USED MODELS AND CRITERIA FOR ASSET YIELDS EXPLANATION[1]

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Abstract

There were compared two known models (CAPM and TPA resuting the model describing better, in case of Romania , cashings and variation of cashings for ensured guarantees.

There were taken into account monthly cashings (1.01.2005-31.12.2010 period) of 60 companies listed at Bucharest Stock Exchange (BVB).

Key words: models: CAPM – Capital Asset Pricing Model; TPA – Arbitrage Pricing Theory; criteria, methodology, results.

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The current study is based on monthly returns for stocks listed on the Bucharest Stock Exchange during the 01.01.2005 – 31.12.2010 interval, with respect to the available information. Logarithmical values are used to ensure the series' stationarity. The data was obtained from the web pages of BVB[2] and of "Kmarket"[3] investment firm. The missing observations were completed with interpolation. All the stock market's categories are taken into account (I,II and III), and they include 76 assets having available data, and from those, some are eliminated due to lack of more than 25% of the observations in the time period. Hence the final sample consists of 60 assets, each of them with 72 observations of monthly return (12 months for 6 years).

For a proxy of the riskless interest rate, the government bonds were used, with respect to the available data. This rate had an annual value of 8.8404% in the 01.01.2005 – 31.12.2010 period, equaling an average monthly value of 0,7367%. The data was obtained from the monthly reports of BNR, on its web site[4].

The entire 6-year working time interval will be divided into 2 equal sub-periods: the first sub-period is 01.01.2005 - 31.12.2007, and the second one is 01.01.2008 – 31.12.2010. The testing will be developed on each sub-

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period, and then also on the whole period of time, for comparative and superior accuracy purposes.

As comparative criteria will be used: the residual analysis, Davidson-MacKinnon equation and the "Posterior odds ratio" (POR) indicator, them being used frequently in previous tests as well. The calculus methodology, the obtained results and their interpretations are detailed separately for each criteria.

The residual analysis

Such a direct comparison was made by Chen[2], with Copeland and Weston[3] presenting the procedure more thoroughly. The 2 considered equations were:

$$R_i = \lambda_0 + \lambda_1 * b_{i1} + \dots\dots\dots + \lambda_n * b_{in} + \varepsilon_i \qquad \text{(APT)}$$

and:

$$R_i = \lambda_0 + \lambda_1 * \beta_i + \eta_i \qquad \text{(CAPM)}$$

which are the classical forms of both models. Then some regressions were applied in the following form: the first one has the residual variables of CAPM (η_i) as dependent variables and the "factor loadings" of APT (b_{ik}) as the independent variables, and the second one having the residual variables of APT (ε_i) as dependent variables and the β_i coefficients of CAPM as independent variables.

The logic behind such a test rests here: if CAPM is correctly specified, then the expected return for any "i" asset will be explained through the β_i coefficient, and the η_i variable will be only a "white noise" having zero mean and constant variance. If the expectations on the market are rational, which is a fundamental condition for the model, the realized return can be expressed like:

$$R_i = E_i + v_i$$

which says that the obtained return is given by the sum of the rational expected return and an error term. But if CAPM is correctly specified, the obtained return can be re-written like:

$$R_i = E_i(CAPM) + \eta_i$$

where $E_i(CAPM)$ represents the expected return given by the model. Hence:

$$E_i(CAPM) + \eta_i = E_i + v_i$$

or:

$$\eta_i = E_i - E_i(CAPM) + v_i$$

If the model is valid, then $E_i = E_i(CAPM)$, and $\eta_i = v_i$ is a "white noise" and it should not prove as having influence on the returns, because if it would, then it means that E_i contains information not captured by $E_i(CAPM)$ and hence CAPM becomes incorrectly specified.

Concluding, the logical testing for determining the superior performance between CAPM and other model (for this case is APT) is the one presented above: regressing the residuals of CAPM (η_i) on "factor loadings" of APT (b_{ik}) and the other way around, regressing the residuals of APT (ε_i) on the β_i coefficients of CAPM. The second regression has a opposed goal as the first one, by analyzing the possibility that CAPM captures information missed by APT.

In Chen's testing, APT was able to explain a significant part of the CAPM residual variance, but CAPM was not able to explain a significant part of the APT residual variance. The present study applies this testing as well in Romania's case, for having a direct comparison between the 2 models. The regression equations have the below form:

$$\eta_i = \lambda_0 + \lambda_1 * b_{i1} + \lambda_2 * b_{i2} + \lambda_3 * b_{i3} + \lambda_4 * b_{i4} + \lambda_5 * b_{i5} + \varepsilon_i \qquad \text{(APT)}$$

and:

$$\varepsilon_i = \lambda_0 + \lambda_1 * \beta_i + \eta_i \qquad \text{(CAPM)}$$

The results for the 2005-2007 sub-interval are:

CAPM residuals/factor loadings of APT

Table 1.

	Coefficients	t Stat	P-value
λ_0	0,9477	0,5978	0,5524
λ_1	-4,4405	-2,2171	0,0308
λ_2	-2,2443	-0,8620	0,3924
λ_3	6,7130	3,0572	0,0034
λ_4	-4,2724	-1,8166	0,0748
λ_5	4,6175	1,7248	0,0902

APT residuals/β coefficients of CAPM

Table 2.

	Coefficients	t Stat	P-value
λ_0	-2,0682	-2,0745	0,0424
λ_1	2,4315	2,4632	0,0167

It can be seen that CAPM residuals are captured by factors 1 and 3 of APT, which are statistically significant, whereas APT residuals are captured by λ_1 of CAPM, which is also significant. In order to decide which model is better, we make use of the "R-square" indicator: 0,3865 for the first regression (equivalent with the fact that APT is explaining 38,65% of CAPM's unexplained variance) and 0,0947 for the second one (CAPM is explaining 9,47% of APT's unexplained variance). Hence it can be established that APT seems to be more appropriate in explaining stocks' returns.

For the 2008-2010 sub-interval the following estimations are obtained:

CAPM residuals/factor loadings of APT

Table 3.

	Coefficients	t Stat	P-value
λ_0	1,7406	1,7592	0,0842
λ_1	-3,4536	-2,8266	0,0065
λ_2	-1,0293	-0,7647	0,4477
λ_3	0,2748	0,2026	0,8401
λ_4	-0,2805	-0,1797	0,8580
λ_5	-1,3308	-0,9652	0,3387

APT residuals/β coefficient of CAPM

Table 4.

	Coefficients	t Stat	P-value
λ_0	-1,2990	-2,0847	0,0415
λ_1	1,4233	2,3461	0,0224

CAPM residuals are captured by factor 1 of APT, and APT residuals are captured by the factor of CAPM. Additionally, "R-square" indicator has a value of 0,1528 for the first regression (equivalent with APT explaining 15,28% of CAPM's unexplained variance) and 0,0866 for the second regression (CAPM explaining 8,66% of APT's unexplained variance). Once again, APT seems like a more appropriate model.

If the entire time period is considered:

CAPM residuals/factor loadings of APT

Table 5.

	Coefficients	t Stat	P-value
λ_0	2,0134	2,6176	0,0114
λ_1	-4,3630	-4,8965	0,0092
λ_2	-2,2860	-1,7950	0,0782
λ_3	2,4038	2,4322	0,0183
λ_4	1,7488	1,3202	0,1923
λ_5	-3,3751	-3,0151	0,0039

APT residuals/β coefficient of CAPM

Table 6.

	Coefficients	t Stat	P-value
λ_0	-2,0688	-3,5081	0,0008
λ_1	2,2608	3,8167	0,0003

CAPM residuals are captured by factors 1,3 and 5 of APT, while

APT residuals are captured by the factor of CAPM. "R-square" indicator has a value of 0,5145 for the first regression (equivalent with the fact that APT is explaining 51,45% of CAPM's unexplained variance) and 0,2007 for the second regression (CAPM is explaining 20,07% of APT's unexplained variance). Both models are performing better in the entire period of time than in the sub-periods, and again APT is more relevant than CAPM.

It can be observed that the APT model is more reasonable in explaining the variations of the stocks returns', both in sub-periods and in the full period of time. This conclusion was found by Chen's study as well as by the majority of the later studies. A further comment is needed here: while Chen's study found CAPM totally not able to explain part of the APT's residuals, this present analysis is finding CAPM able to explain part of APT's residuals, but is a smaller percentage than the capacity of APT to explain part of CAPM's residuals.

Davidson-MacKinnon equation

This method was suggested by economists Davidson and MacKinnon, and the first empirical test to use it was again Chen's. The procedure works like this: first the predicted returns R_{APT} and R_{CAPM} are needed, returns that are foreseen by APT and CAPM models. Then a multiple regression is employed, having the below form:

$$R_i = \alpha * R_{APT} + (1-\alpha) * R_{CAPM},$$

where α measures the efficiency for APT, and (1-α) measures the one for CAPM. If CAPM is more correct than APT, then (1-α) will have an estimated value closer to 1, and α will have an estimated value closer to 0 and the other way around: is APT is more correct than CAPM, then (1-α) will have a value closer to 0, and α, one closer to 1. Chen's study found that the second scenario was valid, and the same analysis will be made for the current situation.

The regression results are:

Regression results

Table 7.

	Coefficients	Standard error
A	0,9063	0,1712
(1-α)	0,1413	0,2047

It can be clearly observed that the estimated value for α is closer to 1, and the one for (1-α) is closer to 0. Thus, APT looks like the more appropriate model compared with CAPM for this sub-Period

The regression shows the below estimations for the 2 indicators:

Regression results

Table 8.

	Coefficients	Standard error
α	0,8058	0,1448
(1-α)	0,4386	0,1817

Although tighter values are available now, the indicator for APT (which is α) continues to be closer to 1, while the one for CAPM (which is 1-α) continues to be closer to zero. So even for the entire time period, the first model seems more appropriate than the latter, using the Davidson-MacKinnon equation criteria. These findings are similar to Chen's, which concluded almost the same in his analysis. Groenewold and Fraser[5], Cagnetti[1], Theriou, Aggelidis and Maditinos[8] and Yuen[9] underline resembling conclusions as well.

Posterior odds ratio

It is a more formal indicator than the Davidson-MacKinnon equation, but was used pretty frequently in various tests and it has a strong theoretical basis, especially a statistical one. Zellner[10] gives the calculus formula for the indicator, having such a general form:

$POR = (SSE_A/SSE_B)^{N/2} * N^{(kA-kB)/2}$

where:

SSE_A = "error sum of squares" indicator for model A;

SSE_B = "error sum of squares" indicator for model B;

N = the number of considered observations for both models;

k_A = the number of independent variables of model A;

k_B = the number of independent variables of model B;

If the above formula offers a result with a value below 1(POR<1), the model A performs better than model B; a value above 1(POR>1) means a better performance from model B.

In this study, model A will be APT, whereas model B will be CAPM. The formula for calculating the "error sum of squares" indicator is:

$$SSE = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

that is the sum of squares of the differences between the real returns and the returns estimated through both models.

The indicators for the first sub-interval are:

SSE_A = 1110,6909 and SSE_B = 1633,8994
and from this:
$POR_{2005\text{-}2007}$ = 0,00478

It can be seen that the result has a value below 1 and, as being said, this means that model A performs better than model B, thus is APT performing better than CAPM in this 3-year sub-period.

The second sub-interval offers the following indicators:
SSE_A = 311,3083 and SSE_B = 356,1947
hence:
$POR_{2008\text{-}2010}$ = 40,6239

This time, the result has a value above 1, showing a superiority of the B model above the A model, which is a superiority of CAPM above APT.
Finally, studying the entire interval, the following values for the indicators arise:

SSE_A = 234,8943 and SSE_B = 327,3897
and from this:
$POR_{2005\text{-}2010}$ = 0,0333

The value of the result being higher than 1, we can conclude that the A model, which is APT, performs better than the B model, which is CAPM, on the entire period of time.

The findings obtained here are again close to the ones from the majority of previous studies (among which Cagnetti; Theriou, Aggelidis and Maditinos and more), which also showed the superiority of APT in most of the cases.

The summary of the comparing between the 2 models, in function of each studied criteria and with the underline of the more appropriate one, is described in table 9:

Comparison between the two models

Table 9.

	Residual analysis	***Davidson-MacKinnon equation***	***Posterior odds ratio***
Sub-period 2005-2007	APT	APT	APT
Sub-period 2008-2010	APT	APT	CAPM
Whole period 2005-2010	APT	APT	APT

Conclusions

With a single exception, given by the 2008-2010 sub-period regarding the "Posterior odds ratio" criteria, APT proved to be the a more appropriate model than CAPM for explaining the stocks' returns. This fact is somehow logical, since the first model permits that those returns to be influenced by multiple factors, whereas the latter states that only one factor, namely the market portfolio can influence them.

Aspects:

• *the* availability and the accuracy of the data is not fully guaranteed, and the missing observations were completed by interpolation, which is not similar with being totally exact;

• using a "proxy" for the riskless interest rate;

• the existence of some possible miscalculating, including some caused by the author;

Notes

2. http://www.bvb.ro
3. http://www.kmarket.ro
4. www.bnr.ro

References

- Cagnetti, A.(2007) – ***"CAPM and APT in the Italian Stock Market: an empirical study."***, Unpublished article;

- Chen, N.F.(1983) – ***"Some empirical tests of the theory of arbitrage pricing."***, Journal of Finance, 1393-1414;

- Copeland, T.; Weston, F.(2004) – ***"Financial Theory and Corporate Policy"***, 4th edition, Addison-Wesley Publishing Company;

- Davidson, R.; MacKinnon, J.(1981) – ***"Several tests for model specification in the presence of alternative hypothesis."***, Econometrica 49(3), 781-793;

- Groenewold, N.; Fraser, P.(1997) – "***Share prices and macroeconomic factors.***", Journal of Business Finance and Accounting 24(9), 1367-1383;

- Pieleanu, F.(2011) – "***Empirical testing of CAPM model on the Romanian capital market.***", Studii şi Cercetări de Calcul Economic şi Cibernetică Economică 45(3-4/2011), 125-146;

- Pieleanu, F.(2011) – "***The APT model and its applicability in Romania's case.***", Paper sent for publication, Academy of Economic Studies Bucharest;

- Theriou, N.; Aggelidis, V.; Maditinos, D.(2007) – ***"Testing the relation between risk and returns using CAPM and APT: the case of Athens stock exchange."***, Paper from TEI of Kavala, School of Business and Economics;

- Yuen, M.(1985) – "***An empirical test of the APT in the Hong Kong stock market.***", Master Thesis and Dissertation;

- Zellner, A.(1979) – "***Statistical analysis of econometric models.***", Journal of the American Statistical Association 74(367), 628-651;