STUDY ON REDUCING THE ABSENCE OF RISK AND PRUDENCE

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Abstract

In this paper the researchers set out to analyze the dependencies of risk aversion, finding after the analyzes that this is determined by the fact that the marginal utility of a person decreases with wealth. It is also interesting to address another issue related to wealth growth. On the other hand, the authors are interested in determining how the risk premium for a certain zero area risk is affected by a change in the initial wealth.
Of the insights from what the literature offers at the moment, we note Arrow's contributions, which argued that intuition implies that wealthier people are generally less willing to pay for eliminating fixed risk.
By risk, we seek to define the notions, to establish the forms of manifestation and especially to try to unravel the perspective and the effects that these risks will have. Of course we can discuss at this time also the vulnerability of the economic systems, although through careful forecasts, by including all the factors of influence, we can make a series of prefigurations of the perspective of the macroeconomic evolution.

Keywords: utility function, risk, capital, variation, average, density, distribution

JEL Classification: E47, G24

Introduction

Approaching the issue from the point of view of how to cover the risks we can say that there are insurable and uninsurable risks, in the sense that there are risks that can be accurately predicted and for eliminating their effects on the economic evolution it is advisable to set up provisions, hedge funds, so that the diminished risks do not have the effects that would be obtained without the possibility of coverage.

It is particularly important to conceptualise the indicators for measuring and analyzing risks, in that they create the possibility for the researcher to find and take the necessary measures to know and seek to influence, if not eliminate, at least to mitigate the effects of risks in the perspective of development.

The expected utility theory, which has many supporters and many detractors, will be addressed in the paper. Thus, we will analyze some generalizations of the expected utility criteria, which satisfy those people who consider the expected utility too restrictive. Researchers in economics and finance have long considered the theory of expected utility, as an acceptable paradigm for decision making under uncertain conditions.

Literature review

Anghel, M.G., Diaconu, A. (2016), they study the equilibrium and self-regressive models they use in economic forecasts. Anghelache C., Niță G., Badiu A. (2016), It addresses some aspects of decision-making at risk, assuming that the risks will appear and develop even when the elements of their occurrence can be predicted, but it is important that these risks be foreseen in order to take measures to diminishing effects. The attitude towards risk and uncertainty has been largely addressed by Kahneman, D (2010) şi Tversky, A. (2000), pointing out that this is a psychological side of human behavior, but that in the future it has consequences if we do not try to put in the used model and the notion of risk that will manifest itself anyway. Marcowitz, H (2010, 2014) şi Tobin, J. (1987), have addressed in the studies carried out the concept of portfolio efficiency and the concept of influence or effect of risks on a future evolution.

Research methodology, data, results and discussions

We will consider a situation in which 100 units with equal probability will be gained or lost, then this situation will have a dangerous potential for an agent with initial wealth w = 101, while it is essentially trivial for an agent with wealth w = 1000 000. The former should be ready to pay more than the latter to eliminate the risk.

We can only verify this aspect if the absolute risk av, the property has the square-root utility function, with $\Pi = 43.4$ when w = 101 and $\Pi = 0.0025$ when w = 1 000 000. If the wealth is measured in euros, the individual would be willing to pay over 43 euros to avoid the risk when the wealth is w = 101, while the same person would not even pay a hundred euros to get rid of that risk when the wealth is one million euros. In the following, we characterize the set of utility functions that have this property.

The risk premium $\Pi = \pi(w)$ according to the initial wealth w can be evaluated by solving:

$$Eu(w + z) = u(w - \pi(w)) \tag{1}$$

For all w. It is fully differentiated (1) in terms of its yield:
$Eu'(w+z) = (1-\pi'(w))u'(w-\pi)$

Or equivalently:

$$\pi'(w) = \frac{u'(w-\pi) - Eu'(w+z)}{u'(w-\pi)} \qquad (2)$$

Thus the risk premium decreases the capital if and only if:

$$Ev(w+z) \leq v(w-(w-\pi)) \qquad (3)$$

Where the function v ≡ -u ,is defined as minus the derivative of the function u. Since the function v is increasing, we can interpret it as another utility function. Condition (3) then states that the risk premium for agent v is higher than the risk premium Π for agent u.

This assumption is true if and only if u is more concave than u in the sense of Arrow-Pratt, that is, if u is a concave transformation of u. For this utility v, the absolute risk aversion measure is Av = A-u ,= -u' ,' / u ,'. This measure has several uses.

Without elaborating on the terminology function at this stage, we will define:

P (w) = $-u'''(w)/u''(w)$, as the absolute prudence of the utility agent u.

It follows from (3) that -u' is more concave than u and only if: P (w) ≥ A (w) for all w. Thus, the condition P ≥ A is uniformly necessary and sufficient to guarantee an increase in wealth that reduces the risk premia.

Using: A ,(w) = A (w) [A (w) - P (w)], the condition P ≥ A is equivalent to the condition A' ≤ 0. Thus, the risk premiums associated with each risk z decrease the efficiency, if and only if, the absolute aversion to risk decreases; or, if and only if, prudence is greater than absolute aversion to risk.

We observe that the function u (w) = √w satisfies this condition.

Indeed, we have Au (w) = ½w-1, which is decreasing. This can be verified alternatively by observing that v (w) = - 1/2 w-1/2 and Av (w) = Pu (w) = 1.5w-1, which is uniformly larger than Au (w)

We noticed that the decrease in the absolute risk aversion (APRA) requires that the third derivative of the utility function be positive. Otherwise, prudence would be negative, which would imply that P <A: a condition that implies that absolute risk aversion would increase in wealth.

Thus, absolute risk aversion (APRA), a very intense condition, requires the necessary (but not sufficient) condition that u'' be positive, or marginal utility be convex.

• **Aversion to relative risk**

The absolute risk aversion is the degradation rate for the marginal utility. Specifically, absolute risk aversion measures the rate at which marginal utility decreases when capital (wealth / wealth) increases by one euro.

In general, the growth rate for a function f (x) is defined as:

$$\frac{df(x)}{dx} - \frac{1}{f(x)}$$

As the marginal utility u' (x) decreases in wealth, the growth rate is negative. The absolute value of this negative growth rate, which is the measure of absolute risk aversion, is called the decay rate.

If the currency unit were the dollar, the absolute risk aversion would be a different number. In other words, absolute risk aversion is not a free unit, as it is measured in euros (per dollar, or per yen, per pound, etc.).

For this purpose, we define the risk aversion index R as the rate at which the marginal utility decreases when the wealth increases by a percentage. As far as standard economic theory is concerned, this measure is simply the elastic richness of marginal utility. The risk aversion index R can be calculated as:

$$R(w) = \frac{\frac{du'(w)}{u'(w)}}{\frac{dw}{w}} = \frac{-wu''(w)}{u'(w)} = wA(w) \qquad (4)$$

The measure of risk aversion is simply the product of wealth and absolute risk aversion.

The absolute risk premium and the absolute risk aversion index are related to the Arrow-Pratt approximation. We can develop analogous types of outcomes for relative risk aversion. Suppose that your initial wealth w is invested in a portfolio whose Z yield over the period is uncertain.

Suppose that Ez = 0. We will follow to see which part of the initial wealth must be paid in order to escape this proportionate risk. The solution to this problem is referred to as the relative risk premium. This measure is also a measure without units, as opposed to the absolute risk premium, which is measured in euro. It is defined by default by the following equation:

$$Eu(w(1 + z)) = u(w(1 - \Pi)) \qquad (5)$$

Obviously, the relative risk premium and the absolute risk premium are equal if we normalize the initial wealth towards unity. In general, the relative risk premium for the proportional risk z is equal to the absolute risk premium for the absolute risk wz, divided by the initial wealth w: Π (z) = Π (wz) / w.

From this observation, we obtain that, if agent u is more risk-averse than the agent with the same initial wealth, agent u will be ready to pay a greater share of his wealth than agent u to insure against a given proportionate risk z. Also, if a σ^2 denotes the variance z, then the variance of wz is equal to $w^2\sigma^2$. Using the Arrow-Pratt approximation it follows that:

$$\Pi(z) = \frac{\Pi(wz)}{w} \cong \frac{-\frac{1}{2} w2\sigma 2A(w)}{w} = \frac{1}{2}\sigma^2 R(w) \qquad (6)$$

The relative risk premium is equal to half the variance of the proportional risk or the index of aversion to the relative risk. This can be used to establish a range for acceptable degrees of risk aversion. Suppose a person's wealth is at risk of 20% gain or loss with equal probability, then we will look to see what is the range that he would find reasonable for the share of wealth Π, for someone would be ready to pay it to get rid of this risk.

Therefore, I found that most people would be willing to pay between 2% and 8% of their wealth. Since the risk z in this experiment has a variance of 0.5 (0.2) 2 + 0.5 (-0.2) 2 = 0.04, using approximation (7) we obtain a range for the risk aversion relative to between 1 and 4.

There is no definitive argument for or against reducing risk aversion. Arrow initially assumed that aversion to relative risk is likely to be constant or likely to increase, although he stated that intuition was not as clear as intuition for lowering absolute aversion to risk. Since then, numerous empirical studies have yielded contradictory results.

There are two contradictory effects that need to be considered. On the one hand, under the intuitive assumption of absolute risk aversion (APRA), becoming richer also means becoming less risk-averse. This effect tends to reduce Π. But on the other hand, getting richer also means having an absolute higher risk wz. This effect tends to rise Π. There is no clear intuition about whether the first or second effects will prevail. For example, many of the classical models in macroeconomics are based on a relatively constant risk aversion on all levels of wealth, which implies that the two effects are mutually exclusive.

Of course, there is no a priori reason to believe that the dominant effect will not change on different wealth levels. For example, some recent empirical evidence points to a possible „U-shape" for risk aversion, lowering R to lower wealth levels, then adjusting somewhat before rising to higher wealth levels.

- **Classic utility functions**

The expected utility theory of the UA has a long and prominent place in the decision-making process under uncertain conditions. Even detractors of theory use UA as a standard for comparing alternative theories. In addition, many of the models in which the AU theory has been applied can be modified, often producing better results.

While the current trend is the generalization of the UA model, researchers often limit the expected utility criterion by considering a certain subset of the utility functions. This is done to obtain tractable solutions for many problems. It is important to look at the implications that derive from choosing a certain utility function. Some findings from the literature may be robust enough to apply for all preferences to avoid risks, while others may be limited to applying a narrow class of preferences only.

We will first continue to present particular types of utility functions that are often encountered in the economic and financial literature. The utility is unique only up to a linear transformation.

Historically, much of finance theory was developed in the 1960s, considering the subset of utility functions that are squared of form:

$u(w) = aw - \frac{1}{2}w^2$, pentru $w \leq a$ (8)

We find that the richness domain that u is defined comes from the required requirement that u not decrease, which is true only if w is smaller than a. The set of functions is useful because the UA generated by the final richness distribution is a function only the first two moments of this distribution:

$Eu(w) = aEw - \frac{1}{2} Ew^2$ (9)

In this case, the expected utility theory (AU) simplifies the average approach to the variance of decision making under uncertain conditions, preferences between different situations should be determined only by the means and their variance.

Above the wealth level, the marginal utility becomes negative. As the quadratic utility decreases in wealth for w> a, many people may feel that this is not suitable as a utility function.

However, it is important to remember that we are trying to model human behavior with mathematical models. For example, if the quadratic utility function models the behavior quite well with a value of = 100 million euros, it is really a problem that this function decreases for high levels of wealth.

The idea is that the quadratic utility could work well for richer levels of wealth, and if it were, we shouldn't worry too much about its properties at unrealistically high levels of wealth. However, the quadratic utility function has another property that is more problematic. Namely, the quadratic utility functions have an absolute aversion to absolute risk:

$$A(w) = \frac{1}{a-w} \Longrightarrow A^{'}(w) = \frac{1}{(a-w)2} > 0 \tag{9}$$

For this reason, quadratic utility functions are no longer so commonly used.
A second set of classic utility functions is the set of so-called constant utility functions - constant risk aversion (FUCA), which are exponential functions characterized by:

$$u(w) = \frac{\exp(-aw)}{a} \tag{10}$$

where a is a positive scalar. The domain of these functions is the real line. The distinguishing feature of these utility functions is that they have an absolute aversion to absolute risk, with A (w) = a for all w. It can be shown that the Arrow-Pratt approximation is accurate when u is exponential and w; is normally distributed with mean μ and the variance <σ^2. Indeed, we can expect to see this:

$$\mathrm{Eu}(w) = \frac{-1}{\sigma a\sqrt{2\pi}} \int \exp(-aw)\exp\left(-\frac{(w-\mu)2}{2\sigma^2}\right)dw =$$

$$= \frac{1}{a}\exp(-a(\mu - \frac{1}{2}a\sigma^2))\left[\frac{1}{\sigma\sqrt{2\pi}}\int \exp(-\frac{\left(w-\left(\mu-\frac{1}{2}a\sigma^2\right)\right)2}{2\sigma^2})dw\right]$$

$$= \frac{1}{a}\exp\left(-a\left(\mu - \frac{1}{2}a\sigma^2\right)\right) = u\left(\mu - \frac{1}{2}a\sigma^2\right) \tag{11}$$

The third equality comes from the fact that the term bracketing is integral to the density of the normal distribution $N(\mu - \frac{1}{2}a\sigma^2, \sigma)$, which must be equal to unity. Thus, the risk premium is indeed equal to $\frac{1}{2}\sigma^2 A\ (w)$. In this very specific case, we get that the Arrow-Pratt approximation is accurate. The fact that risk aversion is constant is often useful in analyzing choices and more alternatives.

As we will see later, this hypothesis eliminates the effect of income when dealing with the decisions to be made regarding a risk whose size is invariable in the change of capital. However, this is often the main criticism of FUCA's utility, as absolute aversion to risk is rather constant than decreasing.

Finally, a set of preferences that was by far the most used in the literature, is the set of electric utility functions. Researchers in the field of finance and macroeconomics are so used to this restriction that many of them do not even mention this when presenting their results. Let's suppose that:

$$u(w) = \frac{w^{1-y}}{1-y} \; pentru \; w > 0 \tag{12}$$

The y-scale is chosen so that y> 0, y ≠ 1. It is easy to show that y is equal to the degree of relative risk aversion, because A (w) = y / w and R (w) = y for all w.

Thus, this set presents the decrease of the absolute risk aversion and the constant relative risk aversion, which are two reasonable hypotheses. For this reason, these utility functions are called preference class for constant - relative - aversion (FUCA).

We can see that our definition does not allow y = 1. However, it is simple to show that the function u (w) = ln (w) satisfies the property that R (w) = 1 for all w. Thus, the set of all FUCA utility functions is completely defined by:

$$u(w) = \begin{cases} \frac{w^{1-y}}{1-y} \; pentru \; y \geq 0, \; y \neq 1 \\ \ln(w) \; pentru \; y = 1 \end{cases} \tag{12}$$

We can also show that u (w) = ln (w) as a limiting case of the electric utility function. For this purpose, we rewrite the electric utility function, using a linear transformation, such as: $u(w) = \frac{1}{1-y} \left(w^{1-y} - 1\right)$.

Conclusions

A first conclusion is that, any economic aggregate evolves in the presence of foreseeable risks, which will occur with certainty but also of more easily identifiable risks, subject to the criterion of uncertainty, in which case, if some factorial conditions are met, they will can manifest.

The economic-financial risks manifest themselves regardless of the measures that are taken, but these risks once known can lead to a well-structured plan of measures that will ensure, if not eliminate, at least diminish their effects.

Another conclusion is that the classic utility functions can eliminate any income effects, when making decisions about risks whose size is commensurate with the wealth level of each. As shown in this paper, the risk premium fI defined by equation (6) is independent of wealth w, and the assumption that the aversion to relative risk is constant, simplifies many of the problems often encountered in macroeconomics and finance.

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