THEORETICAL NOTIONS ON MARKET RISK IN INVESTING IN SECURITIES PORTFOLIOS

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Abstract

A main objective of risk management is to evaluate and improve the performance of financial institutions in the context of risk-taking, in order to make a profit. Therefore, quantifying risk in a portfolio optimization issue is essential, as it is the first step in managing a portfolio's risk. Financial market volatility involves a detailed analysis of risks, as well as the quantification of risks that generate optimal solutions.

The most used training in practice on the issue of efficient portfolio selection is the medium-variance model developed by Markowitz. It is a cornerstone of modern portfolio theory and despite reserved views and several proposals for new risk measures, variance is still the most widely used measure of risk quantification in financial practice. The main approach is that the agents optimally select efficient portfolios using the medium-variance criterion. In practice, this model is widely used to manage portfolio risk. Specific uses include establishing optimal asset allocations, quantifying gains from international diversification, and evaluating the performance of a portfolio.

Models of this type include the distributions of returns characterized and compared with two statistics: expected return and volatility as a measure of risk. Medium-variance models have an intuitive interpretation of the results and in most cases are computationally convenient. Some researchers dispute these advantages because the practice of using a distribution that depends on only two parameters involves neglecting some information, which proves to be necessary.

Keywords: risk, performance, efficient portfolios, securities, statistical-econometric models.

JEL classification: C10, G11

Introduction

In general, the most commonly used measure of market risk is Value-at-Risk-VaR. VaR a risk measure specified according to a quantile, has become one of the most used measures among practitioners. For a certain time horizon t and a confidence level β, the risk value of a portfolio is that loss over time horizon t, which can be guaranteed with a probability 1 - β. VaR is used because it is easy to calculate, analyse and interpret. As a measure of market risk, VaR has certain recognized limits. Some researchers have analysed that VaR does not take into account losses that exceed VaR and that for different confidence levels it can provide contradictory results.

VaR lacks the properties of sub-additivity and convexity, being a risk measure that satisfies these axioms, becoming a coherent risk measure. Since 1999, the concept of coherent risk measurement has become a criterion for assessing risk measures. But, VaR is not a coherent risk measure because it does not respect the axiom of sub-additivity, implying that VaR, for a combination of two portfolios, can be higher than the sum of the VaRs of the individual portfolios. Please note that VaR is a coherent risk measure only when it is based on the standard deviation of normal distributions. It should be noted that VaR is in contrast to portfolio diversification due to non-compliance with the axiom and considers that VaR is not a risk measure because a risk measure cannot violate the axiom of sub-additivity.

It has been shown that the problem of minimizing the VaR of a portfolio can lead to more local minimums. Also, VaR optimization can lead to a difficult non-linear and non-convex problem, very difficult to solve. Therefore, despite a considerable amount of research activity, VaR optimization is still an open issue for study, analysis and interpretation.

For these reasons, another measure of portfolio risk has been recommended in the literature, namely Conditional Risk Value (CVaR). For a certain time horizon t and confidence level β, CVaR is the expected level of anticipated loss conditioned on the fact that it is higher than VaR.

At the same time, CVaR is a risk measure that has very interesting properties, respectively CVaR is attractive because it is a coherent risk measure. CVaR being a convex function, it is relatively easy to control and optimize, as a solution to an optimization problem. Numerical experiments have shown that minimizing CVaR leads to near-optimal solutions and minimizing VaR, given that VaR never exceeds CVaR. Several authors have created a new technique, a minimization formula. Using this technique, the VaR value can be calculated at the same time and the CVaR can be optimized. It was noted that the CVaR measure, as a modeling tool in optimization cases, has appropriate properties in several respects. Also, the reduction of CVaR usually leads to a portfolio with a low VaR.

Literature review

The statistical-econometric models of risk analysis regarding the investment of portfolios on the capital market were approached in their works by specialists in the field, from which we mention some works such as Anghelache C., Anghel M.G., Iacob Ș.V. (2020), in which the authors focused on an econometric model that can be used in the situation of asset accumulation and portfolio decisions that are taken at the risk of inflation, starting from the fact that the capital market is influenced by the effect of price changes. Also, Anghelache C., Dumitru M., Grigorescu D.L. (2020) addresses the problem of obtaining an optimal result starting from estimating the multiplier between two equations, demonstrating that the optimal solution or the optimal contract for an investor is given by two equations that lead to Pareto efficiency, thus highlighting the perspective of substantiating decisions to conclude a optimal contract. In the same sphere is the work of Anghelache C., Anghel M.G., Iacob Ș.V., Pârțachi I. (2020), in which the authors addressed the issue of the effect of operational risk in the context of placing investments in various markets. Regarding the balance of the capital market, this aspect is analysed in his paper by Black, F. (1972). Hagstromer and Binner (2009) also addressed a number of issues regarding the selection of the portfolio of financial instruments. In the same vein, Linton, O. (2016), is concerned with issues related to statistical-econometric modelling, and Okhrimenko and Manaenko (2014) analysed some aspects of the determinants of investment decisions on capital market.

Methodology, data, results and discussions

The determination of the boundaries for efficient asset portfolios can be performed using the VaR model, based on Markowitz. Also, Conditional Risk Value (CVaR) can be used to determine portfolio risk. The problem of investing asset portfolios on the capital market can only be done after the effects of market risk have been determined and analysed.

The advantages of CVaR over VaR, as a measure of risk, have led to the development of concepts that explore the use of CVaR in portfolio optimization. Thus, portfolios with maximum expected returns are characterized in case of different CVaR constraints. At the same time, the occurrence of CVaR constraints can be used to control risk when there is a benchmark asset.

We find that variance and CVaR quantify risk from different perspectives. Variance measures the sharing around the expected value of a random variable, while CVaR measures the expected loss for the most unfavourable possible cases, depending on the established confidence level. Medium-variance models and average-CVaR models could lead to different solutions. A portfolio obtained as a solution in the medium-variance model may be considered unacceptable by a regulator, as it may have an excessively high CVaR, which leads to very high losses in unfavourable scenarios. At the same time, traditional fund managers may consider a portfolio obtained with the medium-CVaR model unacceptable, as it could have an excessively high variance and thus an excessively low Sharpe index.

The study aims to analyse the allocation of assets in assets, as well as to evaluate the implications generated by the imposition of several types of constraints in the issue of portfolio selection. The optimal allocation of wealth is calculated by imposing VaR or CVaR constraints. By integrating a VaR or CVaR constraint into the optimal portfolio building policy, investors can directly optimize risk reduction through diversification. In practice, the concept is often imposed when a historical simulation is used to estimate VaR and CVaR.

We can consider that the addition of a VaR constraint for this model is motivated by the fact that the portfolio management industry uses it more and more often to set certain risk limits. A CVaR constraint in the case of the medium-variance model is motivated by the fact that there are a number of advantages of using CVaR instead of VaR to control risk.

In the practical activity it is appreciated that the medium-variance model has been widely used in the banking field, while VaR is used for the calculation of the minimum capital requirements associated with their exposure to market risk. The regulation of bank capital based on CVaR is, under certain conditions, more efficient than in the case of the VaR model.

The CVaR constraint boundary consists of portfolios that, given the CVaR constraint, minimize variance for a certain expected level of return. When the constraint is reached, the portfolios on this border can be built using mutual funds (K + 3), where K is the number of states for which the portfolios suffer losses equal to VaR. In the case of a VaR constraint, this result simplifies the problem of portfolio selection, which implies that the portfolios on the border with CVaR constraints are inefficient medium-variance.

The main views on the boundary with VaR constraints are as follows: expected return, standard deviation, VaR and CVaR for the portfolio of an agent with VaR constraints are lower than those of an optimal portfolio without restrictions and the distance of an optimal portfolio with VaR constraints compared to the border of unconstrained portfolios is higher for VaR constraints and low for low risk aversion agents.

The result suggests that a CVaR constraint is more effective than a VaR constraint in the process of reducing losses from the mean-variance model. If the VaR for the optimal portfolio with CVaR constraints is close to the value of the optimal portfolio with VaR constraints, the CVaR in the first case is lower than the CVaR in the second situation.

Analysing the implications of imposing a maximum intermediate loss constraint on a portfolio selection issue and comparing, it with those arising from the imposition of VaR or CVaR constraints, contradictory solutions.

Drawdown (MD) constraint is a restriction on the set of portfolios that are available for selection and specifies: the time period for calculating the MD and the MD threshold.

The interest in portfolio optimization is obvious, finding that the medium-variance model may underestimate the risk induced by extreme events. If a portfolio manager has private information and is compensated based on performance against a reference asset, then he will select a portfolio that is not optimal for another investor.

There is a link between VaR, CVaR and the mean-variance model, the theoretical results on the impact of an MD constraint on the mean-variance models. The main theoretical conclusion is to characterize the optimal portfolios in these models when an MD constraint is present. Many authors present practical examples of the implications for the selection of efficient portfolios.

The set of portfolios that minimize variance for different levels of expected return is referred to as the medium-variance boundary, while the set of those portfolios that act similarly due to constraints that are referred to as the medium-variance boundary portfolios with constraints. Optimal portfolios are medium-variance inefficient when a constraint is imposed. The conclusions of this analysis are that practitioners should be aware that such constraints can lead to the selection of inefficient portfolios. Institutional investors and portfolio managers, who are considering the design of a constrained portfolio management model, must consider that the profitability-risk trade-off is very different.

Before developing the optimization problem, it is necessary to analyse a series of theoretical concepts regarding VaR and CVaR. Thus, we will introduce the concept of coherent risk measures, defining a complete set of axioms, which must be fulfilled by a risk measure, in a general sense.

A measure of risk ρ is called coherent if four axioms are met:

- Axiom of translation invariance for a measure of risk. Thus, for all random losses X and constants α, ρ (X + a) = ρ (X) + α.
- Axiom of subadditivity for a measure of risk. Thus, for all random losses X and Y, ρ (X + Y) $\leq$ ρ (X) + ρ (Y).
- Axiom of positive homogeneity for a measure of risk. Thus, for all $\lambda \geq 0$ and random losses X, ρ (λ X) = λ ρ (X).

- Axiom of monotony for a measure of risk. In case of X $\leq$ Y for each scenario, then ρ (X) $\leq \rho$ (Y).

These details are important because they define the statistical properties that an appropriate risk measure must meet. The risk management process has its own scientific rules defined through this new deductive framework. The theory of coherent risk measures is based on the idea that an appropriate risk measure is consistent with finance theory and portfolio theory. The consequence of consistency in the case of a portfolio optimization problem is important because it retains the convexity characteristic.

Considering a certain quantum, in other words for a certain level of confidence α ∈ (0,1) and a random variable X, the corresponding VaR level α is defined by the relationship:

$$VaR_\alpha(X) = -q_\alpha^+(X) = -q_{1-\alpha}^-(X) = inf\{\beta | F_X(\beta) \geq \alpha\} \qquad (1)$$

Please note that z =*f*(x,y) represents the cost function, x is the decision vector, x∈X⊂R^n and y is a random variable defined on a probability space, representing the uncertainties that can affect costs. Probability distribution of y in R^m is considered to have a probabilistic density denoted by p(y). Knowing the probability distribution of y, z is a random variable. In this situation, the distribution of z depends on the decision vector. For everyone x, $F_X(\beta)$ defined on R, is the distribution function for z. When the confidence level is given, the probability that *f*(x, y), not to exceed a certain threshold β is given by:

$$F_X(\beta) = \int_{f(x,y)\leq\beta} p(y)dy \qquad (2)$$

Statistics $VaR_\alpha(X) = -q_\alpha^+(X)$ presents the minimum losses of the constituted portfolio, which can appear in the most unfavourable of cases, for a certain period of time. Under these conditions, VaR is equal to the percentile α of the VaR loss distribution, which is the lowest value so that the probability of losses exceeding or equalling this value is greater than or equal to α.

We know that VaR is based on probabilities, so it cannot be established with certainty, but rather a level of confidence, which is selected in advance by the user. Being a risk measure VaR satisfies the properties of a risk measure, but fails to respect the sub-additivity property and is not a coherent risk measure. Usually portfolio diversification always leads to risk reduction. VaR contrasts with portfolio diversification. VaR is not considered to be a risk measure, as a risk measure cannot violate the axiom of sub-additivity.

In addition, VaR is not a convex risk measure, which is due to the fact that sub-additivity and positive homogeneity together imply the convexity of a function, and VaR does not satisfy the sub-additivity property. In the case of an optimization problem, VaR may have several local minima, which means that VaR is not convex. We note that in the process of minimizing risk, only convex surfaces have the property that the local minimum, which leads to optimal solutions globally. Therefore, VaR is one of the most used tools for risk management, although VaR is difficult to optimize when calculating based on scenarios.

We know that VaR is the minimum loss that can occur in the most unfavourable cases for the portfolio in a certain period of time. At the same time, the CVaR represents the expected loss, which can occur in the most unfavourable cases, measuring how much can be lost, on average, if the losses exceed the VaR. This is a measure to quantify losses lower than VaR. Thus, the CVaR can be defined to be the conditional average of the loss in relation to x, if the loss is equal to or greater than q_α (x), according to the relationship:

$$CVaR = \phi_\alpha(x) = E\{f(x,y): f(x,y) \geq q_\alpha(x)\} = \frac{1}{1-\alpha}\int_{f(x,y)\geq q_\alpha(x)} f(x,y)p(y)dy \quad (3)$$

In the presented equation another distribution function is used which is not decreasing and continuous, and is obtained by resizing the distribution function z = f (x, y) on the interval [α,1]. We specify that CVaR is a coherent risk measure, because it satisfies all four axioms presented above.

VaR and CVaR for a loss function z = f (x, y) can be calculated by solving a convex, one-dimensional optimization problem for a certain confidence level, α. The main approach is to use a special convex function F_α (x,β) to characterize ϕ_α(x) şi q_α(x). The characteristic function for ϕ_α(x) and q_α(x) is defined as follows:

$$F_\alpha(x,\beta) = \beta + (1-\alpha)^{-1}E\{[f(x,y)-\beta]^+\} \quad (4)$$

where $[t]^+ = \max(0,t)$

If minimized F_α (x, β) for all (x, β) ∈ X x R, we obtain the equivalent result of minimizing the value CVaR ϕ_α (x) according to x ∈ X:

$$\min \phi_\alpha(x) = \min F_\alpha(x,\beta) \quad (5)$$

The study starts from the presentation of the optimization problem that focuses on capturing the risk through CVaR. Thus, the optimization problem has the following form:

$$\min_x CVaR_\alpha(x) \quad , x \in S, \quad (6)$$

where α represents the desired confidence level, and

$$S = \{(x_1, x_2, \dots, x_m) | x_i \geq 0 (i = 1,2,\dots,m), x_1 + x_2 + \cdots + x_m = 1\}$$

In this case, x_i is the decision variable for the portfolio share of sub-portfolio i. The optimization problem given by the above relation can be converted into a linear programming problem, as follows:

$$\min_{\beta,x} \beta + (1-\alpha)^{-1} \int_{y\in R^*} [f(x,y)-\beta]^+ p(y)dy, \qquad (7)$$

with $x \in S$

The common probability distribution of returns $p(y)$ is unknown, which makes problem (7) more difficult to solve. Therefore, most of the time, the decision-making process is influenced by parameters with a high degree of uncertainty, and its implementation can be influenced by certain errors.

Conclusions

The aspects contained in the article carried out on the basis of a careful study on the placement of investments in securities portfolios lead to some especially practical conclusions. In general, investing in portfolios in the market is risky. We take this market risk because it is under the influence of certain factors that act differently from one period of time to another. In this sense, in the literature is recommended as a measure of risk the risk value conditioned, because these risks appear under the influence of certain factors.

A conclusion that emerges is that the placement of portfolios of securities on the market will always be accompanied by a market risk and hence the need to assess this market risk in order to anticipate how the investment will be completed, as concrete results, securities portfolios in that market.

There are certain constraints in the placement of these portfolios and therefore the article concludes that the study must be deepened by considering these boundaries, so that we can more accurately anticipate the market risk posed by the placement of securities portfolios.

The placement of asset portfolios (securities) is one that must take into account the market, because if we do not anticipate the evolution of conditions that the market implies we can have a much higher risk and consequently a much lower return.

References

1. Anghelache C., Anghel M.G., Iacob Ș.V. (2020). Statistical-econometric methods and models used in the analysis of the capital market under the risk of inflation, Economic Computation and Economic Cybernetics Studies and Research, volume 54, issue 2/2020, pp. 41-58
2. Anghelache C., Dumitru M., Grigorescu D.L. (2020). Mathematical model used in substantiating optimal contract, Theoretical and Applied Economics, Volume XXVII (2020), No. 2(623), Summer, pp. 5- 18
3. Anghelache C., Anghel M.G., Iacob Ș.V., Pârțachi I. (2020). The study of the effect of the operational risk in the context of moral hazard. Romanian Statistical Review, Supplement, no. 8, pp. 25-45
4. Black, F. (1972), Capital Market Equilibrium with Restricted Borrowing, The Journal of Business, 45.3, 444-55
5. Hagstromer, B., Binner, J.M. (2009). Stock portfolio selection with full-scale optimization and differential evolution. Applied Financial Economics, 1559–1571
6. Linton, O. (2016), Probability, Statistic and Econometrics, Academic Press
7. Okhrimenko, O., Manaenko, I. (2014). Determinants Of Investment Support: Principles, Mechanisms, Efficiency. Economics of Development, 69 (1), 34-40s