THEORETICAL ASPECTS REGARDING RISK AVERSION

Prof. Constantin ANGHELACHE PhD *(actincon@yahoo.com)*
Bucharest University of Economic Studies / „Artifex” University of Bucharest
Alexandru BADIU PhD Student *(badiu@transferrapid.com)*
Bucharest University of Economic Studies

Abstract

In any field of activity, the unforeseen must be taken into account, that is, the situation in which less anticipated elements (aspects) can exert their influence on the future evolution. In general terms we can talk about certainties and uncertainties. Therefore, the certainty is given by the foreshadowing of factors that will act in the future and which may have both positive and negative influences on the economic evolution. Uncertainty is essentially the risk, in the sense that, in the desire to make a forecast as detailed and reliable, new variants appear that may manifest themselves in the future without the guarantee that they will manifest with certainty.

The risk or the decisions taken under risk is a very sensitive problem of the elements of macroeconomic analysis and, especially, of macroeconomic forecasting. In this paper, the authors set out to make a well-structured analysis based on a broad consultation of the profile biography in this field, in relation to the concept of risk, the manifestation of risks, the risk management and the situation in which the decisions under risk must be taken for have a forecast, a guideline on the macroeconomic results that will occur over time.

Keywords: risk, risk aversion, utility function, profit, insurance, payments, yield, concavity

JEL classification: E47, G24

Introduction

The issues regarding risk and risk aversion addressed and analyzed in this study are particularly sensitive, and a number of leading researchers and economists have expressed their views and given a slightly different definition of what it concerns the risk but also the conceptual way in which I propose the realization of models and the methods by which they can be applied so that we can make macroeconomic forecasts with a rich theoretical substrate.

These concepts applied in practice raise some problems regarding databases, the complexity of the databases, the correlation that exists between the indicators that measure the economic growth and many other aspects with statistical.etic content.

Literature review

In the works published by Anghelache C., Niță G., Badiu A. (2016) și Oancea, D., Anghelache, C., Zugravu, B. (2013) , the authors focused on the need to clarify some aspects regarding decision-making at risk. They started from the clear premise that the risks, as a factor of uncertainty, will arise and develop even when the elements of their occurrence can be predicted, but it is important that these risks be foreseen and, consequently, to set up the financial reserves to cover the possible destructive effects of the risks in this area. The attitude towards risk and uncertainty has been largely addressed by Kahneman, D (2010) şi Tversky, A. (2000), pointing out that this is a psychological side of human behavior, but that in the future it has consequences if we do not try to put in the used model and the notion of risk that will manifest itself anyway. Marcowitz, H (2010, 2014) şi Tobin, J. (1987have addressed in the studies carried out the concept of portfolio efficiency and the concept of influence or effect of risks on a future evolution.

Methodology, data, discussions, results

We will assume that the decision-maker only plans for a single period, which means that he immediately uses the resources to purchase and consume goods and services. The final asset comes from the initial resources w corrected with the result of any risk incurred during the transition.

We can specify that if a risk-averse agent is warned against it, regardless of the level of resources w, he will not opt for the option that will bring him zero profit:

$$Ez = 0, Eu(w + z) \leq u(w) \quad (1)$$

We note that any z insurance company with an expected gain, other than zero, can be broken down into its expected gain Ez and zero insurance z-Ez.

Thus, by our definition, a risk-averse agent always prefers to receive with certainty the expected outcome of a situation rather than the challenge itself. For a utility maximizer expected with utility function u, this implies that for any insurance z and for any initial wealth w:

$$Eu(w + z) \leq u(w + Ez) \quad (2)$$

If we consider the decision of the economic agent, in the case of transporting the goods with a single cargo, the initial wealth w is 4000, and the profit Z takes the value of 8000 if the goods arrive with good destination or 0 in the case in which it is lost, both situations with equal production probabilities.

Starting from the premise that the economic decision-maker is risk-averse, then he must follow:

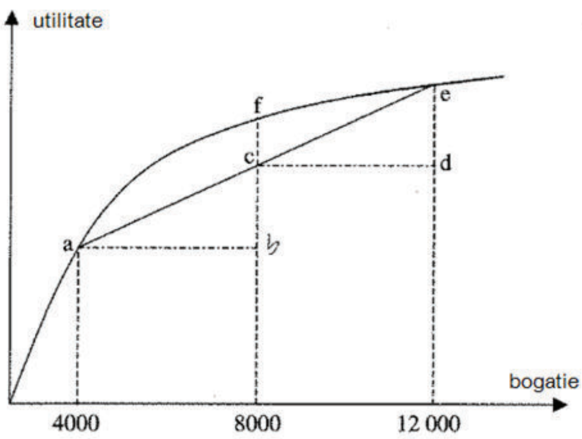
$$1/2\ \ u(12000) + 1/2\ \ u(4000) \leq u(8000) \qquad (3)$$

If the company could find an insurance company that would offer full insurance at a fair actuarial price of Ez = 4000 euros, it would be better to conclude such an insurance policy.

We notice how the inequality (3) is verified in figure 1.

Measuring expected utility (4000, ½: 12000, ½)

Figure 1



The right side of the inequality is represented by the point „f" on the utility curve u. The left side of the inequality is represented by the intermediate point on the arc „ae", that is by the point „c". This can be immediately verified by noting that the two triangles „abc" and „cde" are equivalent, because they have the same base and the same angles. Note that „f" is higher than „c": ex ante, the welfare derived from the z lottery is lower than the welfare obtained if the proper security of the prepayment Ez were obtained.

In short, our decision maker cannot face risks. The intuition of the result is very simple: if the marginal utility is decreasing, then the loss of 4000 Euro reduces the utility more than the increase of the utility generated by the potential gain of 4000 Euro. Seen ex ante, the expected utility is reduced by them as well as weighted potential outcomes.

It is worth noting that relations (1) and (3) are identical. The preference for diversification is intrinsically equivalent to risk aversion, at least in the case of the Bernoullian model of expected utility.

Using the opposite argument, one can easily show that, if u is convex, the inequality in (2) will be reversed. Therefore, the decision-maker prefers risk over his mathematical expectations and thus reveals his inclination to take the risk. How many individual behaviors will be mentioned as risk-loving. Finally, if u is linear, then the welfare of Me is linear in the expected gain of risks. Indeed, if u (x) = a + bx for any x, then we have:

$Eu(w + z) = E[a + b(w + z)] = a+ b(w + Ez) = u(w + Ez)$,

which implies that the decision maker classifies the risks according to the expected result. This individual's behavior is called neutral risk.

In the following sentence, we formally demonstrate that inequality (2) is valid for any z and any initial wealth w if and only if u is concave.

We consider that a decision factor with a utility function u is the risk of avoiding, that is, the inequality (2) is valid for any w and z, if and only if u is concave.

The proof of sufficiency is based on a Taylor second-order extension of u (w + z) around w + Ez. For any z, this yield:

u(w + z) = u(w + Ez) + (z - E z)u ,(w + Ez) + 1/2 (z - E z)2u'' (S(z))

for some S (z) between z and Ez. Since this must be true for any z, it follows that the expectation u (w + z) is equal to:

Eu(w + z) = u(w + Ez) + u'(w + Ez)E(z - Ez) + ½E[(z - Ez)2u ,' (S z))]

We now notice that the second term of the right-hand part above is zero, because E (z - Ez) = Ez - Ez = 0. In addition, if u ,' is uniformly negative, then the third term is waiting of a random variable (zE z) 2 u ,' (S (z)) which is always negative, because it is the result of a square and negative scalar u ,'. Therefore, the sum of these three terms is less than u (w + Ez).

The need is proven by contradiction. Suppose u is not concave. Then there must be values of w and m> 0 for which u „(x) is positive in the range [w - m, w + m]. Considering a small risk close to zero, n, means that supporting the final wealth w + n is completely determined (wm, w + m). Using the same Taylor expansion as in the previous calculation mode, we obtain the yield:

Eu(w + n) = u(w) + 1/(2)E [n 2u'' (S (n))]

Because S (n) has a support that is contained in [w - m, w + m] where u is locally convex, u „(S (n)) is positive for all of n's achievements.

It follows that: E [n2u „(S (n))] is positive, and I (w + n) is greater than u (w). Therefore, accepting the situation n with zero welfare increase values, when the decision-maker is not obverse, it is a contradiction.

The above proposal is nothing more than a rewriting of Jensen's famous inequality. We considered any function with real value ϕ. Jensen's inequality states that Eϕ (y) is smaller than ϕ (Ey) for any random variable y if and ouly if ϕ, is a concave function. It builds a bridge between two alternative definitions of concavity u: negativity u „and the property that any arc that connects two points of the curve u must lie below this curve. Figure 1 illustrates this point. It is intuitive that the marginal utility decrease (u" <0) means risk aversion. In a safe world, the decrease in marginal utility means an increase of wealth by 100 euros which has a positive effect on the utility which is less than the effect of a reduction of wealth by 100 euros.

In an uncertain world, introducing the risk of winning or losing 100 euros with equal probability will have a net negative impact on the expected utility. Pending, the benefit of the prospect of winning 100 euros is overestimated by the cost of the prospect of losing 100 euros with the same odds. In the last two decades, many prominent researchers in the field have challenged the idea that risk aversion comes only from diminishing marginal utility, the idea that there should be a connection between the two.

• **Premium risk level and certainty equivalence**

A risk warning agent is an agent who does not want zero risks. The „zero mean" qualifier is very important. A risk warning agent might want risky lotteries if the expected winnings they make are big enough. Investors facing risk may want to buy risky assets if the expected returns exceed the risk-free rate. People who are at risk may not want to buy insurance if they are too expensive to purchase.

To determine the optimal trade-off between the expected gain and the degree of risk, it is useful to quantify the effect of risk on well-being.

This is especially useful when the agent subrogates the risky decision for others, as is the case when we consider, for example, public safety policy or portfolio management by pension funds.

It is important to quantify the degree of risk aversion in order to help people to know better and to make better decisions in the face of uncertainty. Most of this work concerns precisely this problem.

Obviously, people have different attitudes towards risks. Some are willing to spend more money than others to get rid of a specific risk.

One way to measure the risk aversion of an agent is to ask him how much he is willing to pay to get rid of zero risk. The question will be mentioned as the risk premium fl associated with this risk. For an agent with utility function u and initial wealth w, the risk premium must meet the following conditions:

Eu(w + z) = u(w - Π) (4)

The agent reaches the same welfare either by accepting the risk or by paying the risk premium fl. When risk Z has an expectation that differs from zero, we usually use the concept of certainty equivalent. The certainty equivalent of Z risk is certainly the increase in wealth that has the same effect.

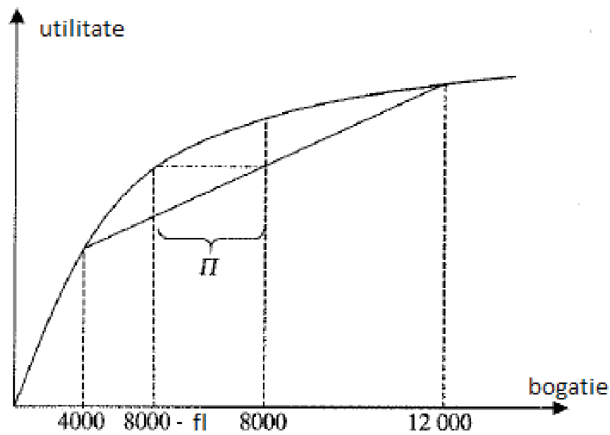
Eu(w +z) = u(w + e) (5)

When Z has a zero value, comparing relations (4) and (5) implies that the certainty equivalent of Z is equal to minus its first risk fl.

A direct consequence of relationship 2 is that the risk premium fl is non-negative. In Figure 2, we measure n for the risk (-4000, ½, 4000, ½) for the initial wealth w = 8000. We observe first that the risk first is zero when u is linear and is non-positive when it is - convex.

Measurement of risk premium P of risk (-4000, ½; 4000, ½)

Figure 2



when the initial wealth is w = 8000

A very convenient property of the risk premium is that it is measured in the same units as the wealth, respectively in euros, in the case of our economic agent. The measure of satisfaction or usefulness is difficult to compare between different individuals.

The risk premium is a complex function of the distribution of Z of the initial wealth w and the utility function u. We can estimate the amount the agent is ready to pay for risk elimination by examining small risks. Suppose Ez = 0.

Using a Taylor second and first order approximation for the left and right sides of the equation (4), we obtain that:

$u(w - \Pi) \cong u(w) - \Pi u'(w)$ și

$Eu(w + z) \cong E[u(w) + zu'(w) + 1/2 \ z2u''(z)]$

$= u(w) + u'(w)Ez + 1/2 \ u''(w)E\, z2$

$= u(w) + 1/2 \ n \ 2u''(w)$,

where Ez: = 0 and n2 = E z2 is the variation of the result. Substituting these two approximations into equation (4) determines yields that Ez = 0.

$$\Pi \cong \frac{1}{2} m^{2} A(w) \quad (6)$$

where function A is defined as:

$$A(w) = \frac{-u''(w)}{u'(w)} \quad (7)$$

Under risk aversion, function A is positive. It would be zero or negative for a risk-neutral or risk-loving agent. We continue to note the function A (-) as the absolute degree of risk aversion. We see that the risk premium associated with the risk for an agent with wealth w is approximately equal to half of the product of variance Z and the degree of aversion of the absolute risk of the agent evaluated at w.

Equation (6) is known as the Arrow-Pratt approximation, as developed independently by Arrow (1963) and Pratt (1964).

The cost of risk, measured by the risk premium, is approximately proportional to the change in its earnings. Thus, the variance may seem to be a good measure of the risk degree of a lottery. This observation led many authors / researchers to use a medium-variation decision criterion for modeling risk behavior.

In a model of average variation, we assume that individual attitudes toward risk depend not only on the average and the variation of the underlying risks. The value of these values depends on the degree of accuracy of the approximation (6), and can be considered as a very small or too big problem.

In such cases, the average variance approach for risky decisions, which has played a very important historical role in the development of finance theory, can be seen as a special case of expected utility theory.

In most cases, however, the risk premium associated with any high risk will also depend on the other moments of the risk distribution, not just the average and its variance. For example, it seems intuitive whether or not X is symmetrically distributed with respect to its average aspects for determining the risk premium. The degree of concealment (ie the third moment) could very well affect the desire for a risk.

Therefore, two risks with the same mean and variance, but one with a distribution that is inclined to the right and the other with a distribution that is inclined to the left, should not be expected to have the same risk premium. A similar argument can also be made about kurtosis (the fourth moment), which is related to the probability mass in the distribution queues.

At this stage, it should be noted that, at least for small risks, the risk premium increases with the size of the risk proportional to the square of this size. To see this, suppose z = ks, with Es = 0. Parameter k can be interpreted as a measure of risk. When k tends to zero, the risk becomes very low.

Of course, the risk premium is a function of the risk size. We can expect this function ll (k) to increase ink. We are interested in describing the functional form that links the risk premium n to the dimension k of the risk. Since the variance z is equal to k2 or the variance of s, we obtain this:

$$\Pi \cong \frac{1}{2}\, k^{2} m_{n}^{2}\, A(w)$$

that is, the risk premium is approximately proportional to the square of the risk size. From this observation we can directly conclude that not only fl (k) approaches zero, since k is close to zero, but also fl ,(0) = 0. This is an important property of the expected utility theory.

At the margin, accepting a low or zero risk has no effect on the well-being of risk-avoiding agents. We say that risk aversion is a second order phenomenon. Tois property in general models, which is not limited to expected utility, is called „secondary risk aversion". With the expected utility model, this property is based on the assumption that the utility function is differentiated. Expected utility maximizers are all risk neutral.

If the utility function is differentiated, the risk premium tends to zero as a square of the risk size.

Next, we will show that IT ,(0) = 0, as suggested by the Arrow-Pratt approximation in our comments above. The relationship between IT and k can be obtained by completely differentiating the equation:

$$E_u(w + ks) = u(w - \mathrm{IT}(k)), \text{ cu privire la } k.$$

$$\Pi'(\mathrm{k}) = \frac{Esu'(w + ks).}{u'(w - \mathrm{IT}(\mathrm{k}))} \qquad (8)$$

We directly deduce that IT ,(0) = 0, because by the assumption Es = 0

• **Aversion to risk**

We will consider the following simple decision problem, in which an economic agent accepts the offer to take the risk of the situation z with the mean μ, and the variance u2. Of course, the optimal decision is to accept the situation if:

$$Eu(w + z) \geq u(w) \qquad (9)$$

or, equivalently, if the certainty equivalent e of Z is positive. In the following, we will examine how this decision is affected by a change in the utility function.

We notice that, the continuous linear transformation of u has no effect on the decision of the decision maker and on the certainty equivalents. Indeed, we consider a function v (•) such that v (x) = a + bu (x) for all x, for a pair of scaling a and b, where b> 0.

Then, obviously, Ev (w + z) ≥ v (w) generates exactly the same restrictions on the distribution z as, condition (9). The same analysis can be done on equation (5) which defines certainty equivalents. The neutrality of the certainty equivalents with the linear transformations of the utility function can be verified in the case of small risks by using the Arrow-Pratt approximation. If v = a + bu, it is obvious that:

$$A(x) = \frac{-v''(x)}{v'(x)} = \frac{-bu''(x)}{bu'(x)} = \frac{-u''(x)}{u'(x)}$$

for all x. Thus, from equation (6), we observe that the risk premiums for the small risks are not affected by the linear transformation. The average risk payment rate minus the risk premium - the neutrality property is equivalent to certainty.

Limiting the analysis to low risks, assumes according to this analysis that agents with an absolute aversion to risk A (w) will be more reluctant to accept small risks. The estimated minimum payment that makes the risk acceptable to them will be higher.

This is why we say that A is a measure of the risk aversion of decision makers.

From a technical point of view, A • = -u „ju ,is a measure of the degree of concavity of the utility function. It measures the speed with which the marginal utility decreases.

We consider the following definition for comparative risk aversion: we assume that agents u and v have the same wealth w, which is arbitrary.

An agent v is more risk averse than another agent u with the same initial wealth if any risk that is undesirable for agent u is also undesirable by agent v. In other words, the risk premium of any risk is higher for agent v than for agent u.

This must be true independent of the initial level of common wealth of the two agents. If this definition were limited to small risks, we know from the above analysis that this would equate with the requirement that this:

$$A_v(w) = \frac{-v''(w)}{v'(w)} = \frac{-u''(w)}{u'(w)} = Au(w)$$

If it is limited to small risks, v is riskier than u if the function Av is uniformly greater than Au. We say in this case that you are more concave than u in the sense of Arrow-Pratt.

It is important to note that this is equivalent to the condition that v is a concave transformation of u, that is, there is an increase and the concave function ef, so that v (w) = <f, (u (w)) for all w.

Indeed, we have this:

v'(w) = <f,'(u(w))u'(w) si

v"(w) = <f," (u(w))(u' (w))2 +<f,'(u(w))u"(w),

Thus, Av is uniformly larger than Au if and only if <f, is concave. This is equivalent to the fact that Av is uniformly larger than Au or that v is a concave transformation of u. It is noted that agent v assesses lower risks than agent u.

We need to impose more restrictions to ensure that agent v evaluates any risk lower than agent u, ie v is more risk averse than u. The following proposal, which is due to Pratt (1964), indicates that no further restrictions are needed.

The following three conditions are equivalent.

(a) Agent v is more risk-averse than agent u, meaning the risk premium for any risk is higher for agent v than for agent u;

(b) For all w, Av (w)?

(c) Works against a concave transformation of function u; ϕ ,> 0 and ϕ' ,≤ 0 such that v (w) = ϕ (u (w)) for all w.

We have already shown that (b) and (c) are equivalent. The fact that (a) implies (b) results directly from the Arrow-Pratt approximation. We will prove that (c) implies (a). Taking into account any situation z. Let IIu and IIv be the risk premium for zero insurance of agent u and agent v respectively.

By definition, we have:

v(w - Πv) = Ev(w +z) = Eϕ,(u(w +z))

We define the random variable as y = u (w + z). Since ¢, is concave, Eϕ, (y) is smaller than ϕ, (Ey) because of Jensen's inequality. It looks like this:
$v(w - \Pi_v) \geq \phi,(Eu(w + z)) = \phi(u(w - \Pi_u)) = v(w - \Pi_u)$

As v is increasing, this implies that Πv is greater than Πu.

In the case of small risks, the only thing we need to know to determine if a risk is desirable is the degree of concavity of the local level at the current level of wealth.

For higher risks, the above proposal shows that we need to know more to make a decision. Namely, we must know the degree of concavity of u at all levels of wealth. The degree of concavity must be increased at all levels of wealth to ensure that a change in u makes the decision-maker more reluctant to accept risks.

If v is more locally concave at some wealth levels and less concave at other wealth levels, the comparative analysis is inherently ambiguous.

To illustrate the sentence, let us return to the singular example of George's ship protection coming from z = (0, ½; 8000, ½), with an initial wealth w0 = 4000 Euro.

If the utility function is u (w) = √w, its certainty equivalent of z is equal to eu = 3464.1, because:

$$\frac{1}{2}\sqrt{4000} + \frac{1}{2}\sqrt{1200} = 86.395 = \sqrt{7464.1}$$

Alternatively, suppose the utility function is v (w) = In (w), which is also increasing and concave. It is easy to see if you are more concave than u in the Arrow-Pratt sense. Indeed, these functions produce:

$$A_v(w) = \frac{1}{w} \geq \frac{1}{2w} = A_u(w)$$

for any w. From the above sentence, this change in utility should reduce the certainty equivalent of any risk. In the case of wo = 4000 and z - (0, ½; 8000, ½), the certainty equivalent of z in v is equal to ev = 2928.5, because
½ ln(4000) + ½ ln(l2 000) = 8.8434 = ln(6928.5)

Thus, ev is smaller than me. We observe that the risk premium Πv = 1071 .5 in v is about twice the risk premium Πu = 535.9. This has been predicted by Arrow-Pratt by approximation, since Av equals 2Au.

Conclusions

One conclusion from this article is that risk aversion results from diminishing marginal utility, which implies that there should be a connection between them.

Also, investors who are at risk seek to buy risky assets when expected returns exceed the risk-free rate, and those who are at risk do not intend to buy insurance if they are too expensive to buy..

Quantifying the degree of risk aversion is a real help to those who want to invest, helping them make better decisions in the face of uncertainty.

Another conclusion that emerges from this paper is regarding the small risks, in which case, the risk premium increases with the size of the risk proportional to the square of this dimension.

References

1. Anghelache C., Niță G., Badiu A. (2016), Remitențele migranților – o sursă externă de fonduri importantă și stabilă, în dezvoltarea economică a unei țări, *Revista Română de Statistică Supliment*, 12, pg. 74-82
2. Anghelache, C., Isaic-Maniu, A., Mitruţ, C., Voineagu, V., Dumbravă, M., Manole, A. (2006), *Analiza macroeconomică – Teorie şi studii de caz*, Editura Economică, București
3. Anghelache C., Manole,A., Anghel, M.G., & Soare, D.V.(2016) Statistical-econometric model used to analyze the operational and insolvency risks, *Theoretical and Applied Economics,* Volume XXIII, No.3 (608), pg. 221-228
4. Kahneman, D (2010), *International Difference in Well-Being*, Editura Penguin Books
5. Marcowitz, H (2010), Portofolio theory: As I still see it, *The Annual Review of Financial Economics*, 2(1), 1-23
6. Marcowitz, H (2014), Mean-variance approximations to expected utility, *European Journal of Operational Research*, 234, 346-355
7. Oancea, D., Anghelache, C., Zugravu, B. (2013) Econometric Model of Risk Forecasting. *Revista Română de Statistică – Supliment*, Vol. 2013, Trim. II, pp.123-127
8. Tobin, J. (1987), Essays in Economics, vol.1: *Macroeconomics*
9. Tversky, A. (2000), *Choices, Values and Frames*, Editura Cambridge University Press