Model of Matrix-based Regression used in Economic Analyses

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Abstract

The emergence of new patterns of conflict with globalization has led to the re-configuration of the security agreement and the emergence of a "new security paradigm" in recent years. Securing could still lead to neglect issues of governance and the creation (or re-creating) the state monopoly of force, becoming the main concern in situations of conflict, often through extraordinary measures leading to fracture and proliferation of conflicts rather than their closure.

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The model relationship for this non-linear variant is the following:

\[ y = Z\beta + u, \]

and we imposed the assumptions:

\[ E'(u|Z) = 0 \]

and

\[ \text{Var}'(u|Z) = \text{Var}'(u) = \sigma^2 I_n. \]
The first condition means that $Z\beta$ is the probability of $y$ conditioned on $Z$ and thus it cannot be altered without fundamentally altering the model nature. In many economic situations, the second condition can be generalized in two modes, first within a setting, making the conditional variation $u_i$ depending on the conditional variables $z_i$ (heteroscedasticity) and, secondly, within a ne-i.i.d., without assuming that the covariance between the rests would be zero. Within a general frame, this can be written as:

$$\text{Var}(u|Z) = \Omega,$$

where $\Omega$ is a matrix which generally depends on $Z$ and on the unknown parameters $\rho$. In this model, the parameters of interest are $\beta$ and $\rho$.

Making the distinction between the case where $\Omega$ is known up to a multiplicative factor and the case where $\Omega$ is a function of unknown parameters, the usual approach of this class of models is of the form $\sigma^2 V\sigma = \Omega$.

First of all, if $\Omega = \sigma^2 V$, where $\sigma^2$ is known and $V$ is a symmetrical positive defined matrix, we can check whether the impartial linear estimator with the smallest dispersion solves the condition of minimum.

This estimator is known as the estimator of the generalized smallest squares (GLS).

The immediate extension of the Gauss-Markov theorem shows in particular that the dispersion of $\beta$ conditional on $Z$ is $\sigma^2 (Z'V^{-1}Z)^{-1}$ and that it is the smallest of the impartial linear estimators. A simple interpretation of this estimator is obtained by realizing that $V^{-1}$ can be factorized in $P^tP$ where $P$ is irreversible.

Assuming the relation $y|Z \sim N(Z\hat{\beta}, \sigma^2 V)$, we can easily verify that $\hat{\beta}$ is MLE of $\beta$, and that $\frac{n-q}{n} \hat{\sigma}^2 = \text{MLE of } \sigma^2$.

Secondly, if $\Omega$ is unknown and depends of a parametrical $\rho$, then the approach consists of two stages:

- We get a preliminary estimate of $\hat{\rho}_a$ of $\rho$ and thus an estimator $\hat{\Omega}_a$ of $\Omega$, replacing $\rho$ with $\hat{\rho}_a$.

We estimate $\beta$ using the formula where $V$ is replaced by $\hat{\Omega}_a$.

Under these conditions, we get the feasible generalized estimator of the smallest squares. This estimator is losing the properties of the small sample of the estimator GLS when $V$ is known and studied from the asymptotic point of view.

Further on, we shall focus, basically, on this study and, meantime, of the heteroscedasticity case and on the extension of the GLS estimators in the multivariate case.
We shall consider a model \( \{ X_\lambda, \Theta, \mathcal{P}_n \} \) and a function \( \psi \) defined on \( X \times \Lambda \times R \) \((\Lambda \subset \mathbb{R}^k, R \subset \mathbb{R}^l)\) with values in \( \mathbb{R}^l \). The function \( \psi(x_i, \lambda, \rho) \) is assumed as inferable for all \( \lambda \) and \( \rho \).

The interpretation is the following: if \( \rho \) is fixed at a certain value, which generally will depend on \( \theta \), \( \rho(\theta) \), then the system is defining a function \( \lambda(\theta) \) of the parameters of interest and the function \( \rho(\theta) \) defines a function of the disturbing parameters. The estimation of this last one is not a priority but approaching them is necessary in order to analyze the parameters of interest. It is noticeable that in the specific situation we are examining, the system contains more unknowns than equations and, thus, it cannot be used alone for estimating \( \rho(\theta) \) and \( \lambda(\theta) \).

Then, we analyze this issue for two situations:

The first case is defined by the assumption that the value of \( \rho \) is known. Generally speaking, this value depends on \( \theta \) and then we assume that \( \rho = \rho(\theta) \). Here, \( \lambda \) can be analyzed by using the known methods. We have a simple system of momentum equations which, in the context of the common conditions of regularity, leads to the \( \tilde{\lambda}_n(\rho(\theta)) \), given as solution of the function:

\[
E^\theta(\psi(x_i, \lambda, \rho)) = 0.
\]

The second case more relevant is that where \( \rho(\theta) \) is unknown but we have an available estimator \( \hat{\rho}_n \) which converges towards \( \rho(\theta) \). We solve the system:

\[
\frac{1}{n} \sum_{i=1}^{n} \psi(x_i, \lambda, \hat{\rho}_n) = 0,
\]

out of which we get the estimator \( \tilde{\lambda}_n \). Then, it is normal to ask whether \( \tilde{\lambda}_n \) keeps the same asymptotic properties as \( \tilde{\lambda}_n(\rho(\theta)) \) and, particularly, whether the asymptotic variation of the estimator is the same when \( \rho(\theta) \) is known or when \( \rho(\theta) \) is estimated. The answer is negative but the following theorem provides a simple criterion according to which both asymptotic distributions are equal.

Let's assume that \( \hat{\rho}_n \) converges to \( \rho(\theta) \) and that \( \sqrt{n}(\hat{\rho}_n - \rho(\theta)) \) has a limit of distribution. If the common conditions of regularity are satisfied, then \( \tilde{\lambda}_n \) converges to \( \lambda(\theta) \). If the condition:

\[
E^\theta\left( \frac{\partial \psi}{\partial \rho}(x_i, \lambda, \rho) \bigg|_{\lambda(\theta) \text{ and } \rho(\theta)} \right) = 0,
\]

is also satisfied, then the asymptotic distribution \( \sqrt{n}(\hat{\lambda}_n - \lambda(\theta)) \) is the same as the one submitted by the previous relations.
The third term of the equality tends towards zero and we see that the solutions we get out of:
\[
\frac{1}{n} \sum_{i=1}^{n} \psi(x_i, \lambda, \hat{\rho}_n) = 0
\]

and
\[
\frac{1}{n} \sum_{i=1}^{n} \psi(x_i, \lambda, \rho(\theta)) = 0
\]

are close arbitrarily and converge towards the same limit \( \lambda(\theta) \).

References


