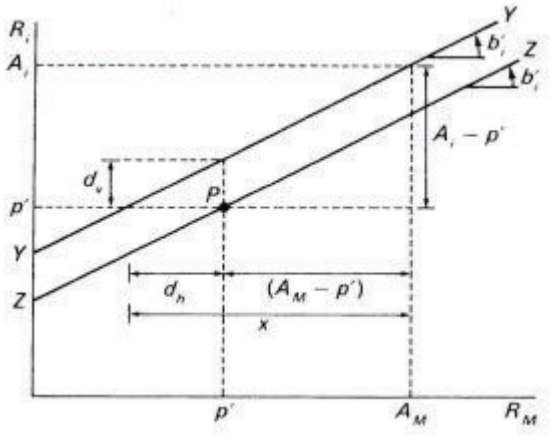
Figure 5



If we consider the title or portfolio whose actual performance is represented by the characteristic *line YY* in Figure 6. The same volatility could have been achieved by selecting an appropriate combination of loan or loan plus investment in the market portfolio. Such an alternative is represented by the characteristic line *ZPZ*, built in such a way that it is parallel to the right *YY* and passes through the point *P*, at which both rates of return are equal to the pure interest rate.

Rate of return equal to interest rate

Figure 6



The slope of the characteristic line of a security or portfolio is equal to its volatility. From Figure 6 we deduce:

$$b_i' = \frac{A_i - p'}{x}$$

Which can be rearranged:

$$x = \frac{A_i - p'}{x b_i'}$$

The phrase on the right is the reward-volatility ratio. Separating the total distance (x) into two components:

$$\frac{r}{b} = d_h + (A_M - p')$$

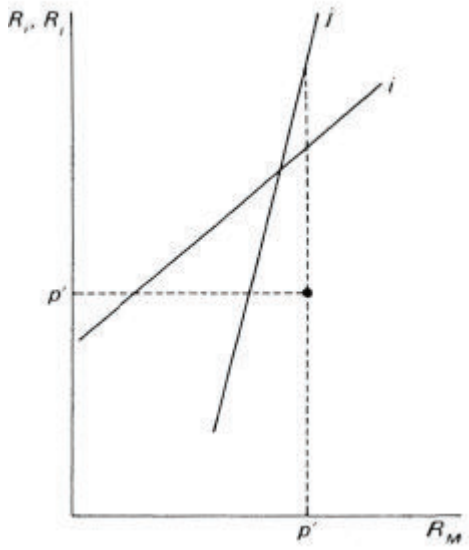
The reward-volatility ratio can thus be viewed as a constant ($A_M - p'$) plus the horizontal distance from point P to the characteristic line of the title or portfolio in question. If this distance (d_h) is positive, the performance was superior to that of a market-based portfolio with comparable volatility.

An alternative measure is the vertical distance from the point P to the characteristic line, indicated in Figure 7 by distance d_v and denoted by the differential efficiency. The meaning should be clear. According to the characteristic line, the title or portfolio in question offered an average differential return of this amount above that derived from a market-based portfolio with comparable volatility.

A positive differential return indicates that the performance was superior to that of a market-based portfolio with comparable volatility.

Positive differential yield

Figure 7

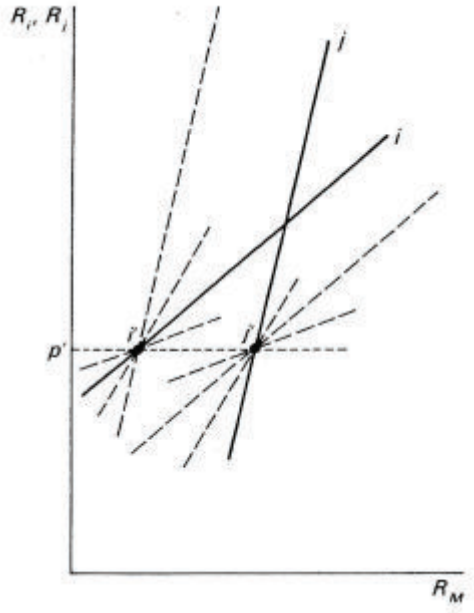


Clearly, the differential yield is closely related to the reward-volatility ratio. Either can be used to compare the performance of a security or portfolio with that of a market-based portfolio with similar volatility. However, they can give different results when used to make other comparisons. Based on differential yields, the performance of the title or portfolio associated with characteristic line *j* is superior to that of the title or portfolio associated with line *i*. But based on the reward-volatility ratios, i's performance was superior to that of *j*.

It is a question of determining which placement is better. The answer is the title or portfolio *i*. Figure 8 highlights why. By combining loans or loans with the investment in *i*, results could have been achieved along any characteristic line desired by point *i'*. By combining the loan or loan with the investment in *j*, results could have been achieved along any characteristic line desired by the point *j'*. Any combination based on *j* could thus have been overcome by a combination of equal volatility based on *i*.

Comparison of securities and best placement

Figure 8



Summing up, the reward-volatility ratio can be used to compare securities or portfolios with each other. They can be used to compare the performance of a title or portfolio to either the reward-volatility ratio or the measure of differential profitability.

Conclusions

The study of the article *Statistical models useful for measuring market performance leads* to a series of theoretical and practical conclusions, since in the analysis we used mathematically illustrated and graphically represented sequences, so that it becomes easy to conclude which is the optimal option to choose.

Any investor when thinking about placing capital market portfolios wants to reach a controllable point of return with which the operation of enrollment, placement, on the capital market will be completed.

Another essential element is that without a careful analysis of the effect of portfolio-specific conditions that can be effective, it can only be ensured by chance that that portfolio of assets is placed on the capital market.

The investor must synthesize by analysis the concrete situation of a portfolio that even if it appears at first ineffective, after the careful study becomes an efficient one. In this sense the reward / variability ratio of the portfolios although they are very diversified, they revolve around a value, which if it is well determined leads to the variant that is to be accepted. Sometimes it is put the evaluation of the performance of a title or portfolio that requires a differentiated analysis. Variability will not always entail a real risk to be borne. The choice must therefore be made under conditions of variability in order to establish the premises and conditions that the capital market on which the portfolio will be placed will face and the effect of that investment being affected to a greater or lesser extent.

The real relationship between the rate of return of a security, a portfolio, and that of a market portfolio must be established, as there is a visible difference between them. The performance of the market portfolio is a premise and a concern of investors, so that any combination of lending to the market portfolio is a definite element for the final performance to be obtained.

Differential return occurs when we study multiple portfolios or assets, but it will indicate that performance may be superior to a market-based portfolio, but with comparable volatility.

In the end the conclusion is that an answer must be found to the problem of determining which placement is better. In this sense, by combining loans and investment one can achieve different results that we must study and choose the most convenient one in terms of the investor's desire.

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