THE CONCEPT OF SECURITIES IN BUILDING PORTFOLIOS

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Abstract

Securities are different and can be included in a portfolio, especially if the term of the guarantee is given a broader meaning, namely a meaning that covers all the possibilities of risks.

The securities are in principle divisible within the specified limits, meaning that any desired amount can be invested in the capital market. As a rule, the real rate of return is not affected by the amount invested, but especially by the structure of the securities considered. The effective rate of return on a portfolio is a weighted average of the price ratio relative to the rates of return on the component securities, using the proportions of the portfolios invested at that time.

One of the major attributes of portfolio theory is the emphasis on the uncertainties that exist and must be anticipated, based on which the relationships between the rates of return of securities are established, which can be established in the form of correlation coefficients, coefficients of determination or covariance. In practice, the problem is to estimate the extent to which the rate of return of each security is linked to the other securities that together make up the portfolio.

The relationship between the rates of return of the two securities is most correctly expressed by means of a correlation coefficient. This correlation coefficient is calculated on the basis of regressions, which give significance and on the basis of which the parameters that can be used in estimating the final results are calculated. In practice we discuss the divergence of the real yield of a security, compared to the value expected to be expressed in standard units. The correlation between the variables we consider is not always causal, it only indicates the extent to which the two securities are correlated. It is sometimes possible to obtain a correlation coefficient from a common probability distribution, which must then be made explicit, and it is preferable to directly estimate the link between the two rates of return of two securities, without specifying the distribution.

The covariation between the rates of return of the two securities is the weighted average of the product of the non-normalized deviations. We say this because the distribution is not always normal.

The real return of a portfolio is the weighted average of the real return of the component securities, established on the basis of the use of the proportions invested in the form of weights. Therefore, the standard deviation of the return rate of a portfolio also depends on the standard deviations of the return for the component securities, expressed by the correlation coefficients and the proportions in which the securities are invested.

Keywords: securities, rate of return, risks, investors, capital market, coefficients, regressions.

JEL classification: C10, G14.

Introduction

The concert of securities used in the construction of portfolios is essential in the sense that it must be based on guarantees seen in a broader sense. In this article, we have set out a series of issues regarding the investment of these securities, assuming that the investor selects a portfolio that includes one or more securities (N).

Considering that the securities are divisible within specific limits, it follows that the real rate of return is not affected by the amount invested, but in particular by the securities considered in the constitution of these portfolios.

We have discussed in this article a series of variants that we have processed from an econometric point of view, resulting in a series of data that give meaning to the appreciations expressed by the authors. Thus, we have shown that if the real rate of return for each security could be accurately predicted, the real rate of return for each portfolio could be predicted, but this is a theoretical element because in practice it turns out that the problem of making predictions about portfolios must first take into account the mean square deviation and the average calculated for each of them.

Through the examples used, we sought to express the fact that it is possible that a yield considered to be associated with a high yield on the other section may not be correlated, and the profitability of securities may sometimes be affected by this mismatch. The correlation is not always causal and therefore from the examples we used in the study, it results that this can be a linear relationship between the profits of two securities or be non-linear if the variability of these securities is different. Also, using a series of econometric criteria and concepts, we expressed that the value of the correlation coefficient always indicates the ratio between the uncertainty attributed to the relationship between the two securities and the total uncertainty associated with only one of them. The studies and data used accurately highlight this very aspect.

Next, after approaching the correlation coefficient, which measures the relationship between the rate of return of the two securities, we addressed some issues regarding the covariance between the rates of return of the two securities. In this regard, we came to the conclusion that the weighted average of the product of non-normalized deviations is the basis for assessing the covariance that exists between the two securities. Covariance is important for the analysis of the product of the correlation coefficient and the standard deviations of the rates of return of the two securities.

The actual return of a portfolio is the weighted average of the actual return on the component securities and this is determined using the proportions invested as weights. The established mathematical relations give essence to this aspect.

From the studies conducted, based on the examples we used, it results that it is not a trivial matter to calculate the standard deviation of profitability for a portfolio, when it contains several securities. To find the effective set of portfolios through an enumeration and elimination process it is necessary to calculate the standard return deviations for a large number of portfolios.

The article is accompanied by a series of simple examples that can highlight the complex aspect of the use of securities in building portfolios.

Literature review

Anghelache C, Anghel M.G., Marinescu A.I., Popovici M. (2019) address some issues related to the allocation of financial resources on the capital market, thus seeking to address some issues raised by forecasts regarding the low frequency evolution of portfolios. In other words, Anghel MG, Anghelache C., Radu I. (2020) study in their paper the interconnections between profitability and volatility, these being important both for quantifying market risk and for evaluating options, in which case they are not formulas for evaluation in GARCH processes and consequently, it is necessary to use simulation methods for the evaluation of financial products. Also, Anghel MG, Petre A., Olteanu C. (2019) are concerned with modeling in the case of portfolios starting from the definition of short-term interest rate processes, and will approximate the variables that enter the model, in continuous time conditions and discreet time. In another line of ideas, Black, F. (1972) approaches in his paper some models of setting the prices of assets on the capital markets in conditions of market equilibrium. Ferreira, M.A., Santa-Clara, P. (2011) addressed issues related to the forecast of returns on portfolios invested in the capital market. Iacob Ș.V., Dumitru D., Popovici M. (2020) address in their paper some aspects regarding the choice of the portfolio and the testing of the model regarding the price of capital assets, starting from the idea that the market is the one that determines the evolution of market prices. Welch, I., Goyal, A. (2008) focused on issues related to the capital premium forecast.

Methodology, data, results and discussions

A lot of different securities can be included in a portfolio, especially if the warranty term is given the broadest meaning. In practice it is considered only a subset of possibilities. If the information and calculation were free, all possibilities should be considered.

We assume that the investor selects a portfolio that includes one or more N securities. The number considered may be small (e.g., N = 10) or large (e.g., N = 10,000), depending on the advantages and disadvantages of a limited selection over a more complete selection. We also consider the securities to be divisible. Within the specified limits, any desired amount may be invested in each security. At the same time, the real rate of return is not affected by the amount invested.

A portfolio can be described according to the proportion invested in each security, according to the data structured in the table below.

Proportion invested in each security

Table 1

Securities	Proportion invested
1	0,10
2	0,50
3	**0,00**
4	0,30
5	**0,00**
6	0,10
	1,00

The proportion invested in security 1 is denoted X_1, the proportion invested in 2 is X_2 and so on. Since the whole is equal to the sum of its parts, the probabilistic proportions must add up to 1, namely:

$$\sum_{i=1}^{N} X_i = 1 \qquad (1)$$

If $X_i = 0$, the portfolio does not offer any guarantee i. A negative value expresses the issuance of an insufficient guarantee. This may or may not be possible, depending on the investor's situation.

A value greater than 1 indicates holdings that require more than the funds provided by the investor. This is possible only if he can obtain additional money by issuing one or more probabilistic securities, each X_i is limited to the range from 0 to 1, inclusive.

A portfolio is a set of X_i values totalling 1. In order to be possible, other restrictions must be met (for example, all $X_i \geq 0$).

The effective rate of return of a portfolio is the weighted average price rate, relative to the rates of return of the component securities, using the proportions of the portfolios invested as shares. R_p allows indicating the effective rate of return of the portfolio and R_i the effective rate of return on the guarantee i. Determination R_p it is done using the relationship:

$$R_p = \sum_{i=1}^{N} X_i R_i \quad (2)$$

Table number 2 provides calculations for the portfolio described above. The real rate of return is 18%. The formula is used for all titles. If an asset is not included in the portfolio, then X_i is equal to zero, thus $X_i R_i$ is equal to zero, and the total is not affected.

Proportion invested and real rate of return

Table 2

Securities (i)	Proportion invested w	Real rate of return	XH 1 1
1	0,10	0,10	0,100
2	0,50	0,20	0,100
3	**0,00**	0,05	**0,000**
4	0,30	0,15	0,045
5	**0,00**	0,07	**0,000**
6	0,10	0,25	0,025
	1,00		0,180

Table number 3 shows the amounts in euros of investments and revenues, assuming a total investment of 100 euros. The overall yield is 18%.

Amount invested and return

Table 3

Securities (/)	investment	return
1	10,00	11,00
2	50,00	60,00
3	**0,00**	**0,00**
4	30,00	34,50
5	**0,00**	**0,00**
6	10,00	12,50
	100,00 euro	118,00 euro

If the actual rate of return for each security could be accurately predicted, the actual rate of return for each portfolio could be predicted. But neither the rate of return of the portfolio nor that of each of its component securities can be predicted with certainty. The problem is to make predictions about securities that can be used to make predictions about portfolios, especially E_p and σ_p from each portfolio.

An estimate of the forecasted or expected rate of return for each security is required. Such an estimate can be provided directly, as the best solution or estimate.

Compared to the expected rate of return, a certain amount of uncertainty is needed (probable divergence of the expected result). Formally, this is considered to be the standard deviation of such a distribution.

In conclusion, two measures are used to clarify the predictions for each of the N titles, namely:

E_i = expected rate of return;

σ_i - standard deviation (uncertainty) of the rate of return of the security *i*.

However, such numbers are assumed to indicate a subjective distribution of probabilities for the rate of return on the security.

One of the major attributes of portfolio theory is the emphasis on uncertainties that exist. The relationships between the rates of return on securities can be established in terms of correlation coefficients, coefficients of determination or covariances.

The problem is to estimate the extent to which the rate of return of each security is related to that of all the others. In practice, such estimates are often derived from a relatively simple model of securities relationships. But the theory provides a separate value for each pair of securities.

Figure number 1a shows an extreme case. Only pairs along the straight line are considered. Yields are correlated.

Figure number 1b shows a case with a higher probability. Any pair in the hatched area is considered possible. High values of R1 are associated with high values of R2, but the association is not exact and the yields are correlated, but not perfectly.

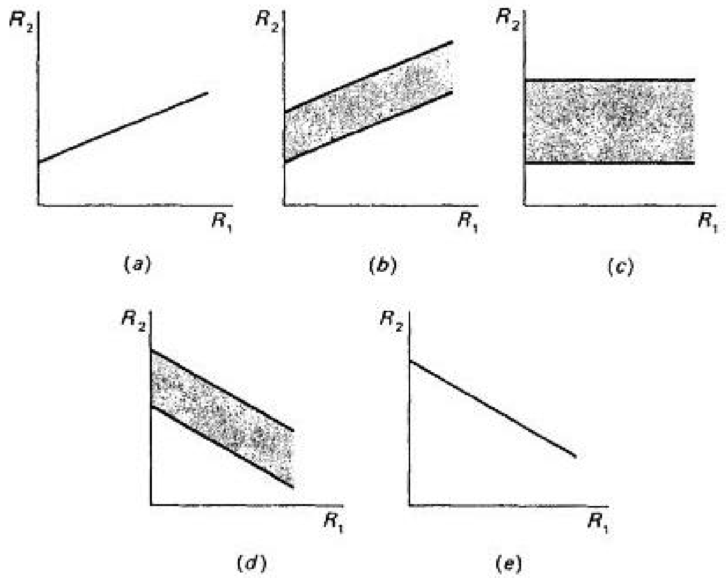
Figure number 1c represents a case in which the yields are uncorrelated.

In Figures 1a and 1b the correlation is positive, being likely that one yield is considered to be associated with a high yield on the other. Figures 1d and 1e show situations in which a high return on one security is probably associated with a low return on the other security.

The securities in figure number 1d are negatively correlated. Those in figure number 1e are perfectly negatively correlated.

Cases of yield correlation

Figure 1



The relationship between the rates of return of the two securities can be expressed by means of a correlation coefficient. A value of +1 indicates a perfect positive correlation (figure number 1a). A value of 0, shows that there is no correlation (figure number 1c). A value of -1 indicates a perfect negative correlation (figure number 1e). In a case such as the one shown in figure number 1c, the value of the correlation coefficient is between 0 and +1; In a case such as the one shown in Figure 1d, the correlation coefficient is between 0 and -1.

The numerical value of the correlation coefficient depends on the similarity of each pair of results. A set of predictions of such probabilities is called a common probability distribution. Table number 4 shows that there is a 0.07 probability that the return on securities will be between 3% and 4% and that the return on securities k will be between 2% and 3%.

Common probability distributions can be present in rough terms or in detail. The ranges can be wide (3% to 4%) or narrow (3.1% to 3.2%). Regardless of the interval, the midpoint is used for calculation purposes.

The common probability distribution contains all the information needed to calculate the expected value and the standard deviation of the rate of return of each security. The sums in the column, shown at the bottom of table number 4, provide the probability distribution for R_j. The line amounts, displayed on the left, ensure the probability distribution for R_k.

Common probability distribution

Table 4

	R_k	R_j: 0–1%	1%–2%	2%–3%	3%–4%	4%–5%
.15	4%–5%	.01	.02	.03	.05	.04
.23	3%–4%	.02	.03	.07	.06	.05
.24	2%–3%	.03	.06	.05	.07	.03
.22	1%–2%	.05	.05	.06	.03	.03
.16	0–1%	.03	.05	.03	.03	.02
		.14	.21	.24	.24	.17

The divergence of the real return of a security from its expected value can be expressed in units of standard deviation. For securities *j*, we use the relation:

$$d_j = \frac{R_j - E_j}{\sigma_j} \tag{3}$$

If R_j represents two standard deviations above the expected value, d_j= +2; if it is below two standard deviations d_j = -2; if it is equal to the expected value d_j = 0.

We will consider a pair of values for R_j and R_k. Each can be expressed as a normalized deviation from the expected value. The product provides a measure of the overall deviation, respectively:

$$d_j d_k = (\frac{R_j - E_j}{\sigma_j})(\frac{R_k - E_k}{\sigma_k}), \quad (4)$$

where: $d_j d_k$ represents normalized deviations from the expected value
R_j, R_k represents deviations from the expected value
E_j, E_k represents rates of return
σ_j, σ_k represents standard deviations of rates of return

The correlation coefficient is the weighted average of all securities, with the probability that each will be used as a weighting:

$$P_{jk} = \sum P_r(d_j, d_k)(d_j d_k) \quad (5)$$

where: P_{jk} represents the correlation coefficient
$P_r(d_j, d_k)$ represents the probability of the pair d_j, d_k. Alternative

$$P_{jk} = \sum P_r(R_j, R_k)(\frac{R_j - E_j}{\sigma_j})(\frac{R_k - E_k}{\sigma_k}) \quad (6)$$

where $P_r(R_j, R_k)$ represents the probability of the pair R_j, R_k.

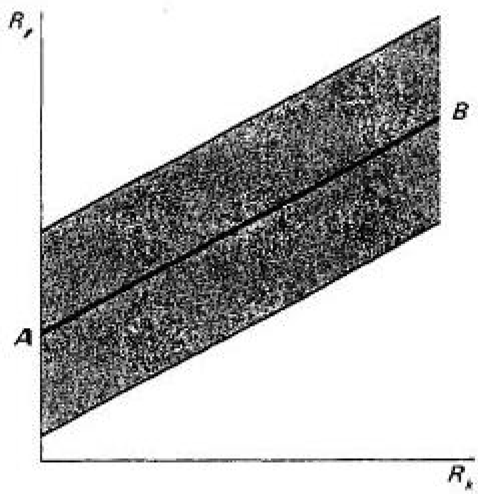
The correlation does not always imply causality. As the name suggests, this only indicates the extent to which the two securities are correlated.

It is possible to obtain a correlation coefficient from a common probability distribution. However, in many cases, it may be preferable to directly estimate the link between the rates of return of two securities without specifying such a distribution. If such an estimate is obtained, the correlation coefficient must be given an intuitive meaning.

In the case of a perfect correlation, there is an exact linear relationship between the profits on two securities. If the correlation is less than perfect, no such relationship is maintained exactly. But it is possible to build a good linear relationship. We assume that the line AB in figure number 2 relates best to R_j la R_k. Uncertainty about the real value of R_j is measured by σ_j. Some are due to the possibility of a result that is not on line AB. But if only values along the AB were possible, there would still be some uncertainty as to which R_j, because there are probably a number of other points along the line. If still R_k would have been known in advance, such uncertaintv would have been removed. This part of the uncertainty about R_j is noted $\sigma_{j \leftarrow k}$.

Correlation between two securities

Figure 2



The value of the correlation coefficient indicates the ratio between the uncertainty attributed to the relationship between two securities and the total uncertainty associated with one of them. In this case we can express the uncertainty through the relationship: $\frac{\sigma_{j\leftarrow k}}{\sigma_j}$

The coefficient of determination is the square of the correlation coefficient, respectively:

$$D_{jk} = \rho_{jk}^2 \text{ or } \rho_{jk} = \sqrt{D_{jk}} \quad (7)$$

It indicates the percentage of the total variation attributed to the relationship between the securities considered:

$$D_{jk} = \rho_{jk}^2 = \left(\frac{\sigma_{j\leftarrow k}}{\sigma_j}\right)^2 = \frac{\sigma_{j\leftarrow k}^2}{\sigma_j^2} \quad (8)$$

Suppose an analyst considers that approximately 60% of the change in the return on share X is attributable to its relationship with its shares and 40% is not. In this situation we get:

$$\rho_{jk}^2 = 0{,}60$$

$$\rho_{jk} = \sqrt{0{,}60} = \pm 0{,}77$$

If the yields are positively correlated we have $\rho_{jk} = +0{,}77$, and if they are negatively correlated we have $\rho_{jk} = -0{,}77$.

The correlation coefficient measures the relationship between the rates of return of two securities. The order in which they are indicated is irrelevant, respectively:

$$\rho_{jk} = \rho_{kj} \text{ and } \rho_{jk}^2 = \rho_{kj}^2 \quad (9)$$

If 60% of the change associated with the return on stock X is attributed to its relationship with that in stock Y, then 60% of the change associated with stock Y must be attributed to its relationship to stock of X.

The covariance between the rates of return of the two securities is the weighted average of the product of the non-normalized deviations, respectively:

$$C_{jk} = \sum P_r(R_j, R_k)(R_j - E_j)(R_k - E_k), \quad (10)$$

where: C_{jk} represents covariance

$P_r(R_j, R_k)$ represents the probability of the pair R_j, R_k

The important covariance for the following analysis is equal to the product of the correlation coefficient and the standard deviations of the rates of return on securities, respectively:

$$C_{jk} = \rho_{jk}\rho_j\rho_k \quad (11)$$

where: C_{jk} represents covariance

ρ_{jk} represents the correlation coefficient

ρ_j, ρ_k represents standard deviations of rates of return

The expected return of a portfolio is the weighted average of the expected returns on its component securities, using the proportions invested as weights:

$$E_p = \sum_{i=1}^{N} X_i E_i \quad (12)$$

where: E_p represents the expected return of a portfolio

E_i represents the expected returns

X_i represents securities

All securities X can be included because $X_i = 0$ if asset i is not included in the portfolio.

The real return of a portfolio is the weighted average of the real return of the component securities, using the invested proportions, as weights. The formula for E_p indicates that a comparable relationship is valid for expected (forecast) returns.

The standard deviation of the return rate of a portfolio depends on the standard deviations of the return for its component securities, the correlation coefficients and the proportions invested:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j=1}^{N} X_i X_j \rho_{ij} \sigma_i \sigma_j \quad (13)$$

where: σ_p^2 represents the standard deviation of the rate of return

X_i, X_j represents securities

ρ_{ij} represents the correlation coefficient

$\sigma_i \sigma_j$ represents standard deviations of rates of return

The formula is valid for all securities because $X_i = 0$ if i guarantee is not included in the portfolio.

This prohibition relationship is not complicated. Double summation indicates that the numbers N^2 must be added together. Each of the numbers is obtained by substituting one of the possible value pairs for i and j. For N = 2, it results:

$$\sigma_p^2 = X_1 X_1 \rho_{11} \sigma_1 \sigma_1 + X_1 X_2 \rho_{12} \sigma_1 \sigma_2 + X_2 X_1 \rho_{21} \sigma_2 \sigma_1 + X_2 X_2 \rho_{22} \sigma_2 \sigma_2 \quad (14)$$

The first and last terms can be simplified. Clearly, profitabilitv can be perfectly (positively) correlated with itself. So $\rho_{11} = 1$, the same as ρ_{22}. The second and fourth terms can be combined because $\rho_{21} = \rho_{12}$. The result is:

$$\sigma_p^2 = X_1^2 \sigma_1^2 + X_2^2 \sigma_2^2 + 2 X_1 X_2 \rho_{12} \sigma_1 \sigma_2 \quad (15)$$

Multiplication $(\rho_{ij} \sigma_i \sigma_j)$ is C_{ij}, that is, the covariance between i and j. The general formula can be written in the form:

$$\sigma_p^2 = \sum_{i=1}^{N} \sum_{j-1}^{N} X_i X_j C_{ij}, \quad (16)$$

where: σ_p^2 represents the standard deviation of the rate of return

$X_{i,} X_j$ represents securities

C_{ij} represents the covariance between i and j

Table number 5 shows the calculations for an example involving three securities. In each part the numerical values are displayed on the right and the variables on the left.

Table number 5a shows the correlation coefficients and standard deviations for the securities. We notice that it exists σ_1 along the diagonal in the table of correlation coefficients. Also, each entry in the lower left triangle is equal to the corresponding entry in the upper right triangle. Of course, this must be true for any table of correlation coefficients.

Each entry in the table is the product of the correlation coefficient, the standard deviation next to its turn and the standard deviation above its column.

Table 5 also shows the proportion invested in each of the three securities. Fifty percent of the portfolio is invested in asset 1, 30% in asset 2 and 20% in asset 3.

Covariances and proportions invested for three securities

Table 5

	σ_1	σ_2	σ_3
σ_1	$\rho_{1,1}$	$\rho_{1,2}$	$\rho_{1,3}$
σ_2	$\rho_{2,1}$	$\rho_{2,2}$	$\rho_{2,3}$
σ_3	$\rho_{3,1}$	$\rho_{3,2}$	$\rho_{3,3}$

	5	15	10
5	1	.5	.6
15	.5	1	.7
10	.6	.7	1

	X_1	X_2	X_3
X_1	$C_{1,1}$	$C_{1,2}$	$C_{1,3}$
X_2	$C_{2,1}$	$C_{2,2}$	$C_{2,3}$
X_3	$C_{3,1}$	$C_{3,2}$	$C_{3,3}$

	.5	.3	.2
.5	25	37.5	30
.3	37.5	225	105
.2	30	105	100

As this example shows, it is not a trivial matter to calculate the standard deviation of profitability for a portfolio, especially one containing many securities. To find the effective set of portfolios through an enumeration and elimination process it would be necessary to calculate the standard return deviations for a large number of portfolios.

Conclusions

In the activity of the capital market, the securities are considered to be indivisible. Within the specified limits any desired amount can be invested in each of the portfolios, and the real rate of return is not only affected by the amount invested. Thus, a portfolio is also defined according to the proportion invested in each security. The effective rate of return on a portfolio is determined by the weighted average of the price rate relative to the rates of return on the securities, using as proportions the proportions of the portfolios invested.

If the actual rate of return for each security could be accurately predicted, then the actual rate of return could be forecast for each portfolio. Given that the construction of alternative portfolios, are often discussed, the one that gives the most certain results in terms of estimation, forecast, final profitability will be chosen.

One of the important attributes of portfolio theory is to consider the uncertainties that may exist. In this regard, we conclude that the relationships between the rates of return on securities can be established in terms of correlation coefficients, coefficients of determination or covariance. In this context, the question that arises is to estimate the extent to which the rate of return of each security is related to the rate of all other securities that form part of the portfolio.

In practical activity such estimates derive from a relatively simple model of the relationships between securities, using an econometric model that leads to accurate results and based on the respective parameters can be made certain estimates.

Another conclusion is that the numerical value of the correlation coefficient depends on the similarity of each pair of results. From this point of view we can appreciate that one of the major attributes of the portfolio theory is the emphasis on the uncertainty that exists. As a rule, the relationships between the rates of return on securities can be established in terms of coefficients of determination and covariation. The correlation does not always imply causality, as the name suggests, it only indicates to the extent that the two securities considered are correlated. In the case of a perfect correlation there is an exact linear relationship between the profits on the two securities, which can be demonstrated using an appropriate econometric model.

As it results from the way the study was presented by the authors, the issue of securities in building the portfolio is of utmost importance, because by calculating the standard deviation of the profitability of a portfolio we can highlight individually which securities and securities to take into account. consideration.

The above analysis can also be expressed by using other econometric models to extend these aspects of securities.

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