General Aspects Regarding the Methodology for Prediction Risk

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Abstract

In order to measure the total risk to which an investor or a financial institution is exposed when they invest in a financial asset, there needs to be a tool to capture this risk. The most widely used tool in measuring the total risk is Value at Risk. The first parameter that must be estimated is represented by the decay factor, because based on its value we will estimate further the volatility and Value at Risk. However, we are not just interested in computing the VaR for all considered models, but moreover we want to test if models used in these estimations are accurate and able to predict the risk. To achieve this objective we will use two types of test: unconditional coverage test and conditional coverage test.

Key words: financial instrument, prediction, risk metrics, risk prediction

JEL Classification: D81, G32

Consider $X$ a financial instrument and $P_1, P_2, P_3, ..., P_T$ - the prices of the financial instrument for a period of $T$ days. Based on this information, the one day continuously compounded returns - $r_t$ ($t$ representing one trading day), are defined as:

$$ r_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \; t = 1, ..., T $$

(1)

Going further, the RiskMetrics model assumes that the continuously compounded returns follow a stochastic process given by:

$$ r_t = \mu_t + \epsilon_t = \mu_t + \sigma_t z_t $$

(2)
\[ \mu_t \equiv 0 \quad \forall t, \quad \mu_t = E[r_t | I_{-t}] \quad \text{and} \quad \sigma_t^2 = E[\varepsilon_t^2 | I_{-t}]; \]

where \( z_t \) is an independent and identically distributed Gaussian random variables with \( E[z_t] = 0 \) and \( V[z_t] = 1 \), moreover the RiskMetrics methodology sets due to difficulties in the estimation of expected returns.

Let \( \lambda \) be the decay factor, such that \( \lambda \in (0,1) \) and the conditional variance modeled based on exponentially weighted moving average (EWMA):

\[ \sigma_t^2(\lambda) = (1 - \lambda) \sum_{i=1}^{t-1} \lambda^{i-1} r_{t-i}^2 \]

(3)

According to the RiskMetrics, (3) is well approximated by the following relation:

\[ \sigma_t^2 = \lambda \sigma_{t-1}^2 + (1 - \lambda) r_{t-1}^2 \]

(4)

In the J. P. Morgan (1996) it is stated that the most appropriate value for the decay factor is \( \lambda = 0.94 \) for one day continuously compounded returns and \( \lambda = 0.97 \) for one month continuously compounded returns. Over time, RiskMetrics improved the assumption related to the errors distribution. In 1994, it was assumed that the probability distribution for the residuals was the Normal distribution. The assumption was changed in 2006, when it was assumed that the probability distribution for the residuals is the Student-\( t \) distribution with 5 degrees of freedom (Zumbach, 2006). This assumption was improved due to the existence of fat tails in the data.

Based on (4) we are able to estimate the volatility, and further the total risk of a financial instrument. However, in order to measure the total risk to which an investor or a financial institution is exposed when they invest in a financial asset, there needs to be a tool to capture this risk. The most widely used tool in measuring the total risk is Value at Risk, defined as conditional \( \alpha \)-percentile:

\[ VaR_{\alpha} \equiv q_{\alpha, t} = k_{\alpha} \sigma_t(\lambda) \]

(5)

where \( k_{\alpha} \) is either the conditional normality - \( \phi^{-1}(\alpha) \) conditional Student-\( t \) -
The goal of this paper is to see if the RiskMetrics model is good enough to forecast the volatility during the financial crisis. In order to see this, it makes use of the rolling window method for back testing.

Figure 1 presents the essentials aspects of rolling window estimation. The analysis uses daily financial data starting with January 1st, 1986 until July 1st, 2009. Based on the rolling window methodology, we divided the total sample in two subsamples: in-sample (ante-financial crisis period) and out-of-sample (financial crisis period). Following the Halbleib and Pohlmeier (2011) methodology, the start date for financial crisis will be considered the 17th of July 2007, the day when FED identified some problems on the subprime loan markets and offered their support and supervision for the subprime mortgage lenders. In the rolling window there will be used $T = M + P$ observation, where $T$ - the total number of observation, $M$ - the number of observation from in-sample period and $P$ - the number of observation from out-of-sample period.

The first parameter that must be estimated is represented by the decay factor, because based on its value we will estimate further the volatility and Value at Risk.

$$H^{-1}(\alpha)\sqrt{\frac{\nu-2}{\nu}}$$ with $\nu$ - degree of freedom parameter.
Despite just applying the RiskMetrics formula and the assumption that the decay factor is equal with 0.94, we will estimate empirical the decay factor. Doing this we will verify which method is most efficient in estimating the Value at Risk.

Moreover, we will consider two types of function in estimating the decay factor empirically. The first will be the initial methodology of RiskMetrics, estimating the decay factor for each window by minimizing the squared error loss function for the conditional variance:

\[
\hat{\lambda}_t = \arg \min_{\lambda \in (0,1)} \frac{1}{T-M} \sum_{j=1}^{T-M} [\sigma_j^2(\lambda) - r_j^2]^2
\]  

(6)

Gonzalez-Riviera et al. (2007) proposed a better methodology for estimating the decay factor. They emphasize the purpose of VaR, and they argue that it is most appropriate to determine the decay factor by minimizing the check error loss function, rather than minimizing the squared error loss function. They used the following check error loss function:

\[
\rho_\alpha(e_j) = \begin{cases} 
\alpha \cdot e_j & e_j \geq 0 \\
(\alpha - 1) \cdot e_j & e_j < 0 
\end{cases}, \text{ where } e_j = r_j - VaR_{j,\alpha}
\]  

(7)

Similarly, this paper will estimate the RiskMetrics model by minimizing the check error loss, estimating the decay factor based on formula below:

\[
\hat{\lambda}_t = \arg \min_{\lambda \in (0,1)} \frac{1}{T-M} \sum_{j=1}^{T-M} \rho_\alpha(e_j)
\]  

(8)

Having all values for the decay factors, we are able now to compute the volatility and further the Value at Risk, for each financial instrument used in our analysis. In order to do this we define three types of RiskMetrics models.

First we will analyze the original RiskMetrics model, for which it is assumed that the decay factor equals 0.94. Moreover we will consider two types of return distribution: Normal distribution (RiskMetrics-1994) and Student-t distribution (RiskMetrics-2006). We will note this model as RM1.

In order to improve the RiskMetrics assumptions, we will construct other two types of models. For the second model, named RM2, we estimate empirically the decay factor based on squared error loss function, considering both Normal distribution (Empirical RiskMetrics-1994) and Student-t distribution (Empirical RiskMetrics-2006).
RiskMetrics-2006). For the last model, we will estimate the decay factor based on check error loss function, under Normal distribution and Student-$t$ distribution.

However, we are not just interested in computing the VaR for all considered models, but moreover we want to test if models used in these estimations are accurate and able to predict the risk. To achieve this objective we will use two types of test: unconditional coverage test and condiţional coverage test.

**Unconditional coverage test**

Unconditional coverage test is represented by LR test proposed by Kupiec (1995). Called also the proportion of failure test, in the case of this test the null hypothesis states that the probability of failure (return is lower than VaR) is equal with the desired significance level - $\alpha$, so the model is "correct" and it is accepted, against the alternative hypothesis, that the probability is different from $\alpha$, which means that the model is not "correct", and it is rejected.

This test is easy to implement, due to fact that the only information required is the total number of observation and number of exceptions, given by:

$$LR_{UC}(\alpha) = -2 \ln \left( \frac{(1-\alpha)^{n_0} \alpha^{n_1}}{1 - \left( \frac{n_1}{n_0 + n_1} \right)^{n_0} \left( \frac{n_1}{n_0 + n_1} \right)^{n_1}} \right) \sim \chi^2_{(1)}$$

where, $n_0$ is the number of observation for which $r_t \geq -VaR$, and $n_1$ is the number of observation for which $r_t \leq -VaR$.

**Conditional coverage test**

$$LR_{CC} = LR_{UC} + LR_{IND}$$

Due to fact that the statistic test presented above does not take into account the variance dynamics, because it simply counts the exceptions, Christoffersen (1998) adapted the log-likelihood testing framework of Kupiec into a better form, which take into account the condiţional coverage. The test is a joint test such that

which is $\chi^2_{(2)}$ distributed.

This test is conducted by first defining the $I_t$ - indicator variable such as:
\[ I_t = \begin{cases} 1 & r_t < \text{VaR}_t \\ 0 & r_t \geq -\text{VaR}_t \end{cases} \] 

(11)

The conditional coverage test proposed by Christoffersen is given by:

\[ LR_{CC} = -2 \ln \left( \frac{(1-\alpha)^{n_\alpha} \alpha^{n_\alpha}}{(1-\hat{\pi}_{01})^{n_{00}} \hat{\pi}_{01} (1-\hat{\pi}_{11})^{n_{10}} \hat{\pi}_{11}} \right) \sim \chi^2_{(2)} \]

(12)

where, \( n_{ij} \) is the number of observation with value \( i \) followed by \( j \),

\[ \pi_y = \Pr(I_i = i | I_{i-1} = j) (i, j=0,1), \hat{\pi}_{01} = \frac{n_{01}}{n_{00} + n_{01}}, \hat{\pi}_{11} = \frac{n_{11}}{n_{10} + n_{11}}. \]

**References**


