Econometric Models for the Analysis of Financial Portfolios

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Abstract
Using a factorial design for explaining the rentabilităților allows to reduce the volume of such calculations as long as the number of factors is less than the number of assets. Under these circumstances, rather than to introduce wording ARCH directly into rentabilităților, estimate it will bring in the estimation of their determinants, once they have been identified. In this article we examined the evolution of the return on the portfolio consisting of the ten titles listed on the Bucharest Stock Exchange with Forecast and it emerged that in the following period, the return on the portfolio considered will be relatively low, no one anticipated the major developments of this indicator.

Key words: econometric models, stock exchange, portfolio of financial assets.

1. Heteroscedastic models used for analysis of a portfolio of financial assets
For the analysis of financial securities which may be included in a portfolio the volume of calculations necessary for the purposes of the estimation of covariantelor becomes significantly higher.

Use of a model for an explanation unrequired rentabilitatilor lets you reduce the size these calculations as long as the number of factors is lower number of assets.
In these circumstances, in place to introduce the formulation ARCH directly into rentabilităților, estimate it will bring in the estimation of their determinants, once they have been identified. Using modeling iterate two stages:

- expression of all rentabilităților financial assets depending on the factors identified as being the determining;
- application of ARCH for modeling these factors.

On the basis of statistical and mathematical model is unifactorial generalizing to a larger number of factors.

Be n number of financial assets being considered for the portfolio, and the vector of size n \( \vec{r}_t \) of rentabilităților associated with them.

Calculate the moments of order 1 and 2, respectively \( E(\vec{r}_t \mid \vec{r}_{t-1}) \) and \( \sigma^2(\vec{r}_t \mid \vec{r}_{t-1}) \), vectors of size n and \( (n, m) \). Use the following model:

\[
\vec{r}_t = \beta \cdot f_t + \vec{u}_t
\]

\[\begin{pmatrix}
\vec{r}_{1t} \\
\vec{r}_{2t} \\
\vdots \\
\vec{r}_{nt}
\end{pmatrix} =
\begin{pmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_m
\end{pmatrix}
\begin{pmatrix}
f_{1t} \\
f_{2t} \\
\vdots \\
f_{nt}
\end{pmatrix}
+ \begin{pmatrix}
\vec{u}_{1t} \\
\vec{u}_{2t} \\
\vdots \\
\vec{u}_{nt}
\end{pmatrix}
\]

\( r_t = \) the vector rentabilities bud by surplus snout without risk at time t for the n active

\( u_t = \) the vector of the n terms of residual econometric regression

\( \beta = \) the vector coefficients of sensitivity of its profitability to factor f

Matrix form, the model can be written as:

\[
\begin{pmatrix}
\vec{r}_{1t} \\
\vec{r}_{2t} \\
\vdots \\
\vec{r}_{nt}
\end{pmatrix} =
\begin{pmatrix}
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_m
\end{pmatrix}
\begin{pmatrix}
f_{1t} \\
f_{2t} \\
\vdots \\
f_{nt}
\end{pmatrix}
+ \begin{pmatrix}
\vec{u}_{1t} \\
\vec{u}_{2t} \\
\vdots \\
\vec{u}_{nt}
\end{pmatrix}
\]

by assumption, the average residual factors is zero and finite dispersion, respectively:
\[
\begin{align*}
E(u_t | I_{t-1}) &= 0 \\
\sigma^2(u_t | I_{t-1}) &= \nu
\end{align*}
\]

\(V\) is a diagonal matrix of size \((n, n)\) the residual terms (errors of estimating the profitability of each asset based on the model respectively) are independent. \(f_t\)

The \(f_t\) factor can be modeled as:

\[
f_t = \mu + e_t
\]

Whereas the variables \(u_t\) and \(e_t\) independent

With the help of this model can be determined early profitability (hope mathematics of the profitability) for the \(n\) assets, on the basis of the information available at the time \((t-1)\). Each of the components vector \(r_t\) will be determined that:

\[
E(r_{it} | I_{t-1}) = E(\beta_t f_t + u_{it} | I_{t-1}) = \beta_t E(f_t | I_{t-1}) + E(u_{it} | I_{t-1}) = \beta_t \mu + E(u_{it} | I_{t-1})
\]

It may be observed that the two terms of this relationship are equal to zero,

where: \(E(r_{it} | I_{t-1}) = \beta_t \mu\)

In matrix form, this equation may be expressed as follows:

\[
E(r_{it} | I_{t-1}) = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_n \end{pmatrix} \cdot \mu
\]

Whereas the variables \(u_t\) and \(e_t\) were assumed independent, The terms \(\text{cov}(f_t, u_{it} | I_{t-1})\) and \(\text{cov}(f_t, u_{jt} | I_{t-1})\) are equal to zero, because:

\[
\text{cov}(f_t, u_{it} | I_{t-1}) = \text{cov}(u_{it}, u_{jt} | I_{t-1}) = 0
\]

Whereas the terms of residual regression function are independent shows that and the term \(\text{cov}(u_{it}, u_{jt} | I_{t-1}) = 0\)
Under these conditions, for any $i \neq j$:

$$\text{cov}(\gamma_i | \gamma_j) = \text{cov}(\beta_j | \beta_j) \cdot \text{var}(\gamma_i | \gamma_j)$$

for any $i = j$:

$$\text{cov}(\gamma_i | \gamma_i) = \beta_1^2 \cdot \text{var}(\gamma_i | \gamma_i) + \text{cov}(\gamma_i | \gamma_i) = \beta_1^2 \cdot \text{var}(\gamma_i | \gamma_i) + \psi$$

In these circumstances, the variance - covariance matrix of will write:

$$V(\gamma_i | \gamma_i) = \begin{pmatrix}
\beta_1^2 & \beta_1 \beta_2 & \beta_1 \beta_n \\
\beta_2 \beta_1 & \beta_2^2 & \beta_2 \beta_n \\
\beta_n \beta_1 & \beta_n \beta_2 & \beta_n \beta_n
\end{pmatrix} \cdot \text{var}(\gamma_i | \gamma_i) + \begin{pmatrix}
\sigma_1^2 & 0 & 0 \\
0 & \sigma_2^2 & 0 \\
0 & 0 & \sigma_n^2
\end{pmatrix}$$

Although heteroscedastic models is not a perfect solution with respect to estimating the profitability of individual securities or portfolios, while maintaining the classic of the classics hypothesis, the use of such models can be on the Modeler to consider a degree of volatility at times of financial market turbulence, for certain periods.

2. Using time-series analysis of a portfolio of financial instruments

To substantiate how effective for the analysis of performance of a portfolio of financial instruments using econometric modeling models type chronological series we have used a series of information on developments in its portfolio consisting of the ten financial securities subject and analysis using the regression model. The values of yield recorded for this portfolio of financial instruments or calculated for the year 2012 (the yield BET index and portfolio).

To determine the parameters econometric model based on chronological series, the series of data relating to developments in portfolio yield considered in the 250 tradings session undergoing analysis has been imported into an application computing managed using Eviews specialized software solution.

Also according to the specific methodology of estimation of ARMA models, with the top pointing out, was generated for the first data series of differentiations, using for this purpose the relationship:

$$d_{\text{ran}_\text{port}} = \text{ran}_\text{port} - \text{ran}_\text{port} (-1)$$

In a first step, we analyzed the initial series with statistics and quizzes generated using the histogram information solution above.
As can be seen, at the level of this data series is there ample and frequent changes, which makes it difficult using preformatted econometric method. A similar analysis was performed and the data relating to the first series differentiation results can be summarized as follows:
The next stage is completed in order to determine a model specific to the ARMA yield on portfolio development consisting of the ten titles deemed to represent staționarității series analysis. For this purpose, we have used both graphics method (stationarității analysis using corelogramei), as well as one of specialized tests implemented in the program Eviews namely test ADF (augmented Dickey-Fuller). The results of these tests on initial series of data shall be submitted as follows:

```
RAN_PORT
Date: 07/30/13      Time: 19:24
Sample: 1 250
Included observations: 250

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<td>AC</td>
<td>PAC</td>
<td>Q-Stat</td>
<td>Prob</td>
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<td>-0.144</td>
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<td>0.063</td>
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<td>-0.094</td>
<td>9.8954</td>
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<td>0.057</td>
<td>-0.001</td>
<td>10.325</td>
<td>0.171</td>
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### Augmented Dickey-Fuller Test

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<tr>
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<tr>
<td>9 -0.045 -0.029 11.267 0.258</td>
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<tr>
<td>10 0.101 0.048 12.636 0.245</td>
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<tr>
<td>-12.59862 0.0000</td>
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<tr>
<td>11 -0.130 -0.077 14.923 0.186</td>
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<tr>
<td>12 0.037 -0.004 15.107 0.236</td>
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<tr>
<td>13 -0.133 -0.121 17.540 0.176</td>
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<tr>
<td>14 0.140 0.150 20.278 0.122</td>
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<tr>
<td>15 -0.019 -0.011 20.331 0.160</td>
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<tr>
<td>16 0.058 0.045 20.815 0.186</td>
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<tr>
<td>17 0.048 0.033 21.151 0.220</td>
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<td>18 -0.080 -0.050 22.084 0.228</td>
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<tr>
<td>19 -0.153 -0.221 25.513 0.144</td>
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<tr>
<td>20 -0.013 0.011 25.537 0.182</td>
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<td>21 -0.088 -0.045 26.680 0.182</td>
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<td>22 0.040 -0.044 26.923 0.214</td>
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<td>23 -0.085 -0.074 28.747 0.246</td>
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<tr>
<td>24 0.047 -0.018 28.783 0.276</td>
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<td>25 -0.049 -0.036 28.763 0.322</td>
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<tr>
<td>26 -0.033 -0.001 28.935 0.364</td>
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<tr>
<td>27 0.078 0.079 29.906 0.368</td>
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<tr>
<td>28 0.078 0.130 30.890 0.371</td>
<td></td>
</tr>
<tr>
<td>29 0.078 0.130 30.890 0.371</td>
<td></td>
</tr>
<tr>
<td>30 -0.100 -0.154 32.538 0.343</td>
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<tr>
<td>31 0.074 0.025 33.446 0.349</td>
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<tr>
<td>32 0.003 0.024 33.448 0.397</td>
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<tr>
<td>33 0.018 0.084 33.505 0.443</td>
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<tr>
<td>34 0.008 -0.040 33.516 0.491</td>
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<tr>
<td>35 -0.124 -0.025 36.166 0.414</td>
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</tr>
<tr>
<td>36 0.098 0.023 37.839 0.385</td>
<td></td>
</tr>
</tbody>
</table>

**Null Hypothesis**: RAN_PORT has a unit root

**Exogenous**: Constant

**Lag Length**: 0 (Automatic based on SIC, MAXLAG=12)

**Augmented Dickey-Fuller test statistic**: -12.59862, Prob.* = 0.0000

**Test critical values**:
- 1% level: -3.485586
- 5% level: -2.885654
- 10% level: -2.579708

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(RAN_PORT)
Method: Least Squares
Date: 07/30/13   Time: 19:52
Sample (adjusted): 2 250
Included observations: 249 after adjustments

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAN_PORT(-1)</td>
<td>-1.143761</td>
<td>0.090785</td>
<td>-12.59862</td>
<td>0.0000</td>
</tr>
<tr>
<td>C</td>
<td>0.173281</td>
<td>0.125095</td>
<td>1.385190</td>
<td>0.1686</td>
</tr>
</tbody>
</table>

R-squared 0.573584    Mean dependent var 0.010861
Adjusted R-squared 0.569971    S.D. dependent var 2.078569
S.E. of regression 1.363055    Akaike info criterion 3.473861
Sum squared resid 219.2345    Schwarz criterion 3.520319
Log likelihood -206.4316    F-statistic 158.7253
Durbin-Watson stat 1.973898    Prob(F-statistic) 0.000000

The two types of tests carried out on the data series that contains the 250 comments on the evolution of the portfolio considered have revealed that it is stationary and can be used in order to estimate the parameters of a ARMA model that describes the phenomenon of subject to analysis.

In these circumstances, we have to estimate ARMA model specification intended for analysis of developments in portfolio yield considered. In this respect, they have been tested various specifications AR and MA, following tests resulting in the ARMA model (4,4 ) shows the highest significance.

Dependent Variable: RAN_PORT
Method: Least Squares
Date: 07/30/13   Time: 20:56
Sample (adjusted): 5 250
Included observations: 246 after adjustments
Convergence achieved after 29 iterations
Backcast: 1 4

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAN_PORT</td>
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</tr>
<tr>
<td>C</td>
<td>0.173281</td>
<td>0.125095</td>
<td>1.385190</td>
<td>0.1686</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.573584</td>
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</table>
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| Durbin-Watson stat | 1.973898 | Prob(F-statistic) 0.000000

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As we can see, the tests validating the significance ARMA econometric model above, tests respectively R², F-statistically and Prob (F-statistic) presents significant value resulting from the idea according to which the model can be accepted and, subsequently, used in forecasting trends in portfolio yield considered.

Also, it may be observed that the parameters to the degree of the highest significance are AR (2) and MA(2).

With the help of the computer programme Eviews has been generated for this model an econometric equation, meaning:

**Estimation Command:**

```
LS(DERIV=AA) RAN_PORT C AR(1) AR(2) AR(3) AR(4) MA(1) MA(2) MA(3) MA(4)
```

**Estimation Equation:**

```
RAN_PORT = C(1) +
[AR(1)=C(2),AR(2)=C(3),AR(3)=C(4),AR(4)=C(5),MA(1)=C(6),MA(2)=C(7),MA(3)=C(8),MA(4)=C(9),BACKCAST=5]
```

Substituted Coefficients:
Based on these elements, we have achieved further progress in the yield of the portfolio consists of the ten titles listed on the stock exchange with Forecast and it emerged that in the following period, the return on the portfolio considered will be relatively low, no one anticipated the major developments of this indicator. I also found that the tendency of this data series to record significant variations from one period to another will keep the forecast period.

**Bibliography**