The APT Model and its Applicability in Romania’s Case

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Abstract
Since the discovery and the development of the financial equilibrium asset pricing models, they were constantly and repeatedly tested mainly for the big markets and scarcely for the smaller or the emerging ones. Romania belongs to the last category, hence empirical testing of these models for its case was almost nonexistent. So, this paper examines the validity and the applicability of the Arbitrage Pricing Theory model for the Romanian stock exchange, conditioned of course by the available data. The data used is represented by monthly returns of 60 companies, listed on the Bucharest Stock Exchange, using a 6-year period, from 01.01.2005 to 31.12.2010. This period was divided into 2 equal sub-periods, and the testing process was conducted for each sub-period, and then again for the whole period.

The findings sustain APT only minimally for the 2 sub-periods, where only one priced factor was obtained, and stronger evidence was found for the entire period of time, where 3 factors proved to be significant in influencing the returns of the selected assets. These results and mainly those for the whole period of time resemble the conclusions of the majority of studies, who found in general 2-4 priced factors, regardless of which market was analyzed.

Key words: APT, Bucharest Stock Exchange, factor loadings, returns, multiple regression

JEL classification: C31, G12

The present paper is organized as follows: sections 1 and 2 introduce the APT model and resume its development; section 3 summarizes the concerns in the field and some of the empirical studies conducted through time; section 4 contains the actual testing of the model on the Romanian market, including the descriptions of used database, methodology and obtained results, along with their interpretations; section 5 resumes the conclusions and outlines some obvious limitations; section 6 presents the references used during the analysis.

Introduction
Arbitrage Pricing Theory, formulated by S.Ross[16] in 1976, offers a testable alternative to CAPM, trying to improve some of the latter’s disadvantages. Whereas CAPM states that the security rates of return will be linearly related to a single common factor, which is the rate of return of the market portfolio, APT is more general, predicting that the rate of return on a stock is a linear function of different multiple factors. Hence:
\[ R_i = E(R_i) + b_{i1} \times F_1 + b_{i2} \times F_2 + \ldots + b_{ik} \times F_k + \epsilon_i \]  

(1)

where:
- \( R_i \) = the random rate of return on the “i”th asset;
- \( E(R_i) \) = the expected rate of return on the “i”th asset;
- \( b_{ik} \) = the sensitivity of the “i”th asset’s return to the \( F_k \) factor;
- \( F_k \) = the \( k \)th factor common to the returns of all considered assets, having mean zero, representing the systematic risk of the “i”th asset;
- \( \epsilon_i \) = a random zero mean noise term for the “i”th asset.

In matrix form:
\[ R_i = E(R_i) + B \times F + \epsilon_i, \]

where:
- \( B \) = the matrix of sensitivities \( b_{i1}, b_{i2}, \ldots, b_{ik} \);
- \( F \) = the matrix of all factors considered, presumable to influence assets’ returns.

**Theoretical and mathematical development**

In this section, aspects from Copeland and Weston[8] or Mishkin[14] will be used. The development of APT begins with equation (1) and has its base on the following idea: in equilibrium, all portfolios than can be selected from the set of assets under consideration and that satisfy the conditions:

a) without additional wealth invested
b) without any risk involved,

must not earn a return on average. These portfolios are called *arbitrage* portfolios, and they will be made so: let \( w_i \) be the change of sum invested in the “i”th asset, as a percentage of an individual’s total invested wealth, aiming to modify the proportions of the stocks in his/her portfolio. In order to obtain an arbitrage portfolio that requires no change in wealth (deposit or withdrawal), the logical action will be selling some assets and then buying others using the gained sum. Mathematically, the zero change in wealth is written as:

\[ \sum_{i=1}^{n} w_i = 0 \]  

(2)

If there are \( n \) assets in the arbitrage portfolio, then the additional return gained by modifying the proportions of the included assets is:

\[ R_p = \sum_{i=1}^{n} w_i \times R_i = \sum_{i=1}^{n} w_i [E(R_i) + b_{i1} \times F_1 + b_{i2} \times F_2 + \ldots + b_{ik} \times F_k + \epsilon_i], \]

so:

\[ R_p = \sum_{i=1}^{n} w_i \times E(R_i) + \sum_{i=1}^{n} w_i \times b_{i1} \times F_1 + \ldots + \sum_{i=1}^{n} w_i \times b_{ik} \times F_k + \epsilon_i \]  

(3)
For obtaining a riskless arbitrage portfolio, it is necessary to eliminate both types of risk, systematic (non-diversifiable) and unsystematic (diversifiable, idiosyncratic). This can be done by meeting the following conditions:

i. the arbitrage portfolio being well-diversified; in other words \( n \) must be a large number

ii. the percentage changes in each asset’s investment must be small and approximating \( 1/n \):

\[ w_i \approx 1/n \]

iii. choosing changes \( w_i \) in such a manner that for each factor “\( k \)” no systematic risk exists; in other words the weighted sum of the systematic risk components \( b_k \) to be zero:

\[ \sum_{i=1}^{n} w_i \times b_i \times k = 0 \]

Because the error terms \( \varepsilon_i \) are independent and \( n \) is a big number, the law of large numbers ensures that \( \sum_{i=1}^{n} w_i \times \varepsilon_i \) approaches zero, hence the last term of equation (3) will disappear, which is equivalent to the disappearance of the idiosyncratic risk. Now the equation takes this form:

\[ R_p = \sum_{i=1}^{n} w_i \times E(R_i) + \sum_{i=1}^{n} w_i \times \beta_1 \times \times 1 + \ldots + \sum_{i=1}^{n} w_i \times F_k \times b_k \]  

(4)

Since the systematic risk doesn’t exist as well (by 4.iii), all the terms besides the first one disappear from equation (4), and such a arbitrage portfolio without both types of risks becomes possible, having a return equal to:

\[ R_p = \sum_{i=1}^{n} w_i \times E(R_i) \]  

(5)

But no portfolio is an equilibrium portfolio if its return can be improved without taking an additional risk and without an additional sum invested; and since the analyzed portfolio didn’t imply any of them, it is necessary that:

\[ R_p = \sum_{i=1}^{n} w_i \times E(R_i) = 0 \]  

(6)

Equations (2), (4.iii) and (6) are confirmed also by linear algebra elements: any vector \( w_i \) (\( i = 1, \ldots, n \)) that is orthogonal to the constant vector “\( e \)”, that is:

\[ (\sum_{i=1}^{n} w_i \times e) = 0 \]

and also orthogonal to each of the coefficient \( b_k \) (\( k = 1, \ldots, n \)), that is:

\[ \sum_{i=1}^{n} w_i \times b_k = 0, \quad (for \ each \ k) \]

must be orthogonal to the vector of expected returns, so:

\[ \sum_{i=1}^{n} w_i \times E(R_i) = 0 \]

The algebraic consequence of this is that the expected return vector must be a linear combination of the constant vector and the coefficient vectors. Mathematically, it is necessary that a set of coefficients \( \lambda_0, \lambda_1, \ldots, \lambda_k \) (in a number of “\( k+1 \)”) exists, such that:

\[ E(R_i) = \lambda_0 + \lambda_1 \times b_{11} + \lambda_2 \times b_{12} + \ldots + \lambda_k \times b_{1k} \]

(7)

where \( b_{ik} \) is the sensitivity of the “\( i \)”th asset’s return to the \( F_k \) factor.
If a riskless asset is available on the market, it will offer a riskless return R_f and so:

\[ b_{0k} = 0 \] \text{ and } \lambda = \lambda_0

So equation (7) can be rewritten in “excess returns form” as:

\[ \lambda_0 \]

\[ E(R_i) - R_f = \lambda_1 \times b_{i1} + \lambda_2 \times b_{i2} + \ldots + \lambda_k \times b_{ik} \] (8)

In equilibrium, all assets must fall on the arbitrage pricing line. \( \lambda_1 \ldots \lambda_k \) represent “risk premiums” (prices of the risk) in equilibrium, being defined as the difference between the expected returns of a portfolio with maximum sensitivity to factor 1, 2, …, k and zero sensitivity to the other factors, and the riskless rate of interest:

\[ \lambda_1 = E(F_1) - R_f \]
\[ \lambda_2 = E(F_2) - R_f \]
\[ \ldots \]
\[ \lambda_k = E(F_k) - R_f \]

Hence, the APT form can also have the following expression:

\[ E(R_i) - R_f = [E(F_1) - R_f] \times b_{i1} + [E(F_2) - R_f] \times b_{i2} + \ldots + [E(F_k) - R_f] \times b_{ik} \] (9)

or:

\[ E(R_i) = R_f + [E(F_1) - R_f] \times b_{i1} + [E(F_2) - R_f] \times b_{i2} + \ldots + [E(F_k) - R_f] \times b_{ik} \]

\( b_{ik} \) are defined in a similar way as \( \beta \) from CAPM, like:

\[ b_{ik} = \text{cov}(R_i,F_k)/\text{var}(F_k) \]

Empirical testing on the Romanian market

The database used for testing:

The current study is based on monthly returns for stocks listed on the Bucharest Stock Exchange during the 01.01.2005 – 31.12.2010 interval, with respect to the available information. Logarithmical values are used to ensure the series’ stationarity. The data was obtained from the web pages of BVB\(^1\) and of “Kmarket”\(^2\) investment firm. The missing observations were completed with interpolation. All the stock market’s categories are taken into account (I, II and III), and they include 76 assets having available data, and from those, some are eliminated due to lack of more than 25% of the observations in the time period. Hence the final sample consists of 60 assets, each of them with 72 observations of monthly return (12 months for 6 years).

For a proxy of the riskless interest rate, the government bonds were used, with respect to the available data. This rate had an annual value of 8.8404% in the

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\(^1\) http://www.bvb.ro

\(^2\) http://www.kmarket.ro
The data was obtained from the monthly reports of BNR, on its web site³.

The entire 6-year working time interval will be divided into 2 equal sub-periods: the first sub-period is 01.01.2005 - 31.12.2007, and the second one is 01.01.2008 – 31.12.2010. The testing will be developed on each sub-period, and then also on the whole period of time, for comparative and superior accuracy purposes.

Software programs Microsoft Excel and SPSS are helpful in this empirical testing. As a method of applying factor analysis in SPSS, the “principal component analysis” procedure was used, followed by an “orthogonal varimax” rotation.

**APT’s testing methodology:**

Steps 1-4 previously presented will be followed. The procedure will be thoroughly shown only for the first sub-period, and then for the second one along with the entire one, only the results with their interpretations will be described.

So, for the first sub-period the next succession of operations occurs: since step 1 was already realized (collecting a time series of periodical returns for the considered assets), step 2 (calculating the variance-covariance matrix using those returns) and step 3 (identifying the number of factors with influence and their “factor loadings”), are immediately resolved using the SPSS software. Step 2 is inherent as an intermediate stage for step 3. For the goal of identifying the number of factors to be retained for testing, SPSS “scree test” criteria will be firstly used, by interpreting figure 1:

![Scree Plot](image)

**Figure 7. SPSS “scree test” criteria**

The number of factors to be retained is the one corresponding to the change of the trend’s trajectory from vertically to horizontally. For the present situation,
this fact happens for a number of 5 factors. Adding this information to the one from the literature that states we can keep a number of factors which explain minimum 45% of the total variance:

<table>
<thead>
<tr>
<th>Component</th>
<th>Rotation Sums of Squared Loadings</th>
<th>Total</th>
<th>% of Variance</th>
<th>Cumulative %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11,518</td>
<td>19,197</td>
<td>19,197</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6,875</td>
<td>11,458</td>
<td>30,655</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>6,077</td>
<td>10,128</td>
<td>40,784</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5,231</td>
<td>8,719</td>
<td>49,502</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4,077</td>
<td>6,795</td>
<td>56,298</td>
<td></td>
</tr>
</tbody>
</table>

...we will keep 5 factors for the testing, them being sufficient to explain in a cumulative way 56,298% of the variance. Once the number of factors is established, step 3 continues with estimating the “factor loadings” (the $b_k$ coefficients from the APT equation) for these factors.

Step 4 includes the actual testing of the model, for determining the number of factors who truly influence the returns. In other words, finding the priced factors, thus the factors who have statistically significant and non-zero associated premiums. For this, we will run a multiple regression, having the following form:

$$R_i = \lambda_0 + \lambda_1 b_{i1} + \lambda_2 b_{i2} + \lambda_3 b_{i3} + \lambda_4 b_{i4} + \lambda_5 b_{i5} + \varepsilon_i$$

...where:

- $R_i$ = average (expected) return of the “i”th asset on the studied interval
- $b_{i1}…b_{i5}$ = “factor loadings”, previously estimated
- $\lambda_1…\lambda_5$ = risk premiums associated with the 5 chosen factors
- $\lambda_0$ = intercept, equals the expected return for a riskless asset = $R_f$
- $\varepsilon_i$ = random zero mean noise term for the “i”th asset

It is possible to observe that the regression has exactly the 5 previously chosen factors (in step 3) as independent variables, among which only those to have a significant risk premium will be considered priced, thus having a real influence on the stocks’ returns. The results of the regression are:

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0$</td>
<td>4,4269</td>
<td>0,0104</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>-2,6667</td>
<td>0,2113</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-1,5200</td>
<td>0,5814</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>7,7765</td>
<td>0,0014</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>-3,7262</td>
<td>0,1381</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>4,2307</td>
<td>0,1391</td>
</tr>
</tbody>
</table>
By interpreting these estimations, we can conclude that only factor 3 is priced, because, by having an absolute value of the “t” test superior to the reference level of 1.96 and a “P-value” inferior to the reference level of 0.05 (for a 95% significance level), it proves to present a statistically significant risk premium. Factors 1,2,4 and 5 don’t give such significant risk premiums and by doing so, they have no influence on the studied stocks’ returns.

The methodology is similar for the second 3 year sub-period of time. The chosen 5 factors cumulative explain 63.549% of the variance:

<table>
<thead>
<tr>
<th>Component</th>
<th>Rotation Sums of Squared Loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>% of Variance</td>
</tr>
<tr>
<td>1</td>
<td>10,315</td>
</tr>
<tr>
<td>2</td>
<td>8,939</td>
</tr>
<tr>
<td>3</td>
<td>7,035</td>
</tr>
<tr>
<td>4</td>
<td>6,043</td>
</tr>
<tr>
<td>5</td>
<td>5,798</td>
</tr>
</tbody>
</table>

The results of the regression are:

<table>
<thead>
<tr>
<th>Component</th>
<th>Coefficients</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0$</td>
<td>-0.2584</td>
<td>-0.2571</td>
<td>0.7980</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>-3.1731</td>
<td>-2.5567</td>
<td>0.0134</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-0.7973</td>
<td>-0.5832</td>
<td>0.5621</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>0.4383</td>
<td>0.3181</td>
<td>0.7516</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>-0.0901</td>
<td>-0.0568</td>
<td>0.9548</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>-1.1939</td>
<td>-0.8525</td>
<td>0.3976</td>
</tr>
</tbody>
</table>

For this second sub-period, only factor 1 proves to have influence, having a statistical significant risk premium. The other factors, namely 2,3,4 and 5 don’t have significant risk premiums, thus being not significant themselves. If the whole 6 year-period is analyzed, 52.548% of the variance is cumulative explained by the 5 chosen factors:

<table>
<thead>
<tr>
<th>Component</th>
<th>Rotation Sums of Squared Loadings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>% of Variance</td>
</tr>
<tr>
<td>1</td>
<td>12,551</td>
</tr>
<tr>
<td>2</td>
<td>6,213</td>
</tr>
<tr>
<td>3</td>
<td>4,741</td>
</tr>
<tr>
<td>4</td>
<td>4,300</td>
</tr>
<tr>
<td>5</td>
<td>3,724</td>
</tr>
</tbody>
</table>
The results of the regression which shows the significance of the factors’ risk premiums are shown below:

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>t Stat</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_0$</td>
<td>1.6699</td>
<td>1.7857</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>-1.8503</td>
<td>-1.7080</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>-1.7813</td>
<td>-1.1504</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>3.9805</td>
<td>3.3128</td>
</tr>
<tr>
<td>$\lambda_4$</td>
<td>3.2053</td>
<td>1.9902</td>
</tr>
<tr>
<td>$\lambda_5$</td>
<td>-2.8449</td>
<td>-2.0904</td>
</tr>
</tbody>
</table>

It can be seen that 3 factors are significant for the entire 6 year-period, specifically factors 3, 4 and 5, whereas factors 1 and 2 don’t have influence, their risk premiums being not significant.

Conclusions and limitations

The conclusions section will review the hypothesis and the assumptions of the APT’s empirical testing, will summarize the obtained results and will specify certain limitations which might have been influencing the findings.

The article examined the Arbitrage Pricing Theory in Romania’s stock market, that is in the Bucharest Stock Exchange. Used data were adjusted monthly returns, derived from monthly prices, for 60 listed stocks, for a time period between 01.01.2005 and 31.12.2010.

The summary of the findings, both on the sub-periods and on the entire period, is:

<table>
<thead>
<tr>
<th>Resulted number of factors having influence on stocks’ returns</th>
<th>First sub-period (3 y)</th>
<th>Second sub-period (3 y)</th>
<th>Entire period (6 y)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 factor</td>
<td>1 factor</td>
<td>3 factors</td>
</tr>
</tbody>
</table>

It can be seen that in the sub-intervals (2005-2007 and 2008-2010), APT offered only a single factor with influence (factor 3 for the first and factor 1 for the second), whereas in the whole interval (2005-2010), the model looks better, giving 3 factors with influence (factors 3, 4 and 5).

Thus we can say that the model can’t be considered as fully accepted for this context, but only on a minimal level for the sub-periods, where a single-factorial model proved to be suitable (only one positive risk premium and hence only one factor having influence). APT can be though accepted, with some reserves due to limitations, on the entire period of time, because here the model becomes multi-factorial. The majority of the previous analysis, including the one made by F.Bilbïie, A.Gherman and M.Tureatcă[1], were presenting also 2-4 significant
factors. So, for the full interval, the obtained results are close to the ones listed in the literature review of this topic. Obviously, the resumed conclusions are subdued to certain below-listed limitations, and further tests are necessary to totally validate/partially validate/invalidate them.

Limitations

The results of this testing and their interpretations can not be necessarily considered as references. The following aspects used in this empirical process could possibly provoke some errors and, through this, some deviations from a set of accurate conclusions:

- the availability and the accuracy of the data is not fully guaranteed, and the missing observations were completed by interpolation, which is not similar with being totally exact;
- using a “proxy” for the riskless interest rate;
- the existence of some possible miscalculating, including some caused by the author;

Acknowledgements:

This paper was co-financed by European Social Fond, through POSDRU 2007-2013, project nr.88/1.5/S/55287, DOESEC.

References