THE TRAMO - SEATS MODEL USED IN THE DYNAMIC SERIES ANALYSIS

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Abstract

The time series are very important in analyzing and comparing macroeconomic indicators internationally. The methodology of processing and analysis is, as a rule, different from one country to another. This is the question of unifying the methodological content of collecting and synthesizing time series. In this respect, Eurostat is concerned with harmonizing the methodology for using dynamic series. The Dynamic Series also provides an analysis of economic growth (Gross Domestic Product) through decomposition on factors of influence. The breakdown of chronological series has been synthesized by Eurostat in the Tramo-Seats methodology (ARIMA Noise, Missing Observations and Outliers in ARIMA Time Series). The theoretical elements underlying this methodology ensure the correct interpretation of trade flows, especially at the product group level. The Tramo-Seats methodology includes several steps as follows: building the ARIMA model; identifying extreme values; linearization and then processing by the Seats method for actual decomposition; using the Seats method as the estimated model density function; estimating the parameters for the considered components, and ultimately introducing extreme values and special effects into the estimated components. Particular aspects regarding the content of this methodology are presented in the article, identifying the mathematical relations specific to each stage and the Tramo-Seats methodology in the end.

Keywords: *ARIMA model, Tramo method, Seats method, methodology, dynamic series, factorial influence.*

JEL Classification: C10, C32, C46

Introduction

Currently, the seasonal time series is the main source of information for economic analysts, politicians and different categories of decision-makers acting in various fields. Due to recent developments in computing and modeling theory, several practical methods of processing and decomposing time series have emerged.

The statistical institutes organized at the intergovernmental and national levels are those responsible for the task of both recording and storing the data obtained from the observation as well as processing them in order to be put in advantageous form to the final user. Therefore, the methods of decomposition of the chronological series represented a real interest for the statistical institutes that took over, systematized and developed these methods, thus ensuring an institutionalized and coherent framework for the future researches in this field.

The European Union Statistics Institute, Eurostat, collects data from the national statistical institutes of the member countries, candidate countries and other countries or economic areas, considered significant trading partners. These data are recorded monthly or quarterly. Annual data are generally obtained by aggregating monthly or quarterly data.

Due to the fact that the methodology of processing and analyzing the national statistical institutes in the member countries and especially the candidate countries are not fully harmonized among the attributions of Eurostat, there are also recommended the methods of processing and analysis in the hopes of being used by as many countries as possible from this space.

Literature review

Anghelache and Anghel (2018) presented and analyzed the fundamental aspects of econometrics. Anghelache (2008) is a reference work in the field of economic statistics, including elements of dynamic series processing. Arcidiacono and Miller (2011) have approached a series of aspects regarding the evaluation of dynamic models. Bosq (2012) analyzed the estimation and prediction of stochastic processes. Corbore, Durlauf and Hansen (2006) studied the theoretical and practical elements of econometrics. Elliott, Müller and Watson (2015) had concerns in the sphere of the null hypothesis. Gach and Pötscher (2011) evaluated nonparametric density. Lohr (2007) studied elements of regression. Pesavento and Rossi (2006) studied the aspect of the confidence interval in the sampling activity.

Research methodology, data, results and discussions

With regard to the methodology of decomposition of chronological series, after a long scientific comparative research activity, the preferred method, officially Eurostat, which was also imposed in the European space, is the TRAMO-SEATS methodology (Time Series Regression with ARIMA Noise, Missing Observations, and Outliers - Signal Extraction in ARIMA Time Series).

We will briefly present the main theoretical elements underlying this method, largely stemming from stochastic processes theory and, of course, the use of the method for analyzing commercial flows especially at product group level.

Synthetically, the TRAMO-SEATS methodology can be described by identifying the following steps:

- an ARIMA model is identified for the data series observed under the TRAMO method;
- the extreme values are automatically identified and other special effects (number of working days, variables of different legal holidays, etc.) are also estimated within the TRAMO method;
- the TRAMO linearized data series is then processed by the SEATS method where actual decomposition takes place;
- using the SEATS method the spectral density function of the estimated model is decomposed into the spectral density functions of the unobserved components that are supposed to be orthogonal;
- the SEATS method also estimates parameters for the two components: the trend-cycle component and the seasonally adjusted component; because the parameters are estimated by the Wiener-Kolmogorov filter the data series is extrapolated to its extremes;
- finally, extreme values and special effects are reintroduced into the estimated components.

Decomposition methods of the dynamic series

The discrete values recorded over time, obtained as a result of the observations made on the different phenomena, are recorded in the form of chronological series, also called time series or dynamic series. A very succinct definition of a chronological series could be a collection of sequentially recorded values over time.

Long time ago, statisticians who have worked in different areas have been concerned with breaking down the chronological series and analyzing the elements that make up them. In the economic field, the classical decomposition in the trend component, the cyclical component, the seasonal component and the aleatory component was mainly justified by the need for analysis and prognosis of business cycles. An important practice has been the removal of the seasonal component, or deseasonalization, in order to obtain a clearer picture of the long-term evolution of the studied economic phenomenon.

Although the initiator of modern methods of decomposition is considered Macaulay (1930), these methods find their way back in the nineteenth century in the field of astrology and meteorology studied in

England at the time. It was then realized that a chronological series observed can be generated by several unobserved components that are at the basis of the observed series, an idea that has been maintained over time.

The first studies focused on the false correlation that may occur between economic variables due to the trend and which was therefore removed before studying the actual correlation. Poynting (1884) and Hooker (1901) tried to remove the seasonal and trending component of price evolution by comparing average prices over many years. Spencer (1904) and Andersen (1914) introduced the use of higher order polynomials in eliminating the trend component. A second wave of works focused on trying to predict the components of an economic cycle by removing the seasonal component and the trend in the idea that the remaining part of the series offers a better estimate of cyclical changes.

A very intense activity in this field was developed in the 1920s and 1930s due to Pearson's work (1919) which considered that a series of time could be represented as the source of its components in the additive case or as a product of its components in the case of multiplicative:

$$X_t = S_t \times T_t \times C_t \times R_t \tag{1}$$

$$X_t = S_t + T_t + C_t + R_t \tag{2}$$

where:

X_t - the chronological series observed;

S_t - the seasonal component;

T_t - the trending component;

C_t - the cyclic component;

R_t - the random component.

Pearson's method implied simple data transformations to remove the trend, and then estimates of the seasonal component are calculated. Although, after Yule (1921), referring to a 1905 paper, Pearson is not the first to introduce the four components of the time series, he is certainly the first to find a simple method to estimate them. Pearson's method uses fictional seasonal factors, although in the literature at that time the idea of fixed seasonality is not valid for any field of research.

Sydensticker and Britten (1922) were the first to introduce the variable seasonal factor in decomposition methods, and Crum (1925) was the one to change Pearson's method to adapt it to variable seasonality.

Macauley's method consists of three essential stages:

- A 12th-order moving average is calculated (for monthly data), and then the values observed are related to the values obtained by the mobile media. The averages for each month are calculated from the values thus obtained that represent seasonality indices

- The trend is estimated with a linear or superior polyline
- The mobile media trend is reported in order to obtain an estimate of the seasonal component.

Many researchers have developed alternative alternatives that are based on mobile medians or adjusted environments. Some contemporary practices are still based on methods whose bases were put in that period.

The most important achievements in the field of time series decomposition belong to the 1950s due to the emergence of exponential leveling methods and the introduction of computer use in statistical analysis. As a result of these two new directions and especially due to the speed of the computer in 1954, the Census II method developed by the US Bureau of the Census, and in 1955 the second version, Census II, appeared. Julius Shiskin has made a major contribution to developing these methods, which have been criticized:

- it is not based on a theory of rigorous mathematical statistics, a common feature of ad hoc models;
- allocate part of the component to the other components;
- distorts components due to the moving average;
- eliminates only very pronounced seasonal variations;
- the repetition of the moving average is not justified most of the time.

These critics have contributed to the emergence of the X-3 and X-10 variants. The subsequent evolution led to the appearance in 1965 of the X-11 version that has found a very wide applicability. This method has contributed Eisenpress (1956), Marris (1960) and Young (1965) and others. The X-11 contains methods, based on regression, working-day adjustment, and allows the choice of seasonality, additive or multiplication.

As a result of the ARIMA methodology developed by Box and Jenkins in the 1970s, a new version, X-11-ARIMA, developed by Dagurn appeared. (1980), Canada's Institute of Statistics. X-11, the new version, the X-11-ARIMA allows predictions and estimates to be made at the end of the series at the beginning of the time series in order to get a better backspace / forecasting. The latest X-12-ARIMA version brings important changes. It uses an ARIMA regression model (REGARIMA) to pre-adjust data for extreme values and other special influence factors and introduces the use of spectrum to specify unobserved components. All this family of methods (X-11, X-11-ARIMA, X-12-ARIMA) are based on the same filtering method used in X-11 and dominated for 40 statistical theory and practice.

All the decomposition methods presented here fall into the category of „ad-hoc" models that do not take into account the structure of the dynamic series, are not based on rigorous mathematical or statistical theorems, do not rely on explicit models and are therefore considered empirical methods.

More recently, a new evolution of decomposition methods has emerged that has gradually given birth to a serious alternative to „ad-hoc" models. Thus a class of methods based on the initial modeling of the series and unobserved components has emerged. This class is divided into two important subclasses: the structural approach and the global approach.

The structural approach is particularly attributed to Engle (1978), Harvey and Todd (1983) and is based on the direct estimation of ARIMA models for each of the unobserved components.

The global approach involves finding an ARIMA model for the initial series and then extracting some models for each component. The TRAMO-SEATS method is part of this last subclass and will be expanded in the following. The X-12-ARIMA method is considered methods that make the transition from empirical methods to those based on stochastic modeling of the series and its components.

• **The stochastic processes and time series**

The chronological series, recorded following the observation of the economic phenomena, can be considered, from a mathematical point of view, as achievements or trajectories of stochastic processes.

A stochastic process can be described as a statistical record that evolves over time in accordance with probabilistic laws. The expression „stochastic" is of Greek origin and has the meaning of „connected with the chance". Therefore, the expression „random process" or „random process" can be used as a synonym for stochastic process. Of course, as we are in the sphere of foreign trade, we can not talk about pure random processes or random processes by their very nature, but we can look at an economic phenomenon, the nature of trade flows, as a random process as we do not observe or analyze the influence factors that determine the evolution of the phenomenon. Even if we try a deterministic, quantitative or qualitative approach, it remains a component of the evolution of that process that either can not be explained (it is more difficult to explain) and can be probed again.

Mathematically, a stochastic process can be defined as a collection of random variables that are ordered over time and defined over a set of points, discrete or continuous. The stochastic processes theory deals with the study of families of random variables defined on the same probability field. If we consider $\{\Omega, K, P\}$ a probability field, and E is the set of random variables (with real values) defined on Ω and T any arbitrary then a stochastic process with the set of parameters T is an application of the form:

Formally, a stochastic process depends on two variables
$t \in T$ și $\omega \in \Omega$

To indicate a stochastic process, simple notation or notation is generally used $\xi(t)$. Therefore, a stochastic process consists of a random variable family $\{\xi(t); t \in T\}$ for which the multidimensional distribution functions of the variables are given. $\{\xi(t_1), \xi(t_{2)......}\xi(t_n\}$

For each $t \in T, \xi(.)$ represents a random variable defined on {Ω,K, P}, and for each achievement $\omega \in \Omega$, it is a function defined on *T*, called the trajectory of the process corresponding to the realization ω.

When the set T is composed of a finite number of elements, the stochastic process $\xi_t(w)$ is equivalent to a random vector. If *T* consists of only a large number of elements, the process term can be replaced with the chain term.

The random variables of E can be considered as states of an economic phenomenon and the set of T parameters can be chosen as a discrete representation of time (years, quarters, months, etc.). Considering that the set of T parameters is a submultium of the actual right-hand time, the stochastic process $\{\xi(t); t \in T\}$ gives rise to another concept much more familiar to economic statistics, that of chronological series (time series or dynamic series). For the designation of a time series, the notation $\{X(t); t \in T\}$ is generally used.

A particularly important method of describing a time series is calculating the process moments, especially the first and second moments, which are represented by the mean, variant and autocorporation functions of the process. It is known that the variant function is a particular case of the autocorporation function for $t_1 = t_2$. To standardize the function of the autocovarian is generally calculated the autocorrelation function that takes values in the interval [-1, 1] .

A dynamic series of some $\{X(t); t \in T\}$ is a general study object to be able to be effectively analyzed. A certain class of series, the dynamic dynamic series, certain properties that make them preferable in the modeling and prognosis of some phenomena.

Unfortunately, time series bearing economic information are generally not stationary and require special processing to be brought to this form. There are two ways to define the stationarity that lead to the concepts of strict staying (stationary in a narrow sense) and poor or second order stationarity (stationarity in a broad sense). Since a normal distribution is fully described by the first two moments, a dynamic staggering series that is normally distributed will also be strictly stationary.

Formally, we say that a series of time is stationary when the observations fluctuate around a constant, time-independent environment, and when the fluctuation variation remains constant over time. We can also see if a series is stationary using the graphical representation of the series. If the graphical representation of a time series does not reveal any significant change on average over time, then we say that the series is static in the media report. If the graphical representation of a time series does not show any obvious change of the variance over time, then we say that the series is stationary in relation to the variant. In real economic activity, there are very few phenomena that can be described by static dynamic series and if they are stationary they are only for a short period of time, so it can be practically spoken only by local stationary.

The time series, as they are actually observed, generally shows a trend (mean variable) either ascending or descending. By various mathematical operations they can be brought to a stationary form. The trend or other non-stationary elements of a time series have the effect of positive autocorrelations that dominate the diagram of autocorrelation.

A way to remove non-stationarity is the method of difference or difference operators. This method is an integral part of the procedure recommended by Box and Jenkins (1970). For non-seasonal data, the first-order differentiation is usually sufficient to obtain a series with a relative stationarity, so the new series $\{y_1, y_{2......} y_{N-1}\}$ is obtained from the initial series $\{x_1, x_{2......} x_N\}$ by $\Delta x_t = x_t - x_{t-1} = y_{t-1}$.

Sometimes it happens that the new series of differences is not stationary and therefore it is necessary to build a series of second order differences. The second order differences are defined as follows:

$$\Delta^2 x_{t+2} = \Delta x_{t+2} - \Delta x_{t+1} = x_{t+2} - x_{t+1} - x_{t+1} + x_t = x_{t+2} - 2x_{t+1} + x_t \quad (3)$$

In practice, it is almost never necessary to use the order division of more than two, because real data implies tendencies generally linear or at most exponential.

If the initial series is non-stationary and contains seasonal correlations, it is necessary to use the seasonal difference operator. A seasonal difference is the difference between an observer and her correspondent from the previous year. So, for monthly data with an annual variation that is repeated at 12 months, we will consider the difference $\Delta_{12}\, x_t = x_t - x_{t-12}$.

In the theoretical and practical research on stationary dynamic processes, it was naturally a question of knowing if the study of these processes could not be performed with satisfactory accuracy, only on the basis of a single realization, but covering a large temporal horizon. Such a working hypothesis has been suggested by the defining features of a stationary prospect whose mean value and dispersion are not time dependent, and the correlation function does not depend on the calculated origin of the calculation. On the other hand, the objective reality and especially the social-economic processes offer us only the unrepeatable unique of the various stochastic processes, so that the practical checking of the right hypothesis of the mentioned work would have the gift to open wide research possibilities.

The theoretical researches of the last decades have led to the formulation of a result of great importance, claiming in essence that a rather large class of stationary dynamic processes enjoys the so-called ergodicity property.

If a dynamic stationary process possesses this property, then it is sufficient to study randomly, and only one realization of it; the realization taken in the study - scientifically processed - can give us a good representation of the typical characteristics of the process as a whole.

As we can see, the ergodicity of a stationary process consists in the fact that each separate realization of it constitutes a representative characteristic of the set of possible realizations. From a mathematical point of view, this means that each of the possible achievements of the process has the same probability of occurrence. This is due to the fact that the dynamic static process exerts the influence of one and the same group of factors.

If for a dynamic dynamic process the probabilities of occurrence of each realization are different then the typical values of each realization are different and the respective process does not enjoy the ergodic property. The cause of the lack of ergodicity lies in the internal heterogeneity of the process, ie each achievement is due to a different group of influence factors.

A practical tool for identifying ergodic stationary processes is the correlogram generated by the autocorrelation function. Generally, the lack of ergodicity can be noticed when the autocorrelation function remains constant at a time fixed in time.

A general result on linear processes that provides a very useful analytical representation of processes is called the fundamental representation of Wald or the fundamental representation theorem that is presented below.

If X_t is a statically linear stackable process, then X_t can be expressed as the sum of a deterministic function and a moving average of an infinite string of independent random variables

$$X_t = c(t) + e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + \cdots = c(t) + \sum_{j=0}^{\infty} \psi_j e_{t-j} = c(t) + \Psi(B) e_t \quad (4)$$

where:

e_t is a „white noise” with zero mean and constant variation V_a

$$\Psi(B) = \sum_{j=0}^{\infty} v_j B^j$$

and fulfills the properties:

1. $v_j \xrightarrow{j \to \infty} 0$

2. $\sum_j^{\infty} = 0 |v_j| \angle \infty$

sufficient for the convergence of the series that defines the polynomial $\varphi(B)$.

The deterministic component generally corresponds to the average of the process, and the average of a stationary process is not difficult to estimate. The stochastic part of the process corresponds to the mobile average $\sum_j^{\infty} = 0 v_j B^j$.

If it is a static linear stackable process of mean 0 or if the mean is not zero but was removed then it can be expressed as an infinite medium medium according to the following relationship

$$X_t = e_t + \psi_1 e_{t-1} + \psi_2 e_{t-2} + \cdots = \\ = \sum_{j=0}^{\infty} \psi_j e_{t-j} = \Psi(B)\alpha_t \tag{5}$$

The two major parts of the decomposition method described in this paper, TRAMO and SEATS, address the two components of a process, the deterministic component and stochastic component respectively. Stochastic components are the ones that suffer the actual decomposition after the time series is adjusted by the TRAMO method. It should be noted, however, that the fundamental representation theory implies the existence of an infinite number of elements which does not correspond to the reality of the phenomena specific to the external trade and in general to the statistically observable phenomena.

ARMA models are a very useful tool for approximating the stochastic component with an infinite number of elements from Wald's fundamental representation. Starting from the relationship of Wald's representation we have:

$$x_t = (1 + \psi_1 B + \psi_2 B^2 + \ldots) e_t \tag{6}$$

If the coefficients Ψ are fixed so that $\psi_i = \psi^i$ and $|\Psi| \prec 1$ then we can write:

$$x_t = \left(1 + \psi^1 B + \psi^2 B^2 + \ldots\right) e_t \tag{7}$$

Using the formula of the sum of a geometrical progression we get:

$$x_t = \lim_{n \to \infty} \frac{1 - \psi^{n+1} B^{n+1}}{1 - \psi B} e_t = \frac{1}{1 - \psi B} e_t \tag{8}$$

where it comes from:

$(1 - \Psi \mathrm{B}) x_t = e_t$

By re-calibration, the formula for an AR can be obtained:

$(1 + \phi B) x_t = e_t$

In cele prezentate mai sus a rezultat ca o anumita clasa de procese care admit reprezentarea fundamentala a lui Wald pot fi scrise ca procese autoregresive. Intr-un mod similar un proces autoregresiv de ordin infinit de forma:

$$e_t = (1 + \theta B + \theta B^2 + \ldots) x_t \quad \text{unde } |\theta| < 1 \tag{9}$$

is equivalent to an MA (1) process: $x_t = (1 - \theta B) e_t$

This property of a MA(1) process to admit an infinite autoregressive representation but convergence is known as the inversibility property and the condition that $|\theta| \prec 1$ represents the inversibility condition.

Both autoregressive models and medium-sized models can be used to make a brief representation of certain processes. There is the possibility to extend their scope by combining them and thus obtaining the ARMA (p, q) models that have the following general form: $\phi(B) x_t = \phi(B) e_t$

The representation of Wald, which is the basic theory of ARMA modeling, and from which some very advantageous properties arise, implies that the observed series is stationary. In practice, very few dynamic series are stationary and therefore need to be brought to a stationary form by the difference method. If the observed dynamic x_t series is a non-stationary series by transforming it into a stationary series, we get it: $z_t = \delta(B) x_t$ where $\delta_t = \Delta^d, d = 0,1,2$.............

In practice, there are generally no situations in which to be greater than 2. Therefore, the initial series will follow an ARIMA process (p, d, q), d representing the order of difference, of the form:

$$\Phi(B) \delta(B) x_t = \Theta(B) e_t \tag{10}$$

The main tool in identifying an ARIMA model is represented by the autocorrelation function and the partial autocorrelation function. Once a model describing the behavior of a time series in an appropriate manner has been identified and estimated, it can form the basis for forecasting. However, it should not be forgotten that the predictions based on such models start from the assumption of maintaining the structure and tendency characteristic of the analyzed phenomenon on the horizon. This premise is often infected by reality, therefore, the necessary reserves must be maintained.

Conclusions

From the study on which this article was conceived, it follows that from a theoretical point of view, the Tramo-Seats methodology provides an efficient processing and decomposition of the chronological series. Through this methodology, it is possible to break down the dynamic series into components such as the observed chronological series, the seasonal component, the trend component, the cyclical component and the random component. Using the Tramo-Seats methodology that highlights the steps to be followed, the essentials of the ARIMA model, the Tramo method, the Seats method and the parameters of the components considered are highlighted.

The Tramo-Seats methodology is effective in ensuring that dynamic data series are processed and analyzed to ensure European / International comparability. Currently, the Member States of the European Union use this methodology, which is similarly used by the Member States.

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