Methodology Concerning the Utilization of the Value at Risk

Professor Gabriela-Victoria ANGHELACHE, PhD  
gabriela.anghelache@gmail.com  
Andreea NEGRU (CIOBANU), PhD Student  
Lorand KRALIK, PhD Student  
lorand17@gmail.com  
Academy of Economic Studies, Bucharest

Abstract

The value at risk (VaR) represents an estimate, at a certain level of probability and under normal conditions of the market, for the maximal level of value loss that may be recorded by a portfolio of financial assets over an established time horizon.

Originally, the VaR methodology has been used by the banks for internal purposes but it acquired an increasing significance after the amendment brought in 1996 to the Basel I Agreement, the surveying authorities encouraging the banks to apply the VaR. The main characteristic of the VaR model consists of the emphasize which it put on the expected losses as a result of the volatility of the market value of the financial assets, and not on the losses generated by the gains volatility.

Key words: value at risk, financial assets, portfolio yields, models, confidence probability

JEL Classification: G32

1. General notions concerning the value at risk (VaR)

The VaR method is utilized as an instrument of the market risk management by both the financial institutions (banks, insurance companies) and the non-financial companies and investment funds. For instance, if for a bank the daily value at risk amounts 50 thousand EUR at a confidence level of 95%, this amount means that, under normal market conditions, the bank cannot record a loss higher than 50 thousand EUR, with a probability of 95%, hence for 5 cases only out of 100, the losses can exceed 50 thousand EUR. This example considers 5% level of risk (which corresponds to a confidence level of 95%). In other terms, it is anticipated that for 5% of the days, the expected loss will exceed the value at risk (or the expected loss will be lower than the VaR level for 95% of the days).

The first formal of the value at risk has been developed by JP Morgan in RiskMetrics™, the fundamental formula being:

$$ \text{VaRx} = Vx \times \frac{dV}{dP} \times Pt $$

where:
- $Vx$ = the market value of the portfolio
- $\frac{dV}{dP}$ = the sensitiveness of the portfolio value as against the fluctuations of the market rates
- $Pt$ = the value of the adverse variation of the market price (expressed by interest rates, exchange rates, shares stock exchange rates or prices for certain commodities), $t$ represents the time horizon; the Basel Agreement recommends a 10 days interval.
As submitted by Heffernan (2005), any model for estimating the value at risk must take into consideration a series of aspects:

- which is the frequency for calculating the VaR (daily, monthly, quarterly? etc.);
- the identification of those positions/portfolios that are affected by the market risk;
- the risk factors which changes the market value of the positions/portfolios; The Basel Committee stipulates four risk factors, namely the interest rates (more precisely, their structure by maturities – yield curves), the exchange rates, the shares stock exchange rates and the commodities prices.
- the confidence intervals. As a rule, these are calculated for a confidence level of 99% (as required by Basel) and unilaterally, since VaR is addressing to the possible losses only, not to the potential gains as well. If the option goes to a confidence interval of 99%, the loss should occur in 2-3 days per year. Fixing a percentage of 99% corresponds to a more cautious, conservative approach.
- the financial assets holding period. The setting up of this one \( t \) will depend on the targets of the exercise. For instance, the banks holding liquid assets will be concerned with the daily VaR, while the pensions funds or the investment funds will prefer to calculate the VaR on a monthly basis. The Basel Committee specifies a 10 working days horizon, justifying it by the fact that a financial institution might need more than 10 days to liquidate its portfolio.
- the choice of the probability distribution corresponding to the yields, aspect which is going to be discussed further on.

As shown by Heffernan (2005), the specialized literature consecrated a number of five methods for the VaR calculation: the delta-normal method (known as the parametric method due to the working hypothesis with normal distribution, or the variance-covariance method), the delta-gamma method (or the Graecisms one), the method of historical simulations, the method of the extreme conditions testing (the scenarios analysis) and the last but not the least, the method of Monte Carlo simulation. Further on, we shall proceed to a review of these approaches for the calculation of the value at risk.

We can say that the fundamental hypothesis of the delta-normal method consists of the normality of the distribution of the analyzed portfolio financial assets yields. The portfolio yields is calculated as an average of the composing financial instruments yields weighted depending the initial value of the instruments, as follows:

\[
E(R_p) = \sum_{i=1}^{n} w_i E(R_i)
\]

which implies the fact that the considered portfolio yield follows, at its turn, a normal probability distribution as well.

The portfolio volatility is calculated as a squared average deviation of the recorded portfolio values from the average, namely – written in a matricidal form:

\[
\sigma_p = \sqrt{w^T \cdot \Sigma \cdot w}
\]

where:

- \( w \) is the vector of the weights of the financial assets in the considered portfolio;
- \( Z \) is the matrix of variance-covariance, namely the squared n-dimensional matrix having the structure:
where \( O_{ij} \) represents the covariance between the yields of the assets \( i \) and \( j \) from the portfolio structure and \( G_j \) represents the variance of the asset \( i \).

The \( \text{VaR} \) calculated at the level of the financial assets portfolio becomes:

\[
\text{VaR}_p = V_0 \cdot c \cdot \sigma_p
\]

where:

- \( V_0 \) = the initial value of the portfolio;
- \( c \) = the confidence coefficient relating to the considered probability distribution (which values are shown by tables).

Due to the utilization of the variance-covariance matrix, this method is called also the variance-covariance method. It can offer the estimate for the market risk associated with a much extended portfolio of assets with gamma null (their payoff function is linear which means that the first differential – delta – the modification relating to the rate modification is a constant, while the second differential – gamma – is null). The delta normal method is a parametric method for estimating the being a partial model of evaluation since it takes into consideration the linear dependences only (delta) and ignore the non-linear factors, such as for instance, the convexity of the bonds and the gamma indicators of the options. The delta-gamma method is demonstrating accuracy when the considered portfolios are composed of traditional financial assets or of linear derivatives, but it cannot be utilized when the non-linearity is present. In order to set up the variance-covariance matrix different models of the heteroscedastic volatility (namely the conditioning or the dependence on the time variable) are used, excepting the stochastic volatility (models ARCH, GARCH, EGARCH, IGARCH, EWMA – the model of the exponential mobile weighted average), which are based on historical rates.

The most utilized models in the international financial practice are those of GARCH type, mainly the GARCH model (1,1). The ARCH model has a practical applicability relatively limited; the EWMA model of the exponential mobile weighted average has been invented and suggested by J. P. Morgan in his RiskMetrics™, while the stochastic volatility is intensely debated among the theoreticians and the market players but with good prospects to get implemented. We shall succinctly submit further on these models.

The original ARCH model proposed a change of the vision on the way to estimate the volatility according to which the squared average deviation, through its way of calculation, grant an equal weight \( (1/n) \) to all the historical observations taken into consideration in the volatility setting up:

\[
\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (r_i - \mu)^2}
\]

The Engle model succeeds to overcome this inconvenient, by granting an increased importance to the most recent observations and by reducing the weights of the most remote observations. In addition, it includes also in the calculation a so-called long-term average \( (V) \), to which it grants a weight noted by \( y \). Thus, the variant (dispersion)
from which square root the volatility is resulting, is expressed with the help of the following formula:

\[ \sigma_t^2 = \gamma \cdot V + \sum_{i=1}^{k} \alpha_i r_{t-i}^2 \]

The GARCH(1,1) model is a particular case of the generalizing GARCH (q,p) models. His model acquired an increased popularity due to its relative simplicity, being similar to the Engle model. The formula for this variant is in this case:

\[ \sigma_t^2 = \gamma \cdot V + \alpha \cdot r_{t-1}^2 + \beta \cdot \sigma_{t-1}^2 \]

where \( \gamma + \alpha + \beta = 1 \).

The GARCH(1,1) model suggests that the predicted variant is based on the most recent observation of the assets yield and on the last calculated value of the variant. The general model GARCH(q,p) is predicting the variant on the basis of the last q observations and the most recent p estimated variants.

The EWMA model is a particular case of the previous model where the weight of the average volatility on long-term is considered null (\( \gamma = 0 \)). In this context, the model can be rewritten as follows:

\[ \sigma_t^2 = (1 - \lambda) \cdot \sigma_{t-1}^2 + \lambda \cdot \sigma_{t-1}^2 \]

The estimation of the correlation coefficients between two different time series is of an equal significance with the volatility estimation. The correlation coefficient (\( \rho_{ij} \)) is expressed by the following formula:

\[ \rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \]

where the covariance is calculated as follows:

\[ \sigma_{ij} = (1 - \lambda) \cdot \sum_{t=1}^{T} \left[ \lambda^{t-1} \cdot (r_{i,t} - \bar{r}_i) \cdot (r_{j,t} - \bar{r}_j) \right] \]

The coefficient lambda or the factor of degradation of the most recent data is determining the importance granted to the last historical data. Thus, the extent to which the volatility is influenced by the last information coming from the market varies depending on the coefficient chosen value. To the extent lambda is smaller, the estimated volatility responses lesser to the most recent evolution of the market. Here, the risk managers are facing the issue of achieving a compromise between granting more importance to the most recent information from the market or to the previous volatility, meaning practically to the “older” data. In respect of the two sides composing the compromise, the specialized literature shows two concepts:

- "the shadows" or "the ghosts" - occur when the predicted volatility is "shadowed", excessively marked by the previous tendencies of the rates. When individualizing for the EWMA model, "the ghosts" are occurring when there is a too big lambda;
- the error of extraction/randomizing – when choosing a too small lambda, the significance of the general tendency of the volatility valorized by the historical data gets reduced, to the benefit of the most recent data.
The advantages of the delta-normal method of calculating the VaR are the following:
- the method is easy to implement;
- in the case of the gamma and vega-null portfolios, having a structure which does not change, the calculated VaR for one day time horizon can be utilized for measuring the VaR for a larger horizon, on the basis of the square root model, namely: \( \sigma_{\Delta t} = \sigma \cdot \sqrt{\Delta t} \), where \( \Delta t \) represents exactly the difference between the two horizons.

There is also a lot of criticism regarding the delta-normal method, respectively:
- it does not take into account the extreme situations that may occur during the analyzed period, without being predicted presently. We have to point out here that this is lack of all the forms of the VaR calculation grounded on historical data;
- there are "fat tails" in the real distributions which the yields of the gamma-zero assets follow, fact which suggests that their prices are more inclined to take extreme values than to indicate the normal distribution hypothesis. That is why, the estimated maximum loss for the risk horizon considering a certain probability is in fact smaller than it is in reality;
- thirdly, the method does not take into consideration other than the delta risk, being thus unable to evaluate the value at risk for the portfolios that include non-linear instruments, namely those which gamma may differ of zero. The incapability of the method arises from the fact that delta may be rapidly modified under the conditions of a gamma relatively large and because of the fact that the largest losses may be recorded also for intermediary values of the equities rate not only for the extreme values.

In order to compensate the disadvantage of the previous method, we can apply to the alternative delta-gamma method, called thus because it takes into consideration both the risks gamma and vega. However, the method bears an essential disadvantage: the excessively complex degree of calculations. The more the types of risks are (delta, gamma and vega) and more instruments exist in the portfolio, the more the complexity of the analysis is increasing geometrically. The best alternative to this method is the Monte Carlo simulation. This is the reason due to which the delta-gamma method has a narrow enough area of implementation.

As mentioned above, a significant lack of the variance-covariance method is generated by the hypothesis of the normal distribution for the financial assets distributions, so that the risk of extreme losses may be underestimated. The approaches based on historical simulations have the merit to bring in their own contribution to counteract this inconvenient.

The advantage of this method consists of the fact that it makes no presumption as regards the yield distribution, as in this case the empirical distribution obtained from the analysis of the past data is used. Meantime, the historical simulation has the advantage of being a relatively simple method that does not require the calculation of the variance-covariance matrix. In exchange, this method has the disadvantage that it predicts the future development based on the past, contradicting thus the theoretical models that consider the assets price as being of the category of the Markov type processes.
As shown by HefTernan (2005), the historical simulation method is a non-parametric method for the VaR calculation and, meantime, a complete model of evaluation since it take into consideration any type of dependence, either linear or not, between the portfolio value and the risk factors. As to the statistic horizon required in order to give consistency to the analysis, the Basel Committee recommends a period of minimum one year of history (in fact, as in the case of the delta normal method).

The method shows a series of specific features that distinguish it from the other four methods. As suggested by the naming, the testing of the extreme conditions could be not utilized by itself as a technique of estimating the value at risk, as it comes to complete the outcomes obtained through an independent method. This is necessary in order to evaluate the effects of the most severe markets, with a small probability of achievement. Afterwards, it is characterized by a high subjectivism comparatively with the other methods, as this method is grounded on adding certain extreme value to the fundamental financial variables, which have been chosen by the analyst with the purpose to see which would be the maximum loss in the situation the risk occurs. This is the reason due to which the method of testing the extreme conditions bears a high degree of operational risk.

The Monte Carlo simulation technique is considered as being the most relevant and efficient in the VaR estimation. In this case, the distribution of probability of the portfolio yield over the next \( h \) days is obtained by generating multiple scenarios for all the risk factors taken into account and by composing the portfolio value under these conditions. The method is flexible, being applicable to all the categories of portfolios but it is characterized by an inherent complexity of the calculations and requires a careful choice of the evaluating models for the financial assets from the portfolio structure.

The actual VaR calculation implies the measurement of the following two parameters: the time horizon \( (h) \) that is considered for the risk estimation and the confidence probability \( (p) \) or the percentage of tolerance at risk \( (1-p) \) at which the risk is calculated. Once these two elements set up, the VaR can be calculated in two variants:

a. as difference between the current value of the selected portfolio and the smallest value of the portfolio (known as quantil), considering \( h \) – the selected time horizon, with the set up probability \( p \);

b. as the difference between the expected value of the portfolio, considering the chosen risk horizon \( h \) with the probability \( p \) and the smallest value of the portfolio (quantil) for the same time horizon \( h \) and the same probability \( p \).

Now, we consider the following notations:

- \( W_0 \) = the present market value of the portfolio;
- \( Wh \) = the expected average value of the portfolio according to the chosen time horizon \( h \);
- \( Rm \) = the average yield of the portfolio over the time horizon \( h \);
- \( W \) = the smallest value (quantil) that can be recorded by the portfolio over the time horizon, corresponding to the established confidence level;
- \( R \) = the yield relating to the quantil.

Applying the above notations, we can write the following two relations:

\[
W \_ = W \_ \cdot (1 + R \_ \cdot ) \quad W \_ = W \_ \cdot (1 + R \_ \cdot )
\]

Using these two relations we can calculate VaR (the maximum expected loss) in the two variants, as follows:

a. as against the current value of the portfolio on the established time horizon:

\[
VaR = W \_ - W \_ = W \_ - W \_ \cdot (1 + R \_ \cdot ) = W \_ \cdot R \_
\]

b. as against the expected average values of the portfolio on the time horizon \( h \):
The smallest value of the portfolio yield at the risk profile, R−, is calculated starting from the function f of the yield R distribution, as follows:

\[ 1 - p = P(x < R^*) = \int_{-\infty}^{R^*} f(x) \, dx \]

If the cumulative function of distribution (the repartition function) is known and mainly in the situation where it is just the cumulative function of the normal distribution, the calculation of the value at risk gets considerably simplified. This happens in our case: the yields follow a normal distribution, so that we can write:

\[ R \cdot R^* \sim N(R_m, \sigma) \]

We shall have, of course:

\[ 1 - p = P(x \leq R^*) = \int_{-\infty}^{R^*} f(x) \, dx = \int_{-\infty}^{\alpha} \varphi(z) \, dz \]

with

\[ \alpha = \Phi^{-1}(1 - p) \]

where from:

\[ \alpha = \frac{R^* - R_m}{\sigma} \quad \Rightarrow \quad R^* = \alpha \cdot \sigma + R_m \]

From the tables concerning the repartition function of the standard normal distribution, the following critical values for the parameter d are resulting, corresponding to the probabilities, such as:

<table>
<thead>
<tr>
<th>Probability(P)</th>
<th>99%</th>
<th>97.5%</th>
<th>95%</th>
<th>90%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold d (x &lt; d)</td>
<td>2.33</td>
<td>1.96</td>
<td>1.65</td>
<td>1.29</td>
</tr>
</tbody>
</table>

By replacing the value of R in the calculating relation of VaR, we get the VaR formula for the hypothesis of the normal distribution:

a. as against the current market value Wo of the portfolio:

\[ \text{VaR} = W \cdot (1 - R^*) = W \cdot (1 - R_m - \alpha \cdot \sigma) = W \cdot (\alpha \cdot \sigma + R_m) \]

It is known that the average yield on very short terms tends to zero, so that Rm = 0.

b. as against the expected average value of the portfolio for the chosen time horizon, Wh:

\[ \text{VaR} = W \cdot (1 - R^*) = W \cdot (\alpha \cdot \sigma + R_m) \]

Consequently, the maximum loss of the portfolio (the value at risk), is calculated according to the relation:

\[ \text{VaR} = \text{Current market value} \times \text{Confidence level} \times \text{Volatility} \]

Taking into consideration the fact that the yield follows a normal distribution, there is a relation through which two VaR estimations can be equated, one of them having a certain degree of tolerance at risk while the other estimate having a different percentage, on the basis of the following formula:

\[ \text{VaR} \cdot a = -W \cdot \alpha \cdot \sigma \]

Noting with a and ^ the two percentages of tolerance at risk, we shall have:
If we consider the hypothesis that the yields of one portfolio values follow a normal distribution, it must be pointed out that there is a relation for equating a VaR estimation, with a percentage of tolerance at risk of 5% and a second estimation having another percentage of 1%:

\[
\text{VaR}_{\alpha_2} = \frac{\text{VaR}_{\alpha_1} \cdot \alpha_2}{\alpha_1}
\]

where:

\[
\text{VaR}_{\alpha} = - W \cdot \alpha \cdot \sigma \int_{1-\alpha}^{1} \text{VaR}_{\alpha} = - W \cdot \alpha \cdot \sigma \cdot \Phi
\]

If we consider the hypothesis that the yields of one portfolio values follow a normal distribution, it must be pointed out that there is a relation for equating a VaR estimation, with a percentage of tolerance at risk of 5% and a second estimation having another percentage of 1%:

\[
\text{VaR}_{0,05} = \frac{2.33}{1.65}
\]

where, according to the standard normal repartition function we have:

\[
\Phi^{-1}(0.95) = 1.65, \quad \Phi^{-1}(0.99) = 2.33
\]

References