PRODUCTION FUNCTION USED IN MICROECONOMIC STUDIES AND ANALYSES

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Abstract

Within the market economy, an important role is played by the economic entities that carry out the actual activity. The law of the market has its fundamental law that of self-regulation based on supply and demand. Of course, this is a normal phenomenon that happens, but which if not analysed and studied in advance produces negative effects on economic developments.

We can imagine that the ratio between supply and demand can be adjusted only to the extent that obvious elements appear that determine the restructuring of production programs. For example, a higher supply due to non-coordination of economic agents leads to stocks, which are often unsalable, in other circumstances are even perishable so that they cause damage to the economic entity. Of course, regulation is done in the field of the market but in a distorted way for some economic entities. Therefore, in microeconomic economic studies, the emphasis should be on identifying the conditions acting on the market, taking into account the perspectives of economic entities in the same field, so that they correlate.

At the macroeconomic level, the destabilization that can be caused by a surplus of market products and services, which far exceed market demand, is even more evident in many circumstances. Of course, here we can discuss a function based on elasticity, but coming back we are talking about a production function that must be appropriate to the conditions imposed by the market for each economic agent. In reality, the growth and profitability of economic entities take into account the way in which management takes into account the changes it has made using a production function, which takes into account the situation in the economy, so that the volume of products by structure quality, price and all others to be accepted by the market.

In international trade this study needs to be done even more rigorously and again the production function is used alongside the regression function. The production function starts from the study of the three factors that each economic entity must have, the way in which these three factors

are most effectively correlated and from this perspective to be able to identify the evolution trends and the emphasis to be placed on one or another of the factors of production.

Of course, we also took a concrete study based on which we built a production function, we analysed and the results were interpreted. They were used as an example of how to analyse such a situation based on the production function. Of course, I am referring to large economic entities I am referring to multinational companies, to large companies in a country and in Romania, in which case this production function acquires a much more obvious use than the studies of small entities (SMEs). , which sometimes act empirically and as a result have results that are inconclusive, which is reflected in the exit of a large number of small and medium-sized enterprises over a period of time. In times of crisis this situation is even more obvious, referring in this regard even to the current crisis in which without a precise study, without an analysis based on statistical-econometric models of the production function, large distortions can occur leading to structural destabilization of companies with effect in macroeconomic destructuring and destabilization.

Keywords: methods, models, variables, regression, production function, elasticity

JEL classification: C10, D20.

Introduction

Any activity involves transforming the inputs so as to obtain the desired outputs to achieve the proposed goal. In the case of an economic activity carried out by an economic entity, the role of the management is to achieve the purpose for which it exists, respectively to maximize the profit in the conditions of efficient use of resources and to achieve the planned strategic objectives.

Under these conditions, the economic-financial decision concerns the inputs, respectively the production factors, which have an influence on the desired output, respectively the production function. The decision involves the combination or substitution of factors of production and involves a change in the amount of factors used, a change in the proportion between the factors or a replacement of one factor by another.

The substitution of factors of production has been and is a field of research in economics, given the trend of dematerialization of production through the development of information and communication technology, ICT, and increasing the importance of services in the knowledge-based society.

Methods and models of study have been developed for the study of the combination and substitution of production factors, such as Cobb-Douglas-

type production functions and instruments for measuring the economic effects related to the yield of factors of production and possible substitutions between them: scale elasticity, elasticity of factors of production, elasticity of substitution, rate of progress.

Literature review

In the specialized literature we meet an important volume of works of some important specialists, who approached the economic modelling using the production functions. We mention some works such as that of the authors Anderson, P.W, Arrow, J.K., Pines, D. (1998) who approached these analyses of economics as a complex evolving system. Also, the production factors, as well as the Cobb-Douglas type production function were extensively treated by Anghelache C., (2008) and Anghelache, C., Mitruț, C., Voineagu, V. (2013). The analysis of the correlations at the level of the economic aggregates showed interest for Anghelache C., Anghel M.G., Căpușneanu S., Topor D.I. (2019). The indicators of the efficiency of the economic potential were studied in their paper Anghel M.G., Anghelache C., Stoica R. (2019). A study on the evolution of the capital market and the effect it has on alternative sources of financing for business was made by Govori, F. (2014). In his paper Gikuang, J.C. (2012) focused on the analysis of multicollinearity. Grassetti, F., Mammana, C., Michetti, E. (2018) used VES production functions in the empirical analysis of the substitutionality between production and growth factors. Variable elasticity of substitution production functions were also addressed by Lu, Y. (1967) in his paper. Nicolae, V., Caracotă, D., Constantin, D.L., Pârlog, C., Grădinaru, I., Slăvescu, V., Tobultoc, V. (2000), approach the analysis of economic phenomena at macroeconomic level. Such approaches, but at the microeconomic level, are those of Peterson, L.W. (1980). Rothschild, M., Stiglitz, J. (1976) are concerned and make an analysis regarding the balance of competitive insurance markets. In the same vein, Stan, F. (2005) is concerned with the study of indicators for assessing efficiency and productivity in particular.

Methodology, data, discussions, results

• ***Using the function in the analysis of some companies***

Let Y be the production function that depends on two factors, capital, K, and labor, L:

$$Y = f(K, L) \tag{1}$$

$$e_{\frac{p}{K}} = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta K}{K}} \quad (2)$$

$$e_{\frac{p}{L}} = \frac{\frac{\Delta Y}{Y}}{\frac{\Delta L}{L}} \quad (3)$$

where

$e_{\frac{p}{K}}$ = Elasticity of production due to capital;

$e_{\frac{p}{L}}$ = Elasticity of production due to labor.

Scale elasticity measures the percentage increase in production when the factors of production increase in the same proportions and is the sum of the elasticity of the factors.

Production elasticity, e_p, is defined as the fractional change of the output (production function Y) relative to the fractional change of an input (factor of production).

Substitution elasticity measures the percentage change in the proportions of factors due to a change in the marginal rate of technical substitution.

Calculation relations are:

$$e_s = \frac{\frac{\Delta R}{R}}{\frac{\Delta r}{r}} \quad (4)$$

$$R = \frac{X_A}{X_B} \quad (5)$$

$$\Delta R = R_i - R_{i-1} \quad (6)$$

$$W_{mA} = \frac{Y_i - Y_{i-1}}{X_{Ai} - X_{Ai-1}} = \frac{\Delta Y}{\Delta X_A} \quad (7)$$

$$W_{mB} = \frac{Y_i - Y_{i-1}}{X_{Bi} - X_{Bi-1}} = \frac{\Delta Y}{\Delta X_B} \quad (8)$$

$$r = \frac{W_{mB}}{W_{mA}} = \frac{\Delta X_A}{\Delta X_B} \quad (9)$$

$$\Delta r = r_i - r_{i-1} \quad (10)$$

where:

e_s = Substitution elasticity;

R = The ratio between factor X_A and factor X_B;

ΔR = The difference of the ratio R, at two time points, i and i-1, respectively;

W_{mA} = Marginal productivity of factor X_A;

W_{mB} = Marginal productivity of factor X_B;

r = Marginal productivity ratio;

Δr = The difference of the ratio r at two time points, i and i-1, respectively.

- If $e_s = 0$, then factors X_A and X_B are complementary and it is not possible to replace them;

- If $e_s \to \infty$, then the substitution of factors is perfect;

- If $e_s = 1$, then $\frac{r}{R} = \frac{\Delta r}{\Delta R}$

For the analysis of the elasticity of the substitution were used:

- Production function Y was turnover, CA;
- The production factor X_A was the total fixed assets, K;
- The production factor X_B was the number of staff, L;
- The time for which the calculations were made was the end of each quarter.

With these data, using the formulas (4) to (1), the economic entities were analyzed.

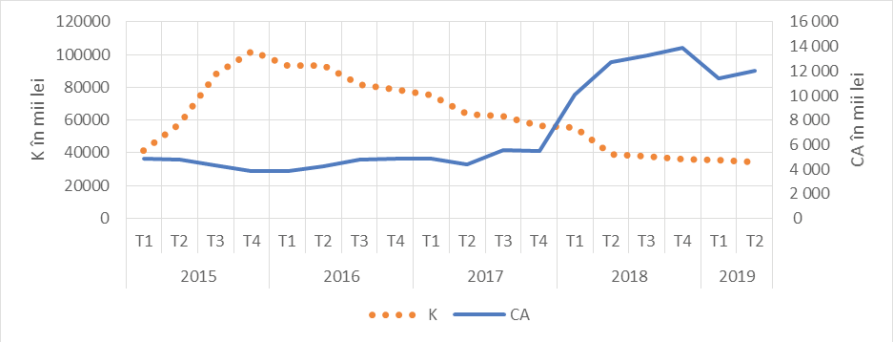
• ***Analysis of the elasticity of the substitution of the economic entity***

The data analysis highlighted the following:

Fixed assets increase significantly in each quarter of 2015, as a result of investments in computing technology and software licenses within the POSCCE project, after which they decrease, as a result of their linear depreciation. Following the investment process carried out in the period 2015-2016, which allowed the approach of new research areas, the turnover shows a significant increase in 2018. The situation is presented in graph 1:

Dynamics of turnover CA and fixed assets K

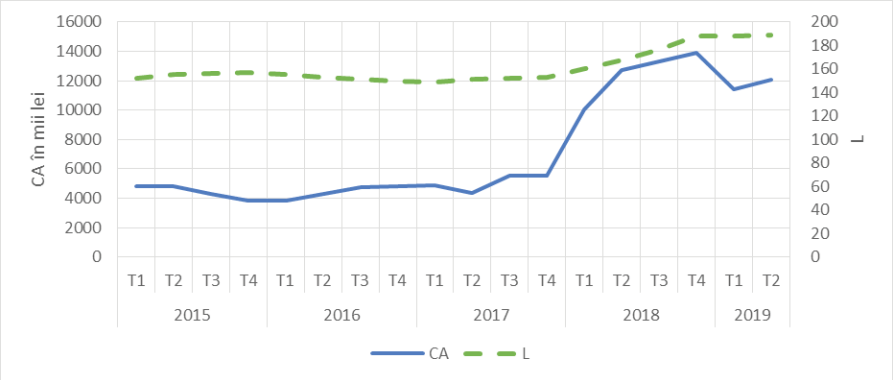
Graph 1



The investments made have allowed the creation of new jobs and the hiring of new research staff. The training of staff in the POSCCE project allowed it to address new areas of research, which justifies the increase in turnover in 2018, as shown in Graph 2:

L and CA turnover dynamics

Chart 2



The value of the modulus of elasticity of the substitution is less than 0.55, with an average of 0.16, which leads to the conclusion that the two factors, K and L, are complementary and cannot substitute for each other. The conclusion is correct, because, in the research-development activity, the man is very important, as in any creative industry, the research staff L, is the one who generates the innovation through creativity. Capital contributes through the technical level of fixed assets and sources of financing to the achievement of physical indicators: patents, new and modernized products, articles, etc.

Fixed assets had productivity between 3.8% and 38.6%, with an average of 15.2%, with great potential for growth, and labor productivity was on average 43,173 lei per quarter / person.

Based on the case studies, it results that there are significant differences both between the values of the indicators CA, K and L, and between the productivity of capital and labor productivity related to the object of activity, the products produced and the services provided. The values of the substitution elasticity calculated by formula (4) highlight the complementarity of the factors of production K and L.

The production function expresses the relationship between the output level and the inputs needed to achieve this output level.

For more accurate modelling of economic reality, various types of production functions have been developed, one of which is the production function with Variable Elasticity Substitution (VES), known as the VES function, proposed by Revankar which has the following form :

$$\boldsymbol{Y = A * K^{\alpha\gamma} * (L + \alpha\beta K)^{\gamma(1-\alpha)}} \tag{11}$$

Where:

K	capital;
L	workforce;
$A > 0$	constant, expression of the integral efficiency of the factors of production;
$0 < \alpha$<1	constant, represents the capital contribution;
$\beta \geq -1\, \beta \neq 0$	constant, represents the variation of the elasticity of the substitution.
$\gamma \neq 0$	constant

If $\beta = 0$ the Cobb-Douglas production function is obtained:

$$Y = A * K^{\alpha\gamma} * L^{\gamma(1-\alpha)} = A * K^{a} * L^{b} \tag{12}$$

The Cobb-Douglas production function is an algometric dependence of the Y output on the K, L inputs and is one of the most popular production functions. By logarithm this function becomes linear of two variables:

$$y = lnY = lnA + \alpha\gamma x_1 + \gamma(1-\alpha)x_2 = lnA + ax_1 + bx_2 \tag{13}$$

$$x_1 = lnK \tag{14}$$

$$x_2 = lnL \tag{15}$$

From the analysis of the Cobb-Douglas production function, the elasticity of the scale can be determined by the relation:

$$e = e_K + e_L = a + b \tag{16}$$

Depending on the value of the scale elasticity, there can be three situations:

a+b<1 Descending scale output when output increases to a lesser extent than input.

a+b=1 Constant scale output, the output increases in proportion to the increase of the two inputs.

a+b>1 As the scale increases, the increase in output is greater than the increase in input.

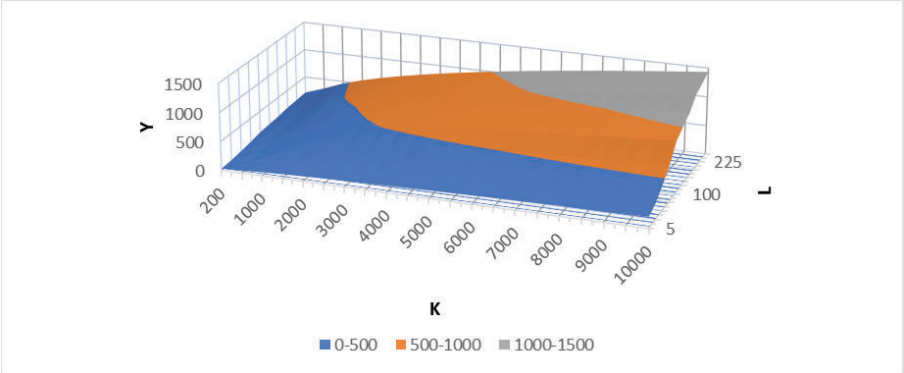
The Cobb-Douglas function used for Romania's economic analysis has the form:

$$Y = 0{,}94 * K^{0{,}49} * L^{0{,}51} \quad (17)$$

Graph 3 of the Cobb-Douglas function described by equation (17) shows isoquant's in areas from 500 to 500.

The Cobb-Douglas function used for the economic analysis of Romania

Graph 3



Source: Data processed by the authors

If $\gamma = 1$ the VES function, described by equation (11), has constant substitution elasticity. The production function with constant substitution elasticity is known in the literature as the CES (Constant Elasticity of Substitution) function, was introduced by Arrow, Chénery, Minhas and Solow (known as SMAC) and is of the form:

$$Y = A * (\alpha K^{-\rho} + (1-\alpha)L^{-\rho})^{-\frac{1}{\rho}} \quad (18)$$

Where:

K capital;

L workforce;

A constant, expression of the integral efficiency of the factors of production;

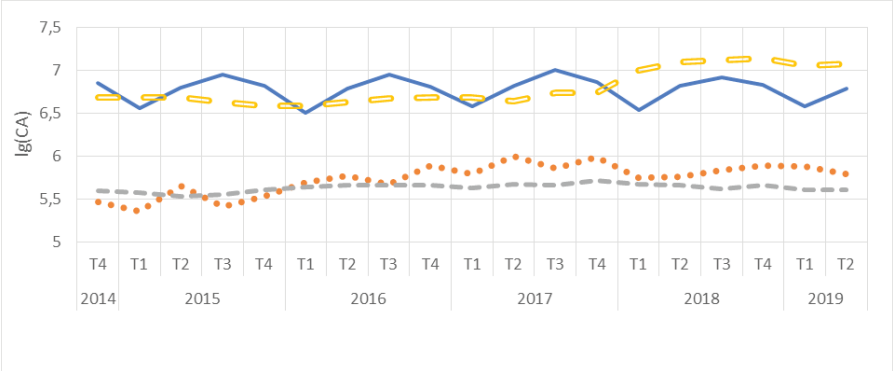
α constant, represents the capital contribution;

ρ Substitution, parameter that allows the calculation of the substitution elasticity.

These production functions are used at the macroeconomic level such as the world, national economy or branches of the national economy with a significant contribution to the realization of the Gross Domestic Product.
The economic entities used by the authors in this study for the Cobb-Douglas production function are very different, both in terms of activity and turnover, fixed assets and number of staff. In order to be able to represent the specific indicators in the same graph for all economic entities, the decimal logarithm was calculated for the turnover represented in graph 4, labor productivity represented in graph 5, fixed assets represented in graph 6 and the number of staff represented in graph 7.

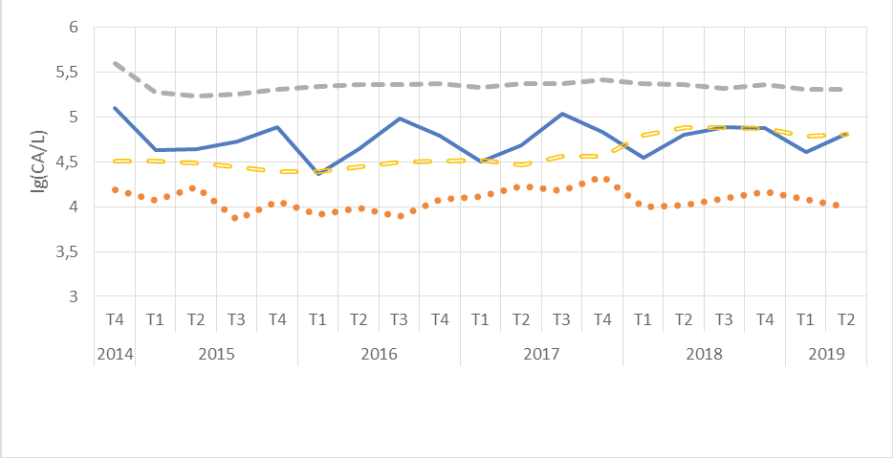
Logarithm in base 10 of Turnover

Graph 4



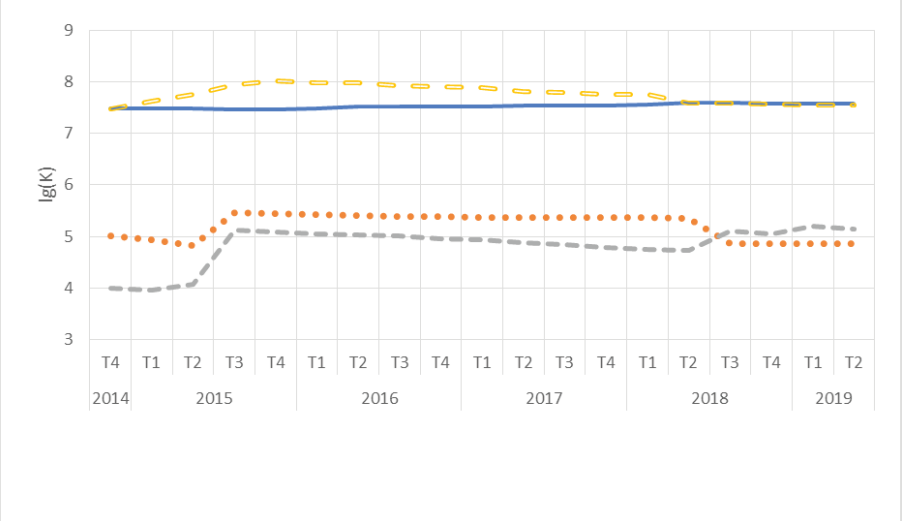
Logarithm in base 10 of Labor Productivity

Graph 5



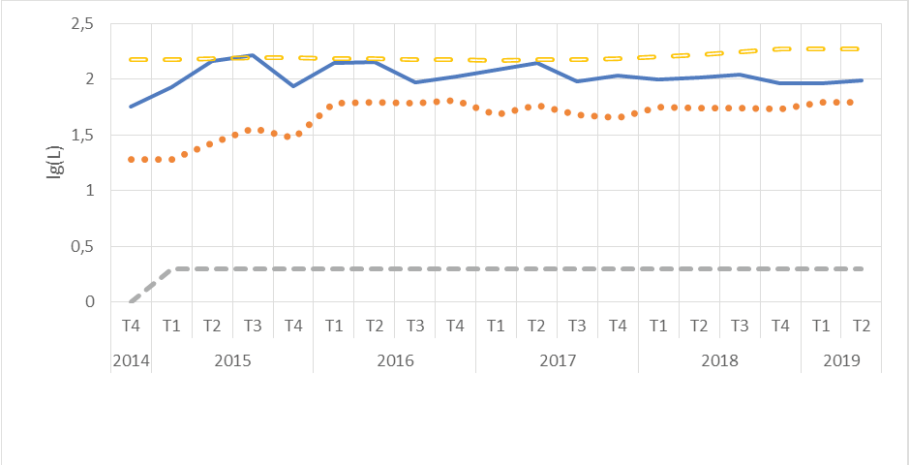
Logarithm in base 10 of Fixed Assets

Graph 6



Logarithm in base 10 of the number of staff

Graph 7



The analysed economic entities fall into the category of SMEs, with between 2 and 200 employees. For the case studies, the Cobb-Douglas function defined by equation (12) was used, which was linearized by logarithm according to equation (15). The regression module in Excel was used to determine the parameters. Three models were used for the Cobb-Douglas production function, in the first model the production function was turnover CA, in the second model the production function was labor productivity, and in the third model the production function was productivity labor and capital assets were used instead of fixed assets. From the three regression functions calculated for each step, the valid model was chosen for which the determination coefficient R2 was maximum and the residual deviation was minimal.

• ***The production function used in the analysis of the economic entity***

Starting from the primary data, the turnover, CA, the fixed assets, K, the number of personnel, L, the turnover per capital equal to the labor productivity, W_L, respectively the fixed assets per capital, W_K were logarithmed. Three models were used for the Cobb-Douglas production function, in the first model the production function was turnover CA, in the second model the production function was labor productivity, and in the third model the production function was productivity labor and capital assets were used instead of fixed assets. The statistical indicators and regression parameters determined with the regression module in Excel are presented in table 1:

Statistical indicators for the three production functions

Table 1

Indicator	$CA=A*K^a*L^b$	$W_L=A*K^a*L^b$	$W_L=A*W_K{}^a*L^b$
R	0,903887	0,864888	0,864888
R^2	0,817011	0,748031	0,748031
Rc^2	0,794138	0,716535	0,716535
s_r	0,215077	0,215074	0,215074
n	19	19	19
k	2	2	2
SPE	3,304540	2,197194	2,197194
SPR	0,740129	0,740109	0,740109
SPT	4,044668	2,937303	2,937303
Fc	35,718551	23,749950	23,749950
ln(A)	2,748148	2,748366	2,748366
A	15,61	15,62	15,62
a	-0,372572	-0,372573	-0,372573
b	3,852528	2,852489	2,479916
Valid model	YES, Fc>3,633723	YES, Fc>3,633723	YES, Fc>3,633723

From the three valid models, the model for which the determination coefficient R is higher was chosen, respectively the model in which the production function is the turnover, described by equation (19):

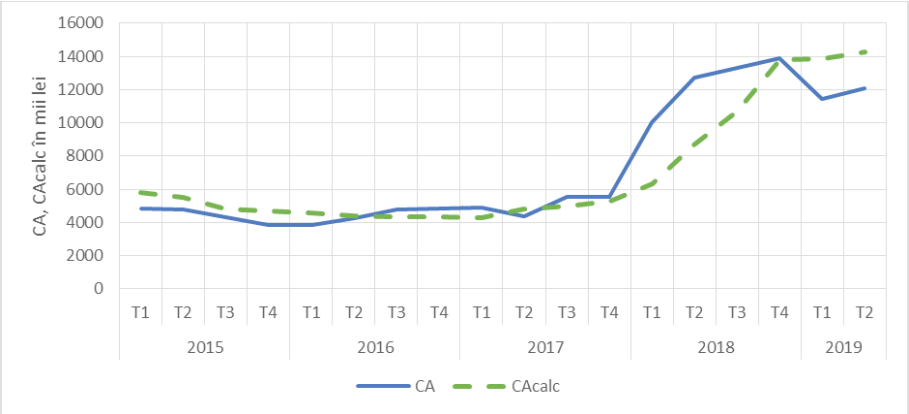
$$CA = 15{,}61 * K^{-0{,}372572} * L^{3{,}852528} \quad (19)$$

The elasticity of the scale is positive and super unitary the increase of the production factor L produces a higher increase of the turnover.

Graph 8 shows the Cobb-Douglas production function described by equation (19).

Cobb-Douglas function for turnover

Graph 8



The production function defined by equation (19) follows the figure of turnover and smooth the jumps. The production function can be used for the short-term forecast of turnover when drawing up the revenue and expenditure budget.

Conclusions

From the article *The production function used in microeconomic studies and analyses*, a series of conclusions can be drawn, both theoretical and practical. Thus, a conclusion that emerges from this study is that the use of the production function in the analysis of economic entities has an important role, in the sense that as Cobb-Douglas noted, between the three factors of production, but primarily between there must be a close correlation between labor and capital. This correlation will have a positive effect on the efficient use of capital and its technological improvement, but also on the quality of the workforce that will be called upon to use these achievements of science and technology, based on research and innovation in concrete activities.

Another conclusion that emerges is that man is very important in research and development and that in any creative industry, research staff is the one who generates innovation through creativity.

In other words, the Cobb-Douglas production function used in this study is one of the most popular production functions, and by logarithm this function becomes linear of two variables and scale elasticity can be determined. Also, following the use of the production function in the analysis of the economic entity, it was found that the scale elasticity is positive and super unitary, and the increase of the production factor will produce a higher increase of the turnover. In the same vein, the production function can be used for the short-term forecast of turnover when preparing the revenue and expenditure budget.

Last but not least, we can conclude that the crisis periods, as will be the case today in the perspective of the economic and financial crisis, will have to carry out many such studies in order to cover the slippage and distortion of macroeconomic correlations and proportions, in order to reinsuring the return to macro stability that gives efficiency to economic activity as a whole.

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