Growth with Endogenous Capital, Knowledge, and Renewable Resources

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ABSTRACT

This paper proposes a dynamic economic model with endogenous technological change, physical capital and renewable resources. The model is a synthesis of the neo-classical growth theory, Arrow's learning by doing, and some traditional dynamic models of renewable resources with an alternative approach to household behavior. The model describes a dynamic interdependence between technological change, physical accumulation, resource change, and division of labor under perfect competition. Because of its refined economic structure, the model analyzes some interactions between economic variables which are not found in the existing literature of economic growth. We simulate the model to demonstrate existence of equilibrium points and motion of the dynamic system. Our comparative dynamic analysis shows, for instance, that a rise in the capacity of the renewable resource increases the stock and reduces the price of the resource of the resource over time; the output levels of the two sectors, the total capital stock, and capital inputs of the two sectors are all increased; the labor distribution between the two sectors is slightly affected initially but is not affected in the long term; the rate of interest rises initially rise and is almost not affected in the long term; the per capita consumption levels of the good and the resource and the wage rate are increased.

Keywords: *renewable resource, harvesting, knowledge, Arrow's learning by doing, capital accumulation, economic growth*

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INTRODUCTION

The purpose of this study is to build a dynamic model to describe interdependence between wealth accumulation, knowledge creation and utilization, and resource dynamics with a new approach to consumers' behavior proposed by Zhang (1993). The model is built upon Solow's one-sector growth model, Arrow's learning by doing model, and some dynamic models of renewable resources. The main mechanisms of economic growth in these theories are integrated into a single framework.

The neoclassical growth theory model is extensions and generalizations of the pioneering works of Solow (1956). The Solow model is sometimes referred as to the Solow-Swan model because Swan (1956) proposed a model similar to the Solow model. The one-sector neoclassical growth model has played an important role in the development of economic growth theory by using the neoclassical production function and neoclassical production theory. The model has been extended and generalized in numerous directions (e.g., Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995; Zhang, 2005). As far as economic structure is concerned, our model is based on the Solow model. Our approach to technological change is based on Arrow's learning-by-doing. One of the first seminal attempts to render technical progress endogenous in growth models was made by Arrow (1962). He takes account of learning by doing in modelling knowledge accumulation. Another earlier contribution to modelling knowledge accumulation is carried out by Uzawa (1965) who introduces education section to the growth theory. There are many other studies on endogenous technical progresses. But on the whole theoretical economists had been relatively silent on the topic from the end of the 70s until the publication of Romer's 1986 paper. The literature on endogenous knowledge and economic growth have increasingly expanded since Romer's 1986's paper (Romer, 1986; Lucas, 1988; Grossman and Helpman, 1991; Aghion and Howitt, 1998; Zhang, 2005). Various other issues related to education, trade, R&D policies, entrepreneurship, division of labor, learning through trading, brain drain, economic geography, innovation, diffusion of technology, and behavior of economic agents under various institutions have been discussed in the literature. Nevertheless, there are only a few growth models with endogenous renewable resources, capital and knowledge.

Changes of renewable resource are now considered as a part of economic evolution in many studies in the literature of economic dynamics. But there are only a few models of growth and renewable resources which treat the renewable resource as both input of production and a source of utility (e.g., Beltratti, et al., 1994, Solow, 1999, Ayong Le Kama, 2001). Our model contains renewable resources as sources of utility and input of production. It has been empirically demonstrated that natural resources may have either an adverse or positive effect on the equilibrium growth rate (e.g., Habbakuk, 1962; Gylfason, *et al.* 1999; Sachs and Warner, 2001; and Chen and Lu, 2009). It is argued that if population growth is faster and capital and labor show jointly diminishing returns, the limited resource may reduce per capita growth. Abundance of natural resources may also cause economies to reallocate resources and inputs from efficient use to less efficient use. If the government controls resources, the distribution of resources may encourage rent-seeking rather than growth-enhancing behavior. It is also suggested that resource-rich countries are likely to have a life-style which is beyond its means during a transitional phase when the resource is depleted. But some economists argue that there are positive interactions between resources and economic development (e.g., Habbakuk, 1962). To examine dynamic interdependence between renewable resource and economic growth, we introduce renewable resource and endogenous knowledge into the neoclassical growth theory. We demonstrate that if the economic system functions effectively, an economy with richer natural resources should have faster economic growth and better steady state. It should be noted that the paper is based on Zhang (2011). It generalizes Zhang's model in that knowledge is endogenous, while Zhang's model does not explicitly consider knowledge. The paper is organized as follows. Section 2 introduces the basic model with wealth accumulation, technological change and dynamics of renewable resources. Section 3 examines dynamic properties of the model. Section 4 conducts comparative dynamic analysis with regard to some parameters. Section 5 concludes the study.

THE BASIC MODEL

The economy has one production sector and one resource sector. Most aspects of the production sector are similar to the standard one-sector growth model (Burmeister and Dobell, 1970; Azariadis, 1993; Barro and Sala-i-Martin, 1995). It is assumed that there is only one (durable) good in the economy. Households own assets of the economy and distribute their incomes to consume and save. Production sectors or firms use inputs such as labor with varied levels of human capital, different kinds of capital, knowledge and natural resources to produce material goods or services. Exchanges take place in perfectly competitive markets. Factor markets work well; factors are inelastically supplied and the available factors are fully utilized at every moment. Saving is undertaken only by households. All earnings of firms are distributed in the form of payments to factors of production, labor, managerial skill and capital ownership. We assume a homogenous and fixed population, N. The labor force is distributed between the two sectors. We select commodity to serve as numeraire. The price of commodity is normalized to 1, with all the other prices being measured relative to its price. We assume that wage rates are identical between professions.

The production sector

We assume that production is to combine labor force $N_i(t)$, and physical capital $K_i(t)$, and renewable resource, $K_R(t)$. The production is also affected by knowledge. Let $Z(t)\,(>0)$ stand for the knowledge stock at time t. We use the conventional production function to describe a relationship between inputs and output. The production function $F_i(t)$ is specified as follows

$$F_i(t) = A_i Z^{m_i}(t) K_i^{\alpha_i}(t) N_i^{\beta_i}(t) K_R^{\gamma_i}(t), \quad A_i, m_i, \alpha_i, \beta_i, \gamma_i > 0, \quad \alpha_i + \beta_i + \gamma_i = 1, \quad (1)$$

where A_i, m_i, α_i, β_i and γ_i are positive parameters. If we interpret $Z^{m_i/\beta_i}(t)$ as the level of human capital, then the term $Z^{m_i/\beta_i} N_i$ is the human capital or qualified labor force employed by the industrial sector. The production function is a neoclassical one and homogeneous of degree one with the inputs. Here, we call m_j country j's knowledge utilization efficiency parameter.

Markets are competitive; thus labor and capital earn their marginal products. The rate of interest $r(t)$ and wage rate $w(t)$, and the price of the resource $p(t)$ are determined by markets. The marginal conditions are given by

$$r(t) + \delta_k = \frac{\alpha_i F_i(t)}{K_i(t)}, \quad w(t) = \frac{\beta_i F_i(t)}{N_i(t)}, \quad p(t) = \frac{\gamma_i F_i(t)}{K_R(t)}, \quad (2)$$

where δ_k is the given depreciation rate of physical capital.

Change of renewable resources

We now model dynamics of renewable resources. It is well known that the logistic model has been frequently used in the literature of growth with renewable resource (e.g., Brander and Taylor, 1997; Brown, 2000; Hannesson, 2000; Cairns and Tian, 2010; Farmer and Bednar-Friedl, 2011). It was proposed early in the nineteenth century. Its wide success in different fields of biological and social sciences is its apparent empirical success. Let $X(t)$ stand for the stock of the resource. The natural growth rate of the resource is assumed to be a logistic function of the existing stock

$$\phi_0 X(t) \left(1 - \frac{X(t)}{\phi}\right),$$

where the variable ϕ is the maximum possible size for the resource stock, called the carrying capacity of the resource, and , the variable ϕ_0 is "uncongested" or "intrinsic" growth rate of the renewable resource. If the stock is equal to ϕ, then the growth rate should equal zero. If the carrying capacity is much larger than the current stock, then the growth rate per unit of the stock is approximately equal to the intrinsic growth rate. That is, the congestion effect is negligible. There are some alternative approaches to renewable resources. For instance, Tornell and Velasco (1992), Long and Wang (2009), and Fujiwara (2011) use linear resource dynamics. In this study, for simplicity we assume both the carrying capacity and the intrinsic growth rate constant. This is a strict assumption as the two variables may change due to changes in other conditions. For instance, in Jinni (2006), the carrying capacity changes as a function of the stock of a renewable resource. Benchekroun (2003, 2008) assumes an inversed-V shaped dynamics of resource accumulation, namely, the resource decreases if its stock is sufficiently large. We may consider the capacity dependent on some factors such as efforts. For instance, in the case of forestry fertilizers or cleaning activities of the soil may affect the parameter. With aquaculture, we can also refer to feedings schemes, water temperature, or oxygen levels (Long, 1977; Berck, 1981; Levhari and Withagen, 1992; Ayong Le Kama, 2001; Wirl, 2004). It should also be mentioned that Munro and Scott (1985), Koskela et al. (2002) and Uzawa (2005: Chap. 2) use a more general growth function in their analysis of renewable resources in growth models.

Let $F_x(t)$ stand for the harvest rate of the resource. The change rate in the stock is then equal to the natural growth rate minus the harvest rate, that is

$$\dot{X}(t) = \phi_0 X(t)\left(1 - \frac{X(t)}{\phi}\right) - F_x(t). \tag{3}$$

We assume a nationally owned open-access renewable resource. The open-access case was initially examined by Gordon (1954). With open access, harvesting occurs up to the point at which the current return to a representative entrant equals the entrant's cost. This condition may not be satisfied, for instance, when property rights of the resource are incomplete. Aside from the stock of the renewable resources, like the good sector there are two factors of production. We use $N_x(t)$ and $K_x(t)$ to stand for the labor force and capital stocks employed by the resource sector. We assume that harvesting of the resource is carried out according to the following harvesting production function

$$F_x(t) = A_x Z^{m_x}(t) X^b(t) K_x^{\alpha_x}(t) N_x^{\beta_x}(t), \quad A_x,\, m_x,\, b,\, \alpha_x,\, \beta_x > 0, \quad \alpha_x + \beta_x = 1, \tag{4}$$

where A_x, m_x, b, α_x and β_x are parameters. The specified form implies that if the capital (like machine) and labor inputs are simultaneously doubled, then harvest is also doubled for a given stock of the resource at a given time. It should be noted that there are other approaches to growth with renewable resources with different property-rights regimes (e.g., Alvarez-Guadrado and VonLong, 2011). Schaefer (1957) uses the following Schaefer harvesting production function to describe the production process

$$F_x(t) = A_x X(t) N_x(t).$$

This is a special case of (4). The Schaefer production function does not take account of capital. The function with fixed capital and technology is widely applied to fishing (e.g., Paterson and Wilen, 1977; Milner-Gulland

and Leader-Williams, 1992; Bulter and van Kooten, 1999). As machines and knowledge are important inputs in harvesting, we explicitly take account of knowledge and capital inputs.

Harvesting is carried out by competitive profit-maximizing firms. The profit is

$$p(t)F_x(t) - (r(t) + \delta_k)K_x(t) - w(t)N_x(t).$$

Firms choose the capital and labor inputs. The marginal conditions are

$$r(t) + \delta_k = \frac{\alpha_x p(t)F_x(t)}{K_x(t)}, \quad w(t) = \frac{\beta_x p(t)F_x(t)}{N_x(t)}. \tag{5}$$

Full employment of capital and labor

Let N and $K(t)$ stand for respectively the (fixed) the population and total capital stock. The labor force is allocated between the two sectors. As full employment of labor and capital is assumed, we have

$$K_i(t) + K_x(t) = K(t), \quad N_i(t) + N_x(t) = N. \tag{6}$$

Consumer behaviors

We apply an alternative approach to household behavior proposed by Zhang (1993, 2005). Consumers decide consumption levels of resources and commodities as well as on how much to save. We denote per capita wealth by $k(t)$, where $k(t) \equiv K(t)/N$. Per capita current income from the interest payment $r(t)k(t)$ and the wage payment $w(t)$ is given by

$$y(t) = r(t)k(t) + w(t).$$

We call $y(t)$ the current income in the sense that it comes from consumers' daily work and consumers' current earnings from ownership of wealth. The total value of wealth that consumers can sell to purchase goods and then to save is equal to $k(t)$. Here, we assume that selling and buying wealth can be conducted instantaneously without any transaction cost. The per capita disposable income is then given by

$$\hat{y}(t) = y(t) + k(t) = (1 + r(t))k(t) + w(t). \tag{7}$$

The disposable income is used for saving and consumption. At each point in time, a consumer would distribute the disposable income between saving $s(t)$, consumption of commodities $c(t)$, and consumption of resources $c_x(t)$. The budget constraint is

$$c(t) + s(t) + p(t)c_x(t) = \hat{y}(t). \tag{8}$$

In our model, at each point in time, consumers have three variables, $s(t)$, $c(t)$, and $c_x(t)$, to decide. We assume that consumers' utility function is a function of $s(t)$, $c(t)$, and $c_x(t)$ as follows

$$U(t) = U(c(t), s(t), c_x(t)).$$

For simplicity of analysis, we specify the utility function as follows

$$U(t) = c^{\xi_0}(t)s^{\lambda_0}(t)c_x^{\chi_0}(t), \quad \xi_0, \lambda_0, \chi_0 > 0, \tag{9}$$

where ξ_0 is called the propensity to consume commodities, λ_0 the propensity to own wealth, and χ_0 the propensity to consume resources. Maximizing $U(t)$ in (9) subject to the budget constraint (8) yields

$$c(t) = \xi \hat{y}(t), \quad s(t) = \lambda \hat{y}(t), \quad p(t)c_x(t) = \chi \hat{y}(t), \tag{10}$$

where

$$\xi \equiv \rho \xi_0, \quad \lambda \equiv \rho \lambda_0, \quad \chi \equiv \rho \chi_0, \quad \rho \equiv \frac{1}{\xi_0 + \lambda_0 + \chi_0}.$$

The demand for resources is given by $c_x(t) = \chi \hat{y}(t)/ p(t)$. The demand decreases in its price and increases in the disposable income. An increase in the propensity to consume resources increases the consumption when the other conditions are fixed.

We now find dynamics of capital accumulation. According to the definition of $s(t)$, the change in the household's wealth is given by

$$\dot{k}(t) = s(t) - k(t). \tag{11}$$

The equation simply states that the change in wealth is equal to saving minus dissaving. The demand for and supply of resource balance at any point in time

$$c_x(t)N + K_R(t) = F_x(t). \tag{12}$$

Knowledge creation with learning by doing

Like capital, a refined classification of knowledge and technologies tend to lead new conceptions and modeling strategies. Some major new knowledge and inventions that had far reaching and prolonged implications, such as Newton's mechanics, Einstein's theory of relativity, steam engine, electricity, and computer. Small improvements and non-lasting improvements take place everywhere, serendipitously and intentionally. Innovations may also happen in a drastic, discontinuous fashion or in a slow, continuous manner. The introduction of the first steam engine rapidly triggered a sequence of innovations. The same is true about electricity and computer. Bresnahan and Trajtenberg (1995) argued that technologies have a treelike structure, with a few prime movers located at the top and all other technologies radiating out from them. They characterize general purpose technologies by pervasiveness (which means that such a technology can be used in many downstream sectors), technological dynamism (which means that it can support continuous innovational efforts and learning), and innovational complementarities (which exist because productivity of R&D in downstream sectors increases as a consequence of innovation in the general purpose technology, and vice versa). This study uses knowledge in a highly aggregated sense. We assume that knowledge growth is through the so-called learning by doing. We propose the following equation for knowledge growth (Zhang, 1993)

$$\dot{Z}(t) = \frac{\tau_i F_i(t)}{Z^{\varepsilon_i}(t)} + \frac{\tau_x F_x(t)}{Z^{\varepsilon_x}(t)} - \delta_z Z(t), \tag{13}$$

in which δ_z (≥ 0) is the depreciation rate of knowledge, and ε_j, and τ_j, $j = i, x$, are parameters. Equation (13) implies that knowledge accumulation is through learning by doing. The parameters τ_j and δ_z are non-negative. We interpret, for instance, $\tau_i F_i / Z^{\varepsilon_i}$ as the contribution to knowledge accumulation through learning by doing by the industrial sector. To see how learning by doing occurs, assume that knowledge is a function of the sector's total industrial output during some period

$$Z(t) = a_1 \left\{ \int_0^t F_i(\theta) d\theta \right\}^{a_2} + a_3,$$

in which a_1, a_2 and a_3 are positive parameters. The above equation implies that the knowledge accumulation through learning by doing exhibits decreasing (increasing) returns to scale in the case of $a_2 < (>) 1$. We interpret a_1 and a_3 as the measurements of the efficiency of learning by doing by the production sector. Taking the derivatives of the equation yields $\dot{Z} = \tau_i F_i / Z^{\varepsilon_i}$, in which $\tau_i \equiv a_1 a_2$ and $\varepsilon_i \equiv 1 - a_2$.

We have thus built the dynamic model. We now examine dynamic properties of the model.

THE DYNAMICS AND ITS PROPERTIES

This section examines dynamic properties of the model. First, we introduce a new variable by $z(t) \equiv K_i(t) / K_x(t)$. We now show that the dynamics can be expressed by the three differential equations with $z(t)$, $Z(t)$ and $X(t)$ as the variables.

Lemma

The motion of the system is determined by the 3 differential equations

$$\dot{z}(t) = \left[\lambda N \hat{y}(t) - \Lambda(t) - \widetilde{\Lambda}(t) \frac{\partial \Lambda(t)}{\partial X(t)} - \overline{\Lambda}(t) \frac{\partial \Lambda(t)}{\partial X(t)} \right] \left(\frac{\partial \Lambda(t)}{\partial z(t)} \right)^{-1},$$

$$\dot{X}(t) = \overline{\Lambda}(z(t), Z(t), X(t)), \tag{14}$$

$$\dot{Z}(t) = \widetilde{\Lambda}(z(t), Z(t), X(t)),$$

where the functions in (14) are functions of $z(t)$, $Z(t)$ and $X(t)$ defined in the appendix. Moreover, all the other variables can be determined as functions of $z(t)$, $Z(t)$ and $X(t)$ at any point in time by the following procedure: $K(t) = \Lambda(z(t), Z(t), X(t)) \rightarrow K_i(t)$ and $K_x(t)$ by (A2) $\rightarrow N_i(t)$ and $N_x(t)$ by (A3) $\rightarrow F_x(t)$ by (4) $\rightarrow K_R(t)$ by (A9) $\rightarrow F_i(t)$ by (1) $\rightarrow r(t)$ and $w(t)$ by (2) $\rightarrow p(t)$ by (5) $\rightarrow \hat{y}(t)$ by (7) $\rightarrow c(t)$, $c_x(t)$ and $s(t)$ by (10).

The differential equations system (14) contains three variables, $z(t)$, $X(t)$, and $Z(t)$. The lemma is important as it provides a procedure to follow the motion of the system with computer with a given initial condition. A steady state of (14) is determined by

$$\lambda N \hat{y} - \Lambda = 0,$$

$$\overline{\Lambda}(z, Z, X) = 0, \tag{15}$$

$$\widetilde{\Lambda}(z, Z, X) = 0,$$

As the expressions of the analytical results are tedious, for illustration we specify the parameter values and simulate the model. We specify the parameters as follows

$$N_0 = 5,\ \alpha_i = 0.3,\ \beta_i = 0.6,\ A_i = 1,\ \alpha_x = 0.3,\ A_x = 0.5,\ \phi = 1,\ \phi_0 = 3,\ m_0 = 0.4,$$
$$m_x = 0.2,\ \tau_i = 0.03,\ \varepsilon_i = 0.3,\ \tau_x = 0.01,\ \varepsilon_x = 0.6,\ \lambda_0 = 0.6,\ \xi_0 = 0.15,\ \chi_0 = 0.03, \tag{16}$$
$$b = 0.7,\ \delta_k = 0.05,\ \delta_z = 0.04.$$

The capacity is unity and the adjustment speed ϕ_0 is fixed at 3. The population is fixed at 5. The propensity to save is much higher than the propensity to consume the commodity and the propensity to consume the renewable resource. Some empirical studies on the US economy demonstrate that the value of the parameter, α, in the Cobb-Douglas production is approximately equal to 0.3 (for instance, Miles and Scott, 2005, Abel et al, 2007). The knowledge utility efficiency parameters of the industrial and environmental sectors are respectively 0.4 and 0.2. With regard to the technological parameters, what are important in our study are their relative values.

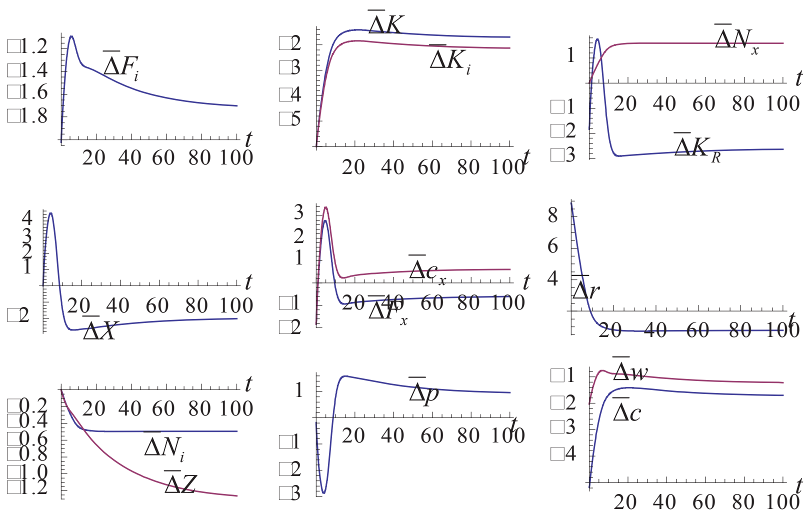
Under (16), the dynamic system has a unique equilibrium point. The equilibrium values of the variables are given as follows

$$K = 34.01, \; Z = 4.86, \; X = 0.61, \; F_i = 10.20, \; F_x = 0.98, \; N_i = 3.81, \; N_x = 1.19,$$
$$K_i = 26.85, \; K_x = 7.16, \; K_R = 0.37, \; p = 2.77, \; r = 0.064, \; w = 1.61,$$
$$c_x = 0.12, \; c = 1.70.$$

With the initial conditions, $z(0) = 3.4$, $Z(0) = 4.6$, and $X(0) = 0.7$, we plot the motion of the system as in Figure 1. We see that the level of the resource stocks falls initially and then rises in the long term; correspondingly its price rises initially and falls in the long term. The knowledge stock falls over time. The total capital and capital input employed by the industrial sector fall over time. The rate of interest rises over time. The capital stock employed by the resource sector falls initially and then rises in the long term. The labor input employed by the resource sector falls and the labor input employed by the industrial sector rises over time. The wage rate and consumption levels of the resource and goods fall over time. It is straightforward to calculate the three eigenvalues as: $\{-1.74, -0.17, -0.03\}$. This guarantees the stability of the steady state. Hence, the dynamic system has a unique stable steady state.

Motion of the Economic System

Figure 1



COMPARATIVE DYNAMIC ANALYSIS

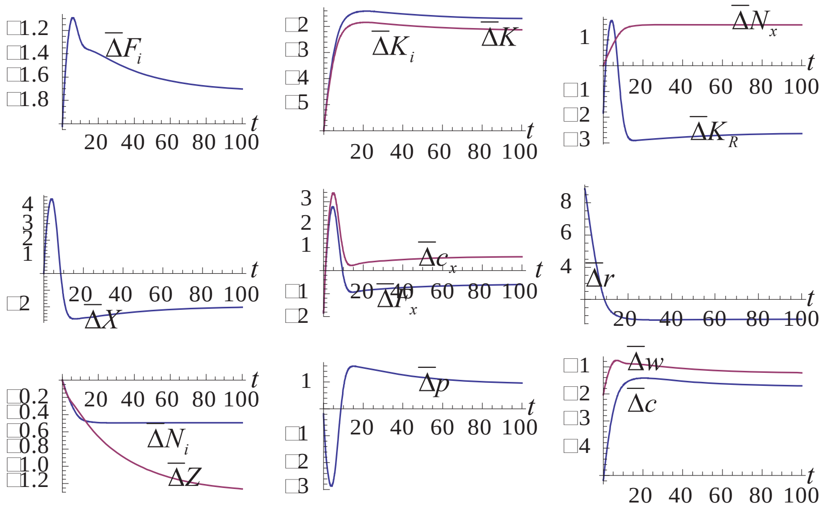
We now examine effects of changes in some parameters on the motion of the economic system. We introduce a variable $\overline{\Delta} x(t)$ to stand for the change of the variable $x(t)$ in percentage due to the change in a parameter value.

A Rise in the propensity to consume resources

First, we study the case that all the parameters, except the propensity to consume resources, are the same as in (16). The propensity to consume resources is increased as follows: $\chi_0 = 0.03 \Rightarrow 0.031$. We plot the simulation result in Figure 2. The rise in the propensity to consume resources reduces the industrial sector's output and capital input, the total capital, the wage rate and level of the consumption good. The interest rate rises over time. The level of the resource stock rises initially but falls in the long term. The consumption level of the resource is increased over time. The output of the resource sector rises initially but falls in the long term. The price of the renewable resource is reduced initially but increased in the long term. Some of the workers employed by the industrial sector are shifted to the resource sector. The wage rate and the consumption level of commodities fall over time.

A Rise in the Propensity to Consume Resources

Figure 2

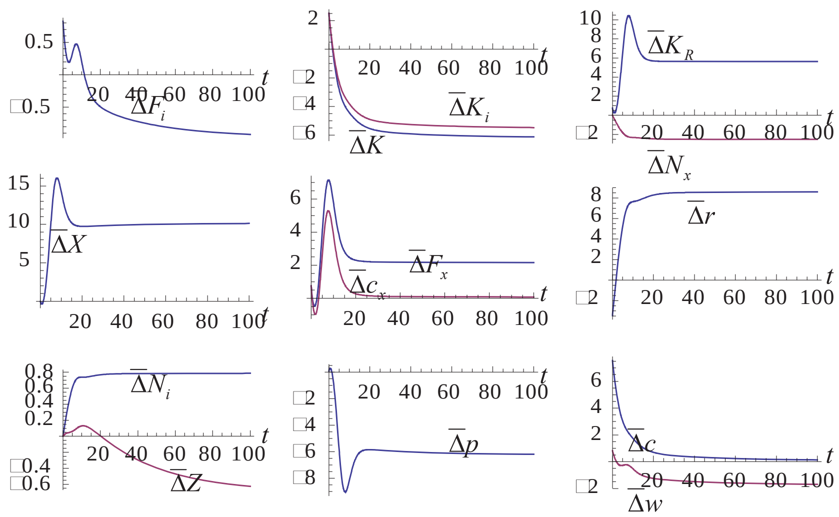


A rise in the propensity to consume commodities

We now allow the propensity to consume commodities to be increased as follows: $\xi_0 = 0.15 \Rightarrow 0.16$. The rise in the propensity to consume commodities initially increases the industrial sector's output and capital input and national capital stock and reduces these variables in the long term. The capital input employed by the resource sector is increased over time. The price of the resource is reduced. The per capita consumption level of the resource falls initially, then rises, and finally approaches to its original value in the long term. The wage rate rises initially but falls late on. The interest rate falls initially but rises in the long term. The level of the resource stock rises over time. Some workers move their jobs from the industrial sector to the resource sector. The knowledge stock rises initially but soon begins to fall.

A Rise in the Propensity to Consume Commodities

Figure 3

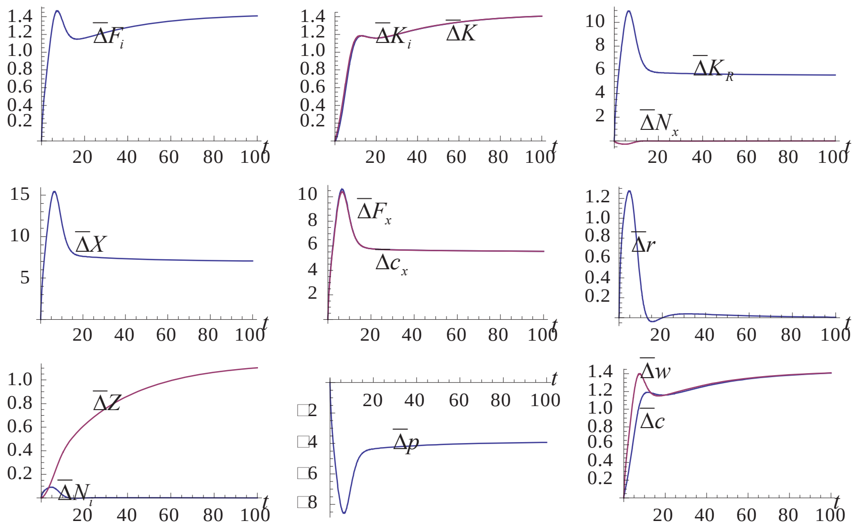


A rise in the resource capacity

We now allow the resource capacity to be increases as follows: $\phi = 1 \Rightarrow 1.05$. As the capacity is increased, the stock of the resource is increased. In association with the increase in the resource stock, the price of the resource is reduced. The output levels of the two sectors, the total capital stock, and capital inputs of the two sectors are all increased. The labor distribution between the two sectors is slightly affected initially but is not affected in the long term. The rate of interest rises initially rise and is almost not affected in the long term. The per capita consumption levels of the good and the resource and the wage rate are increased. As mentioned in the introduction, some empirical studies demonstrate that natural resources have an adverse effect on the equilibrium growth rate. If we interpret a rise in the capacity as a rise of natural resources, our result implies that if we don't neglect possible effects of rent-seeking and misallocation of natural resources, then economies may benefit from rich natural resources.

A Rise in the Resource Capacity

Figure 4



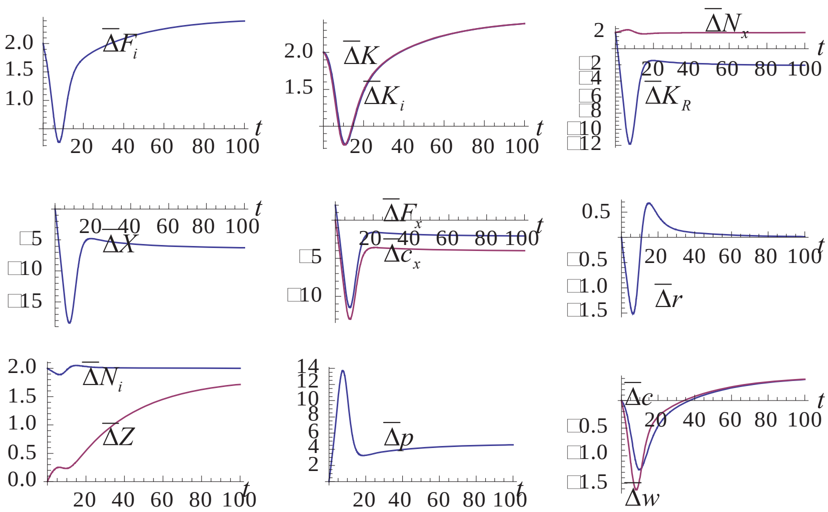
A rise in the population

We increase the population as follows: $N = 5 \Rightarrow 5.1$. As the population is increased, the labor inputs of the both sectors are increased. The total capital stock, the capital input employed by the industrial sector and the industrial sector's output are all increased. The stock of the resource is reduced and the price of the resource is increased. The rate of interest rises initially, and falls late on, and is not affected in the long term. The output level of the resource sector and per capita consumption level of the resource are reduced. The wage rate and per capita consumption of the good are reduced initially but increased in the long term. It should be noted that in the Solow growth theory, a rise in the population reduces the per capita consumption and wage rate, while our model predicts that a rise in the population reduces the consumption level and wage rate initially, but the variables are increased in the long term.

We see that the increase in the population reduces the consumption level of the resource but increases the per capita consumption level of commodities in the long term.

A Rise in the Population

Figure 5



CONCLUDING REMARKS

This study built a dynamic economic model with wealth accumulation, change of renewable resource, and technological change. The economic system consists of one production sector and one resource sector. Our approach is different from most of the neoclassical growth models with renewable resources based on microeconomic foundation which neglect physical capital accumulation and technological change. The model is a synthesis of the neoclassical growth theory, Arrow's learning by doing, and the traditional dynamic models of renewable resources with an alternative approach to household behavior. The study examines the interdependence among economic variables which are not found in the existing literature of economic growth with renewable resources. We also simulated the model to demonstrate existence of equilibrium points, stability and motion of the dynamic system. The model may be extended in some directions. For instance, we may introduce economic structure and research into the model.

Appendix: Proving the Lemma

The appendix shows that the dynamics can be expressed by the three differential equations in the lemma. From (2) and (5), we obtain

$$z = \frac{K_i}{K_x} = \alpha \frac{N_i}{N_x}, \tag{A1}$$

where we omit time index and $\alpha \equiv \beta_x \alpha_i / \alpha_x \beta_i$. By (A1) and (6), we solve

$$K_i = \frac{zK}{z+1}, \quad K_x = \frac{K}{z+1}, \tag{A2}$$

$$N_i = \frac{zN}{z+\alpha}, \quad N_x = \frac{\alpha N}{z+\alpha}. \tag{A3}$$

By (2), (12) and $p c_x = \chi \hat{y}$ in (10), we have

$$\chi N \hat{y} + \gamma_i F_i = p F_x. \tag{A4}$$

By the definition of $\hat{y}$, we have

$$N \hat{y} = \left(\delta + \frac{\alpha_x p F_x}{K_x} \right) K + \frac{\beta_x p F_x N}{N_x}, \tag{A5}$$

where we use (5) and $\delta \equiv 1 - \delta_k$. By (2) and (5), we have

$$p F_x = \frac{\alpha \beta_i F_i}{\beta_x z}, \tag{A6}$$

where we also use (A1). Insert the above equation and (A5) in (A4)

$$\left[\frac{1}{\chi} - (1+z)\alpha_x - \frac{(z+\alpha)\beta_x}{\alpha} - \frac{\beta_x \gamma_i z}{\alpha \chi \beta_i} \right] p F_x = \delta K, \tag{A7}$$

where we use (A2) and (A3). Substituting (A6) into (A7) yields

$$\left[\frac{1}{\chi} - (1+z)\alpha_x - \frac{(z+\alpha)\beta_x}{\alpha} - \frac{\beta_x \gamma_i z}{\alpha \chi \beta_i} \right] \frac{\alpha \beta_i F_i}{\beta_x z} = \delta K. \tag{A8}$$

From (2), (5) and (A1), we solve

$$K_R = \frac{\gamma_i \alpha_x z F_x}{\alpha_i}. \tag{A9}$$

Substituting (1), (A3) and (A2) into (A8), we solve

$$K = \Lambda(z, Z, X) \equiv \frac{\tilde{n}\, Z^{(m_i + \gamma_i m_x)\gamma_0} X^{\gamma_0 \gamma_i b} \left(\chi_1 - \chi_2 z\right)^{\gamma_0}}{(z+1)^{(\gamma_i \alpha_x + \alpha_i)\gamma_0} (z+\alpha)^{(\gamma_i \beta_x + \beta_i)\gamma_0}}, \tag{A10}$$

where we use (A9) and

$$\gamma_0 \equiv \frac{1}{1 - \gamma_i \alpha_x - \alpha_i}, \quad \tilde{n} \equiv \left(\frac{\gamma_i \alpha_x}{\alpha_i}\right)^{\gamma_i \gamma_0} \left(\frac{A_i A_x^{\gamma_i} \alpha^{1+\gamma_i \beta_x} \beta_i N^{\gamma_i \beta_x + \beta_i}}{\delta \beta_x}\right)^{\gamma_0},$$

$$\chi_1 \equiv \frac{1}{\chi} - 1, \quad \chi_2 \equiv \alpha_x + \frac{\beta_x}{\alpha} + \frac{\beta_x \gamma_i}{\alpha \chi \beta_i}.$$

We express K as a function of z, Z and X. From (A2), K_i and K_x are functions of z, Z and X. From (A3), N_i and N_x are functions of z, Z and X. By the following procedure, we can express other variables as functions of $z(t)$, $Z(t)$ and $X(t)$ at any point of time: F_i by (1) $\rightarrow$ r and w by (2) $\rightarrow$ F_x by (4) $\rightarrow$ p by (5) $\rightarrow$ $\hat{y}$ by (7) $\rightarrow$ c, c_x and s by (10). It is straightforward to see that the right-hand side of (3) is a function of $z(t)$, $Z(t)$ and $X(t)$. Hence, we have

$$\dot{X}(t) = \overline{\Lambda}(z, Z, X), \tag{A11}$$

where we do not explicitly express $\overline{\Lambda}(z, Z, X)$ as it straightforward but its expression is tedious. The right-hand side of (13) is a function of $z(t)$, $Z(t)$ and $X(t)$. We have

$$\dot{Z}(t) = \tilde{\Lambda}(z, Z, X), \tag{A12}$$

Taking derivatives of (A10) with respect to t yields

$$\dot{K} = \frac{\partial \Lambda}{\partial z}\dot{z} + \frac{\partial \Lambda}{\partial Z}\dot{Z} + \frac{\partial \Lambda}{\partial X}\dot{X}, \tag{A13}$$

where

$$\frac{\partial \Lambda}{\partial z} = -\left(\frac{\chi_2}{\chi_1 - \chi_2 z} + \frac{\gamma_0 (\alpha_i + \gamma_i \alpha_x)}{z+1} + \frac{\gamma_0 (\gamma_i \beta_x + \beta_i)}{z+\alpha}\right)\Lambda,$$

$$\frac{\partial \Lambda}{\partial Z} = \frac{(m_i + \gamma_i m_x)\gamma_0 \Lambda}{Z}, \quad \frac{\partial \Lambda}{\partial X} = \frac{\gamma_0 \gamma_i b \Lambda}{X}.$$

Multiplying the two sides of (11) with N and using (10), we have

$$\dot{K} = \lambda N \hat{y}(z, Z, X) - K. \tag{A14}$$

From (A13) and (A14), we solve

$$\dot{z} = \left[\lambda N \hat{y} - \Lambda - \tilde{\Lambda} \frac{\partial \Lambda}{\partial X} - \bar{\Lambda} \frac{\partial \Lambda}{\partial X} \right] \left(\frac{\partial \Lambda}{\partial z} \right)^{-1} . \quad \text{(A15)}$$

where we also use (A11) and (A12). We have thus proved the lemma.

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