Empirical Results of Modeling EUR/RON Exchange Rate using ARCH, GARCH, EGARCH, TARCH and PARCH models

Andreea - Cristina PETRICĂ PhD. Student
(Andreea.petrica@yahoo.com)
Bucharest University of Economic Studies

Prof. Stelian STANCU PhD.
(stelian.stancu@csie.ase.ro)
Bucharest University of Economic Studies

ABSTRACT
The aim of this study consists in examining the changes in the volatility of daily returns of EUR/RON exchange rate using on the one hand symmetric GARCH models (ARCH and GARCH) and on the other hand the asymmetric GARCH models (EGARCH, TARCH and PARCH), since the conditional variance is time-varying. The analysis takes into account daily quotations of EUR/RON exchange rate over the period of 04th January 1999 to 13th June 2016. Thus, we are modeling heteroscedasticity by applying different specifications of GARCH models followed by looking for significant parameters and low information criteria (minimum Akaike Information Criterion). All models are estimated using the maximum likelihood method under the assumption of several distributions of the innovation terms such as: Normal (Gaussian) distribution, Student’s t distribution, Generalized Error distribution (GED), Student’s with fixed df. Distribution, and GED with fixed parameter distribution. The predominant models turned out to be EGARCH and PARCH models, and the empirical results point out that the best model for estimating daily returns of EUR/RON exchange rate is EGARCH(2, 1) with Asymmetric order 2 under the assumption of Student’s t distributed innovation terms. This can be explained by the fact that in case of EGARCH model, the restriction regarding the positivity of the conditional variance is automatically satisfied.

Keywords: Exchange Rate Volatility, Heteroscedasticity, Symmetric GARCH Models, Asymmetric GARCH Models, Fat-tails, Volatility Clustering, Leverage Effect

JEL Classification: C13, C22, C26, C51, C52, C55, C58
1. INTRODUCTION

Nowadays, volatility represents an important tool in Economy and plays an important role in the area of risk management. A disadvantage of stationary linear models is given by not taking into account the fact that volatility is time-varying, and so the length of the forecast intervals remains constant as long as the model parameters are not changed, even if new data become disposable, something that is unlikely in the case of ARCH family models where the forecast intervals include any sudden changes in volatility, keeping all the parameters unchanged. This characteristic makes ARCH family models such important tool in analyzing financial and monetary time series.

From this point of view, the main aim of this paper is to investigate the volatility of EUR/RON exchange rate using non-linear processes, so we proceed to apply ARMA models with different Generalized Autoregressive Conditionally Heteroscedastic (GARCH) errors, including both symmetric and asymmetric models. The main feature of analyzing the conditional variance of exchange rates - and financial time series in general - is given by the fact that it is not directly observable. Starting with the 1980s, there were developed many models for estimating the conditional volatility of financial series. Thus, the starting point is given by Engle (1982), who introduced in the literature the Autoregressive Conditionally Heteroscedastic process (ARCH). Four years later, the ARCH model is extended by Bollerslev (1986) to the so-called the Generalized Autoregressive Conditionally Heteroscedastic model (GARCH). Since both processes have symmetric distribution, so they fail with respect to leverage effect, Nelson (1991), Zakoian (1994), Glosten et al. (1993) and Ding et al. (1993) provide an extended methodological framework to allow for asymmetric effects of positive and negative innovations. For a short overview of recent literature, we bring up the following recent studies: Elsheikh and Zakaria (2011) estimate the volatility of Khartoum Stock Exchange over the period January 2006 – November 2010 using both symmetric and asymmetric GARCH models. The results reveal the presence of leverage effect leading to the better fit of the asymmetric models. Begu et al. (2012) estimated the daily returns of RON/EUR exchange rates from 05.01.2009 to 12.10.2012 using the following models: ARCH, GARCH, EGARCH and TGARCH. They have reached the conclusion that GARCH model is the most promising for investigating the volatility and “fit the sample data good enough”.

1. The Khartoum Stock Exchange represents the principal stock exchange of Sudan.
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(2013) pointed out the effects of currency depreciation in case of Romania, where the exchange rate is considered an influencing factor of the economy used by some countries to correct certain disequilibrium produced as a result of the financial crisis who “hit” in many areas considered previously to be infallible. Damianova (2014) applies GARCH methodology using the dollar-adjusted daily return of the Bucharest Exchange Trading Index and finds out the market inefficiency from the Bucharest Stock Exchange. Dutta (2014) investigated the U.S. – Japan exchange rate (daily observations) by using and comparing the symmetric and asymmetric GARCH models taking into account two distributions (normal and heavy-tailed) and the results of the analysis indicate that the second distribution gives a reduced persistence. Dineată and Anghelache (2015) analyzed the Romanian capital market sensitivity to exchange rate fluctuation through a GARCH model, using daily returns of BET-C Index and EUR/RON exchange rate over the period of 03th January 2005 to 20th June 2014. The empirical results showed that the exchange rate has a strong impact on both the average return on capital market, and on its volatility.

2. METHODOLOGY

The goal of this section is to describe the volatility modeling techniques in terms of symmetry (ARCH and GARCH processes) and asymmetry (EGARCH, TARCH/GJR-GARCH and PARCH processes) concepts, giving an overview of the processes used in this paper for modeling the daily returns of EUR/RON exchange rate. Thus, the key concept in autoregressive conditionally heteroscedastic model and its extensions is represented by the conditional variance. Furthermore, their mathematical representation is given by two equations, one for conditional mean (the predictable component) - where the Box-Jenkins approach can be used, and another one for conditional variance which represents the unpredictable component – where the innovation terms from the conditional mean are modeled using symmetric and asymmetric GARCH models. The first equation is a function of the expected value conditional on the information available at time t-1 and the current value of the innovation term. The second equation can be described as a quadratic function of past values of the innovations \{e_t\}. Another thing worth mentioning is that the difference between the symmetric and asymmetric GARCH models is given by the impact of good and bad news on volatility. In case of symmetric models good and bad news have the same impact on volatility, while in case of asymmetric models bad news affect more the volatility than the good news. In other words, the symmetric models can’t explain the leverage effect.
2.1. The Autoregressive Conditionally Heteroscedastic Model (ARCH)

The Autoregressive Conditionally Heteroscedastic process for the series \( \{e_i\} \), \( \text{ARCH}(q) \), is expressed by allowing the conditional variance to be described by its past \( q \) squared innovations as follows:

\[
\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 = \omega + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2) \tag{1}
\]

where:
- \( \sigma_t^2 \) – the conditional variance of the innovations (errors) at time \( t \);
- \( \omega \) – the constant term;
- \( \varepsilon_{t-q}^2 \) – the squared error at time \( t-q \);
- \( \alpha_1, \ldots, \alpha_q \) – ARCH terms i.e. volatility shocks from prior periods.

To ensure that \( \sigma_t^2 \) is positive we have to impose some restrictions with respect to the parameters in the conditional variance equation: \( \omega > 0 \) and \( \alpha_i \geq 0 \) for \( i = 1, 2, \ldots, q \), and \( \sum_{i=1}^{q} \alpha_i < 1 \). If we rewrite eqn[1] in terms of the lag operator \( (L) \), we get the following representation of the ARCH\((q)\) model:

\[
\sigma_t^2 = \omega + \left( \sum_{i=1}^{q} \alpha_i L^i \right) \varepsilon_t^2 \tag{2}
\]

2.2. The Generalized Autoregressive Conditionally Heteroscedastic Model (GARCH)

The generalized autoregressive conditionally heteroscedastic process, \( \text{GARCH}(p,q) \), is expressed by allowing the conditional variance to be described by both past \( q \) squared errors and \( p \) conditional variances as follows:

\[
\sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i \varepsilon_{t-i}^2) + \sum_{j=1}^{p} (\beta_j \sigma_{t-j}^2) \tag{3}
\]

where:
- \( \omega \) – the constant term;
- \( \alpha_i \) – ARCH terms i.e. volatility shocks from prior periods.
- \( \beta_j \) – GARCH terms i.e. the persistence of volatility;
- \( p \) – the number of lagged conditional variance terms \( \sigma^2 \);
- \( q \) – the number of lagged errors \( \varepsilon^2 \).

To ensure that \( \sigma_t^2 \) is strictly positive we have to impose some restrictions with respect to the parameters in the conditional variance equation: \( \omega > 0 \), \( \alpha_i \geq 0 \), for \( i = 1, q \), \( \beta_j \geq 0 \), for \( j = 1, p \) and \( \sum_{i=1}^{q} \alpha_i + \sum_{j=1}^{p} \beta_j < 1 \).
(i.e. good news and bad news have a declining impact on future volatility). If we rewrite eqn[3] in terms of the lag operator \((L)\), we get:

\[
\sigma_t^2 = \omega + \left( \sum_{i=1}^{q} \alpha_i L^i \right) \varepsilon_t^2 + \left( \sum_{j=1}^{p} \beta_j L^j \right) \sigma_{t-j}^2
\]  

[4]

2.3. The Exponential Generalized Autoregressive Conditionally Heteroscedastic Model (EGARCH)

The Exponential GARCH model (EGARCH) introduced by Nelson (1991) models the logarithm of conditional variance and it captures asymmetric responses of the conditional variance to good and bad news. The equation of the EGARCH model can be written in the following form:

\[
\log(\sigma_t^2) = \omega + \sum_{i=1}^{q} \alpha_i \frac{|\varepsilon_{t-i}|}{\sigma_{t-i}} + \sum_{k=1}^{r} \gamma_k \frac{\varepsilon_{t-k}}{\sigma_{t-k}} + \sum_{j=1}^{p} \beta_j \log(\sigma_{t-j}^2)
\]  

[5]

where \(\gamma_k\) represents the asymmetry parameter (leverage effect). \(\gamma_k \neq 0\) points out the presence of asymmetry, while \(\gamma_k < 0\) shows that volatility rises more after bad news \((\varepsilon_{t-1} < 0)\) than after good news \((\varepsilon_{t-1} > 0)\).

2.4. The Exponential Generalized Autoregressive Conditionally Heteroscedastic Model (TARCH and GJR-GARCH)

The difference between the Threshold ARCH (TARCH) model introduced by Zakoian (1994) and the GJR-GARCH model introduced by Glosten et al. (1993) is given by using the specification on the conditional standard deviation instead of conditional variance. Hence, the conditional variance for the TARCH model is represented by eqn[6], while the conditional variance for the GJR-GARCH model is represented by eqn[7]:

\[
\sigma_t = \omega + \sum_{i=1}^{q} (\alpha_i^+ \cdot \varepsilon_{t-i}^+ + \alpha_i^- \cdot \varepsilon_{t-i}^-) + \sum_{j=1}^{p} \beta_j \cdot \sigma_{t-j}
\]  

[6]

where \(\varepsilon^+ = \{\varepsilon, \varepsilon > 0\text{ and }\varepsilon^- = \{0, \varepsilon < 0, \varepsilon > 0\}\).  

\[
\sigma_t^2 = \omega + \sum_{i=1}^{q} (\alpha_i + \gamma_i \cdot 1_{\varepsilon_{t-i}}) \cdot \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2
\]  

[7]
2.5. Power ARCH (PARCH)

The Power ARCH model (PARCH) introduced by Ding et al. (1993) is an asymmetric model and it has the following representation:

$$\sigma_t^\delta = \omega + \sum_{i=1}^{q} \alpha_i f_i(\varepsilon_{t-i}) + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^{\delta}$$

[8]

where:

- $f_i(\varepsilon_{t-i}) \equiv (|\varepsilon_{t-i}| - \gamma_i \cdot \varepsilon_{t-i})^{\delta}$, $i = 1, q$;
- $\alpha_i$ - the standard ARCH term;
- $\beta_j$ - the standard GARCH term;
- $\gamma_i$ - the leverage parameter ($|\gamma_i| < 1$);
- $\delta$ - the parameter for the power term ($\delta > 0$).

To ensure that $\sigma_t^\delta$ is strictly positive we have to impose some restrictions with respect to the parameters in the conditional variance equation: $\omega > 0$, $\alpha_i \geq 0$, with at least one $\alpha_i > 0$, $(i = 1, q)$ and $\beta_j \geq 0$ $(j = 1, p)$.

It is worth mentioning that for $\delta = 2$, PARCH model becomes a classic GARCH model that allows for asymmetry, while $\delta = 1$ leads to the estimation of the conditional standard deviation.

3. DATA AND EMPIRICAL RESULTS

This section reveals and provides with graphs the empirical results obtained by employing the symmetric and asymmetric GARCH models to the percentage daily returns of the EUR/RON exchange rate $\left(Return_t = \log \left( \frac{Quotation_t}{Quotation_{t-1}} \right) * 100 \right)$, respectively: ARCH, GARCH, TARCH, EGARCH and PARCH models. The data were collected from the official website of the National Bank of Romania and consist of 4439 daily quotations of EUR/RON exchange rate over the period of 04th January 1999 to 13th June 2016. The evolution of daily price and return series for EUR/RON exchange rate during the analyzed period is presented in Figure 1.
The evolution of the daily EUR/RON exchange rate presented in Figure 1 denotes that the series has constant and trend. Likewise, the correlogram reveals very high autocorrelations for all lags, with values decreasing very slowly and reaching a value of 0.959 at the 36th lag. This leads together with High Q-Stat test values (Prob. < 0.05) to the conclusion that the aforementioned series is non-stationary. On the other hand, the ACF and PACF coefficients of daily returns of EUR/RON exchange rate are close to zero at all lags, starting from 0.104 at lag 1 and decreasing quickly at lag 2 (ACF= -0.068 and PACF= -0.080), this indicates that the series of daily returns is generated by a random walk process. Thus, to investigate stationarity, besides visual inspection given by graphs which represents the first step of making an idea if our series are stationary or not, we also have statistical tools for investigating the stationarity of the series. Taking these into account, Table 1 summarizes the findings of the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests which confirm that daily EUR/RON exchange rate series is not stationary, but in terms of daily percentage returns we find stationarity at all conventional confidence levels (1%, 5% and 10%).
Augmented Dickey-Fuller and Phillips-Perron Tests for both series – prices and returns

Table 1

<table>
<thead>
<tr>
<th>ADF Test (Probability)</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>Daily EUR/RON exchange rate</td>
<td>-2.377248 (0.3915)</td>
</tr>
<tr>
<td>Daily EUR/RON return series</td>
<td>-41.39923 (0.0000)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>PP Test (Probability)</th>
<th>Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1%</td>
</tr>
<tr>
<td>Daily EUR/RON exchange rate</td>
<td>-2.371825 (0.3944)</td>
</tr>
<tr>
<td>Daily EUR/RON return series</td>
<td>-59.48531 (0.0001)</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.

Note: The results presented above correspond to the ADF and PP Tests including intercept and trend in case of EUR/RON exchange rate series and ADF and PP Tests without intercept and trend in case of daily returns of EUR/RON exchange rate.

The descriptive statistics reported in Figures 2-3 show:
- negative asymmetry (Skewness = -1.107702) for daily EUR/RON exchange rates and positive asymmetry (Skewness = 0.884889) for returns;
- leptokurtic distribution for both daily EUR/RON exchange rates and returns (Kurtosis = 3.235483 and 17.01422, respectively);
- non-normal distribution for both series (Jarque-Bera = 918.03 and 36896.51, respectively, with zero probability in both cases).

Descriptive Statistics for daily EUR/RON exchange rate

![Descriptive Statistics for daily EUR/RON exchange rate](image)
As we said in the Introduction section, the main disadvantage of stationary linear models is given by not taking into account the fact that volatility is time-varying, which in the case of ARCH family models the forecast intervals include any sudden changes in volatility, keeping all the parameters unchanged. This characteristic makes these models such important tool in analyzing time series. Therefore, using Box-Jenkins approach we employ the Autoregressive Moving Average models (ARMA) for the conditional mean in order to find the adequate model. Based on the correlogram of daily returns the maximum orders of the AR and MA processes is 3, so using EViews 9 software we estimated 10 models and the results are presented in Table 2. As can be seen, according to the all three used criterions (Akaike Information Criterion, Schwarz Criterion and Hannan-Quinn Criterion) the adequate model achieved for modeling the conditional mean is AR(3) model (AIC=1.463956).
The estimated ARMA models for the conditional mean of the daily returns of EUR/RON exchange rate

<table>
<thead>
<tr>
<th>The estimated model</th>
<th>Adjusted R-squared</th>
<th>Akaike info criterion</th>
<th>Schwarz criterion</th>
<th>Hannan-Quinn criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>AR(1)</td>
<td>0.010474</td>
<td>1.473531</td>
<td>1.477856</td>
<td>1.475056</td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.016628</td>
<td>1.467521</td>
<td>1.473287</td>
<td>1.469554</td>
</tr>
<tr>
<td>AR(3)</td>
<td>0.020350</td>
<td>1.463956</td>
<td>1.471164</td>
<td>1.466498</td>
</tr>
<tr>
<td>MA(1)</td>
<td>0.012118</td>
<td>1.471869</td>
<td>1.476193</td>
<td>1.473394</td>
</tr>
<tr>
<td>MA(2)</td>
<td>0.014555</td>
<td>1.469625</td>
<td>1.475391</td>
<td>1.471658</td>
</tr>
<tr>
<td>MA(3)</td>
<td>0.020064</td>
<td>1.464248</td>
<td>1.471456</td>
<td>1.466789</td>
</tr>
<tr>
<td>ARMA(1,1)</td>
<td>0.012982</td>
<td>1.471219</td>
<td>1.476985</td>
<td>1.473252</td>
</tr>
<tr>
<td>ARMA(1,2)</td>
<td>0.017356</td>
<td>1.467005</td>
<td>1.474213</td>
<td>1.469546</td>
</tr>
<tr>
<td>ARMA(2,1)</td>
<td>0.018880</td>
<td>1.465455</td>
<td>1.472663</td>
<td>1.467996</td>
</tr>
<tr>
<td>ARMA(2,2)</td>
<td>0.020182</td>
<td>1.464353</td>
<td>1.473003</td>
<td>1.467403</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.

Consider the mean equation as being given by the econometric model AR(3) since previous results. Given the fact that we are interested in analyzing the volatility of daily returns of EUR/RON exchange rate, we use the residuals of the autoregressive process of order 3 to test for ARCH effects using ARCH LM Test. The number of observations times R-squared is evaluated against Chi-Square distribution, and the Obs*R-squared of 88.14303 with a probability less than 5% indicates that the null hypothesis is rejected, case in which we can run the ARCH family models. This means that we have established that the conditional variance is time-varying. The results are summarized in Table 3.

Testing residuals for ARCH effects

<table>
<thead>
<tr>
<th>Obs*R-squared</th>
<th>88.14303</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prob. Chi-Square(1)</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.
Note: $H_0$: There are no ARCH effects in the residual series.

Next, the difficulty is given by the construction of the models that “accommodate heteroskedasticity so that valid coefficient estimates and models are obtained for the variance of the error terms”.¹ We continue this study by examining the changes in the volatility of daily returns of EUR/RON exchange rate using symmetric GARCH models (ARCH and GARCH) and asymmetric GARCH models (EGARCH, TARCH and PARCH). The first

¹ Rachev et al. (2007), pp. 279
class treats the shocks/errors/innovations as symmetric, meaning that shocks affect the conditional variance in the same way whether they are positive or negative, whereas the second class treats them as having an asymmetric effect on the conditional variance.

Therefore, having set up that the conditional variance is time-varying, the next step in this study consists in modeling this heteroscedasticity by applying the symmetric and asymmetric GARCH models. For this, we are testing different specifications followed by looking for significant parameters and low information criteria (minimum Akaike Information Criterion). All models are estimated using the maximum likelihood method under the assumption of several distributions of the innovation terms such as: Normal (Gaussian) distribution, Student’s t distribution, Generalized Error distribution (GED), Student’s with fixed df. Distribution, and GED with fixed parameter distribution.

**Akaike Information Criterion results of different GARCH models**

<table>
<thead>
<tr>
<th>Error Distribution</th>
<th>Normal (Gaussian)</th>
<th>Student’s t</th>
<th>Generalized Error (GED)</th>
<th>Student’s with fixed df.</th>
<th>GED with fixed parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARCH(1)</td>
<td>1.233917</td>
<td>1.007439</td>
<td>1.001994</td>
<td>1.093013</td>
<td>1.094990</td>
</tr>
<tr>
<td>ARCH(2)</td>
<td>1.135639</td>
<td>0.948401</td>
<td>0.948893</td>
<td>1.011676</td>
<td>1.015676</td>
</tr>
<tr>
<td>GARCH(1,1)</td>
<td>0.882523*</td>
<td>0.797746</td>
<td>0.803225</td>
<td>0.813491*</td>
<td>0.817117</td>
</tr>
<tr>
<td>GARCH(1,2)</td>
<td>0.876557**</td>
<td>0.795905</td>
<td>0.800829</td>
<td>0.810915</td>
<td>0.813683</td>
</tr>
<tr>
<td>GARCH(2,1)</td>
<td>0.876426***</td>
<td>0.795377</td>
<td>0.800473</td>
<td>0.810694*</td>
<td>0.813423</td>
</tr>
<tr>
<td>GARCH(2,2)</td>
<td>0.876780***</td>
<td>0.795822</td>
<td>0.800924*</td>
<td>0.811143*</td>
<td>0.813872*</td>
</tr>
<tr>
<td>EGARCH(1,1) Asymmetric order 1</td>
<td>0.884665</td>
<td>0.794197</td>
<td>0.801765</td>
<td>0.810639</td>
<td>0.816702****</td>
</tr>
<tr>
<td>EGARCH(1,2) Asymmetric order 2</td>
<td>0.881407</td>
<td>0.791276</td>
<td>0.799337</td>
<td>0.807424</td>
<td>0.813924</td>
</tr>
<tr>
<td>EGARCH(2,1) Asymmetric order 2</td>
<td>0.875815</td>
<td>0.789429****</td>
<td>0.797095</td>
<td>0.805045****</td>
<td>0.810718**</td>
</tr>
<tr>
<td>EGARCH(2,2) Asymmetric order 2</td>
<td>0.875409</td>
<td>0.787957</td>
<td>0.796127****</td>
<td>0.803903</td>
<td>0.809881</td>
</tr>
<tr>
<td>TARCH(1,1) Threshold order 1</td>
<td>0.881618</td>
<td>0.797612****</td>
<td>0.802645</td>
<td>0.813216</td>
<td>0.816440</td>
</tr>
<tr>
<td>TARCH(1,2) Threshold order 1</td>
<td>0.876095***</td>
<td>0.795939****</td>
<td>0.800526****</td>
<td>0.810863****</td>
<td>0.813339</td>
</tr>
<tr>
<td>PARCH(1,1)</td>
<td>0.880366*</td>
<td>0.794487</td>
<td>0.800920</td>
<td>0.810265</td>
<td>0.814731</td>
</tr>
<tr>
<td>PARCH(1,2)</td>
<td>0.872975</td>
<td>0.791853</td>
<td>0.797705</td>
<td>0.806716</td>
<td>0.810271</td>
</tr>
<tr>
<td>PARCH(2,1)</td>
<td>0.872857*</td>
<td>0.790706</td>
<td>0.797979</td>
<td>0.806048*</td>
<td>0.809813</td>
</tr>
<tr>
<td>PARCH(2,2)</td>
<td>0.873193</td>
<td>0.790876</td>
<td>0.797517****</td>
<td>0.805896*</td>
<td>0.810264****</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.
Note: * The coefficient of AR(2) term in the conditional mean equation is statistically significant at 10% confidence level.

** The coefficient of AR(3) term in the conditional mean equation is statistically significant at 10% confidence level.

*** The coefficients of AR(2) and AR(3) terms in the conditional mean equation is statistically significant at 10% confidence level.

* The coefficient of GARCH(-2) term is not statistically significant at any confidence level.

*** The coefficient of RESID(-1)/@SQRT(GARCH(-1)) term is not statistically significant at any confidence level.

**** The coefficient of RESID(-1)/@SQRT(GARCH(-1)) term is statistically significant at 10% confidence level.

* The coefficient of RESID(-1)/@SQRT(GARCH(-1)) term is statistically significant at 10% confidence level.

*** The coefficient of RESID(-1)^2 (resid<0) term is not statistically significant at any confidence level.

** The coefficient of RESID(-1)^2 (resid<0) term is statistically significant at 10% confidence level.

**** The coefficient of @SQRT(GARCH(-2))^C(10) term is not statistically significant at any confidence level.

As reported in Table 4, we achieved based on Akaike Information Criterion that the best models have proved to be EGARCH and PARCH. Moreover, Table 5 summarizes the best GARCH model obtained for each error distribution separately. Comparing all the results presented in Table 5, we notice that daily returns of EUR/RON exchange rate could be modeled using EGARCH(2,1) with Asymmetric order 2 under the assumption of Student’s t distributed innovation terms.

### The best GARCH model achieved under different error distribution

**Table 5**

<table>
<thead>
<tr>
<th>Error Distribution</th>
<th>Normal (Gaussian)</th>
<th>Student’s t</th>
<th>Generalized Error (GED)</th>
<th>Student’s with fixed df.</th>
<th>GED with fixed parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>PARCH(1,2)</td>
<td>EGARCH(2,1) asymmetric order 2</td>
<td>PARCH(2,1)</td>
<td>EGARCH(2,1)</td>
<td>PARCH(2,1)</td>
</tr>
<tr>
<td>Minimum AIC</td>
<td>0.872975</td>
<td>0.787957</td>
<td>0.797079</td>
<td>0.803903</td>
<td>0.809813</td>
</tr>
</tbody>
</table>

Source: Authors’ calculations.

Note: All the estimated parameters in the conditional variance equation are statistically significant at all confidence levels (1%, 5% and 10%).

Table 6 points out that AR(3)-EGARCH(2,1) with Asymmetric order 2 and Student’s t error distribution is the most adequate model for estimating
the conditional variance (having the minimum AIC, which is 0.787957). The presence of asymmetry in the returns of EUR/RON exchange rate is confirmed by the non-zero leverage parameters \( \gamma_{k,k=1,2} \) (\( \gamma_1 = 0.059061 \) and \( \gamma_2 = -0.078136 \)). A negative leverage parameter indicates an asymmetric response for positive returns in the conditional variance equation, while a positive leverage parameter indicates that bad news leads to increased volatility. In case of volatility persistence, Table 6 reveals a very large value of the estimated persistence parameter (\( \beta = 0.993305 \)) resulting in a slowly decreasing of the rises in the conditional variance due to shocks.

### Estimation results of EGARCH(2,1) Asymmetric order 2 – Student’s t error distribution

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conditional Mean Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>0.001358</td>
<td>0.003377</td>
<td>0.402083</td>
<td>0.6876*</td>
</tr>
<tr>
<td>AR(1)</td>
<td>0.047834</td>
<td>0.016246</td>
<td>2.944421</td>
<td>0.0032</td>
</tr>
<tr>
<td>AR(2)</td>
<td>-0.036291</td>
<td>0.014342</td>
<td>-2.530440</td>
<td>0.0114</td>
</tr>
<tr>
<td>AR(3)</td>
<td>-0.041527</td>
<td>0.014480</td>
<td>-2.867885</td>
<td>0.0041</td>
</tr>
<tr>
<td>Conditional Variance Equation</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.193853</td>
<td>0.015252</td>
<td>-12.71001</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>0.416476</td>
<td>0.043840</td>
<td>9.499823</td>
<td>0.0000</td>
<td></td>
</tr>
<tr>
<td>-0.167695</td>
<td>0.042571</td>
<td>-3.939215</td>
<td>0.0001</td>
<td></td>
</tr>
<tr>
<td>0.059061</td>
<td>0.029140</td>
<td>2.026764</td>
<td>0.0427</td>
<td></td>
</tr>
<tr>
<td>-0.078136</td>
<td>0.028240</td>
<td>-2.766824</td>
<td>0.0057</td>
<td></td>
</tr>
<tr>
<td>0.993305</td>
<td>0.002386</td>
<td>416.3580</td>
<td>0.0000</td>
<td></td>
</tr>
</tbody>
</table>

*The coefficient is not significant at any confidence level (1%, 5% and 10%).
Source: Authors’ calculations.

Withal, ARCH-LM Test indicates that the conditional variance equation found is well specified as there is no ARCH effect left in the innovations (Obs.* R-squared= 0.238921with Prob. Chi-Square(1)= 0.6250).

### 4. CONCLUSIONS

This study consists in examining the changes in the volatility of daily returns of EUR/RON exchange rate using both symmetric and asymmetric GARCH models. The analysis takes into account daily quotations of EUR/RON exchange rate over the period of 04th January 1999 to 13th June 2016. Thus, we are modeling heteroscedasticity by applying different specifications of GARCH models (16 models as can be seen from Table 4) followed by looking for significant parameters and low information criteria.
(minimum Akaike Information Criterion). All models are estimated using the maximum likelihood method under the assumption of several distributions of the innovation terms such as: Normal (Gaussian) distribution, Student’s t distribution, Generalized Error distribution (GED), Student’s with fixed df. Distribution, and GED with fixed parameter distribution. The predominant models turned out to be EGARCH and PARCH models, but the empirical results point out that the best model for estimating daily returns of EUR/RON exchange rate is EGARCH(2,1) with Asymmetric order 2 under the assumption of Student’s t distributed innovation terms (having the minimum AIC, which is 0.787957). This can be explained on the one hand by the fact that the volatility reacts asymmetrically to the good and bad news and on the other hand by the fact that in case of EGARCH model, the restriction regarding the positivity of the conditional variance is automatically satisfied. The presence of leverage effect in the returns of EUR/RON exchange rate is confirmed by the non-zero asymmetry coefficients $\gamma_1 = 0.059061$ and $\gamma_2 = -0.078136$. Thus, a negative asymmetry coefficient indicates an asymmetric response for positive returns in the conditional variance equation, while a positive asymmetry coefficient indicates that bad news leads to increased volatility. Moreover, volatility persistence reveals a slowly decreasing of the rises in the conditional variance due to shocks given the value of the $b$ parameter of 0.993305, and the ARCH-LM Test indicates that there is no ARCH effect left in the innovations - so the conditional variance equation found is well specified.

**REFERENCES**


