THE SPECTRAL MODEL - GENERAL ELEMENTS

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Abstract

The study of the autocorrelations of an economic process provides information on the dynamics of a time series and synthesizes the link between a variable and historical values. In practice, stationarity is a feature that requires preliminary conversion of the initial series. In order to avoid the wrong conclusions about the economic nature of the variance, the estimation and isolation of the variation of chronological nature is ensured. An economic analysis must start from the removal of as far as possible the irregularities caused by the temporary extreme value. In this regard, the types of values are defined, namely the extreme additive values, the temporary extreme values and the level changes. Time series decomposition is achieved by using the ARIMA model and the Tramo-Seats procedure. In economic analyzes, we have to identify the trend and seasonality that are persistent and regular in time being associated with the non-stationarity concept, and the cyclic transient component and the random component are associated with conceptual stationarity. The model used is based on qualitative elements, the signal representing the unobserved component that is to be estimated.

Keywords: *Spectral model, time series, extreme value, ARIMA model, qualitative factor, factor decomposition, estimation.*

Classification JEL: C13, C44, C50

Introduction

In this article, based on the study, the authors sought to present the main elements of the spectral model and the extent to which it can be used in the analysis of the chronological series. In order to model the deterministic effects, it is necessary that the observed series be mathematically formalized to the stationary series. The authors explain extensively the concepts of extreme additive values, temporary extreme values and level changes. The article discusses the essential issues regarding the decomposition of time series using the ARIMA model. Also highlight the qualitative elements underlying the model. The study focuses on concrete aspects of admissible decomposition and canonical decomposition. Finally, we are dealing with theoretical aspects regarding the estimation of the components aimed at achieving a complete and finite series of the analyzed economic process. The elements underlying the spectral model are highlighted in the article. All these aspects are formalized in mathematical relationships that can be used successfully in economic analyzes. The study underlying this article can be expanded by applying to concrete data on which to analyze and implement pertinent decisions.

Literature review

Anghelache și Anghel (2016), precum și Newbold, Karlson și Thorne (2010) au prezentat aspecte fundamentale ale statisticilor economice. Anghelache (2008) a analizat indicatorii folosiți în studiul seriilor cronologice. Anghelache și Anghelache (2009) au abordat aspecte ale utilizării seriilor cronologice în stochastică proceselor. Bardsen și colaboratorii (2005) au prezentat elemente ale modelării macroeconomice. Benjamin, Herrard, Houee-Bigot și Tavéra (2010) au studiat modalitățile de prognozare pe baza modelelor econometrice. Ghysels și Osborn (2001) se referă la analiza econometrică a seriilor de timp sezoniere. Müller (2007) a analizat estimarea variațiilor de lungă durată. Phillips, Sun și Jin (2006) au cercetat aspecte ale estimării densității spectrale și testarea ipotezei robuste.

Research methodology, data, results and discussion

• Introductory notions

The study of the process's autocorrelations provides information about the dynamics of a time series and synthesises the link between x_t and historical values.

Besides the medium and constant conditions, the weak stationarity also implies the condition no.2 which can be written:

$$Cov(x_t, x_{t-k}) = \gamma_k \tag{1}$$

From this relationship it results that for a certain distance K the autocovarian is constant, ie the relation between the values is constant. The values of the autocovers for different k distances can be represented in a concentrated form by means of the AutoCorporate Generating Function (ACGF) which has the following form:

$$\gamma(B,F) = \sum_{-\infty=1}^{\infty} \gamma_i B^{i} \tag{2}$$

The ACGF function being symmetric can be written:

$$\gamma(B,F) = \gamma_0 + \sum_{ij=1}^{\infty} \gamma_i \, (B^{\mathrm{i}} + F^{\mathrm{i}}) \tag{3}$$

An important result that comes out of Wald's fundamental theorem is the representation of ACGF function with infinite polynomial $\psi(B)$

$$\gamma(B,F) = \Psi(B)\Psi(F)V_\alpha \tag{4}$$

From the fundamental representation of Wald

$$X_t = \alpha_t + v_1\alpha_{t-1} + v_2\alpha_{t-2} + \cdots = \sum_{j=0}^{\infty} v_j\alpha_{t-j} = \Psi(B)\alpha_t \tag{5}$$

we can see that it can not be used in numerical estimation problems because it contains an infinite number of terms. Therefore, the following rational approximation will be used:

$$\Psi(B) = \frac{\theta(B)}{\Phi(B)} \tag{6}$$

From Relationships (4) and (6) there is a new form of ACGF function for an ARMA process:

$$\gamma(B,F) = \frac{\theta(B)\theta(F)}{\Phi(B)\Phi(F)} V_\alpha \tag{7}$$

B is a complex number with module 1 that can be written as $e^{i\omega}$. Therefore, if in relation (3) we replace operators F and B by their complex representation we obtain:

$$g(\omega) = \gamma_0 + \textstyle\sum_{j=1}^{\infty} \gamma_i \, (e^{-\mathrm{i}\omega \mathrm{j}} + e^{\mathrm{i}\omega \mathrm{j}}) \tag{8}$$

and if we replace in relation (7) we obtain:

$$g(\omega) = \frac{\theta(e^{-\mathrm{i}\omega})\theta(e^{\mathrm{i}\overline{\omega}})}{\Phi(e^{-\mathrm{i}\overline{\omega}})\Phi(e^{\mathrm{i}\overline{\omega}})} V_\alpha \tag{9}$$

If we use the identity

$e^{-\mathrm{i}\omega\mathrm{j}} + e^{\mathrm{i}\overline{\omega}\mathrm{j}} = 2\cos(\mathrm{j}\overline{\omega})$ the relationship (8) becomes:

$$g(\omega) = \gamma_0 + 2\sum_{j=1}^{\infty} \gamma_i \cos(j\omega) \tag{10}$$

The function $g(\varpi)$ represents the spectrum generating function that is used, depending on the situation, in one of its above-mentioned forms.
If we divide the relation (10) prin 2π, we obtain the power of the spectrum:

$$f(\omega) = \frac{1}{2\pi}\left[\gamma_0 + 2\sum_{j=1}^{\infty}\gamma_i \cos(j\omega)\right] \quad (11)$$

The transition from the generating function of the autocovers to the power of the spectrum is called the Fourier transform.

Obtaining autocovers from the power of the spectrum is done by the inverse of the transformed Fourier data through the relationship:

$$\gamma_k = \int_{-\pi}^{\pi} \cos(\bar{\omega}k)\, f(\bar{\omega})d\bar{\omega} \quad (12)$$

If the generating function of the autocovers is replaced by the correlation generating function, by division with the series variation γ_0, the spectral density function is obtained**.**

• **Pre-adjustment of time series**

To model the deterministic effects we assume first that the observed series is stationary so we can write:

$$X_t = \mu_t + Z_t \quad (13)$$

Where μ_t are the average of the process such as:

$$\mu_t = E(X_t) = Y_t'\beta \quad (14)$$

The variable Y_t is a vector of regressive variables, $Y'_t = (Y_{1t}......Y_{rt})$, weighted by the vector of coefficients $\beta, \beta' = (\beta_1.......\beta_r)$, Z^t follows a general ARMA process, of form $\Phi(B)Z_t = \theta(B)a_t$, in which $\phi(B)$ and $\theta(B)$ satisfy the conditions of stationarity and inversibility. The variance of the process Y^t is given by:

$$V(X_t) = V(Z_t) \quad (15)$$

In practice, stationarity is a feature that often requires a preliminary transformation of the initial series. We note with $\delta(B)$ the differentiating operator of the order d that makes the necessary transformation to obtain stationarity. Then

$x_t = \delta(B)Y_t$ *și* $z_t = \delta(B)Z_t$ therefore we obtain:

$$x_t = \delta(B)Y_t'\beta + z_t \quad (16)$$

This type of model is known as linear regression models with ARIMA errors or, in short, REG-ARIMA.

In most cases, statistical agencies offer quarterly or monthly data. If data are recorded monthly, for example, it is natural to have a monthly variation due to the number of days in a month and, implicitly, the number of working days in each month. In order to avoid misconceptions about the economic nature of the variation, it is desirable to estimate and isolate the variation of the calendar nature.

If we express the series observed according to the number of days of each month, considering each day of the week a possible influence factor, we can write:

$$X_t = \beta_1 Y_{1t} + \beta_2 Y_{2t} + \ldots + \beta_7 Y_{7t} + Z_t \qquad (17)$$

where Y_{it} is the number of days for each day of the week, for example Y_{1t} represents the number of months in months of month t, Y_{2t} the number of days and so on and the coefficients δ_i represent the average amount of the respective day.

The amount $\sum_{i=1}^{7} \beta_i Y_{it}$ calculated for each month of registration t, represents an average value calculated from the assumption that on a certain day of the week the same value is observed throughout the year. Therefore, the values observed on a monthly basis will be different only because of the number of days, thus eliminating the calendar day effect of the number of working days.

In practice, the coefficients β_i are strongly correlated and hence recursive. If we calculate the daily average, $\overline{\beta} = 1/7\sum_{i=1}^{7} \beta_i$ and we consider m_t as the length in the days of the month t so $m_t = \sum_{i-1}^{7} Y_{it}$ then we can rewrite:

$$\sum_{i=1}^{7} \beta_i Y_{it} = \sum_{i=1}^{7} (\beta_i - \bar{\beta}) Y_{it} + \sum_{i=1}^{7} \bar{\beta}\, Y_{it} = \sum_{i=1}^{7} b_i (Y_{it} - Y_{7t}) + \bar{\beta} m_t \qquad (18)$$

The economic activity varies depending on certain special moments of the year, such as Christmas or Easter holidays, which are usually associated with strong sales growth. While Christmas holidays take place every year at a fixed date, the Easter effect can manifest in both April and May. Therefore, choosing each year of the date when Easter is celebrated can induce a certain variability in the data series from one year to the next.

It is often useful to remove and, as far as possible, explain the irregularities in the data series. These can be of several types: extremely additive values, temporary extreme values, and level changes. Assuming that a model has been identified for the series then we have the residual values e_t. Considering is a „white noise" $I_{t_{o(t)}}$ it is an alternative variable so $I_{t_{o(t)}} = 1$ if $t = t_0$ and $I_{t_0}(t) = 0$ if $t \neq t_0$ then we can define the following types of values:

- Extreme additive values that are very low or very high values that affect a single recorded value. These values may be recorded due to exceptional events. For example, if we have a set of time that expresses the monthly traffic on a particular route when the road is closed for repair for one month in the time series, we will have a value of 0 for that month.

$$e_t = \alpha_t + \omega_A I_{t_0}(t) \quad (19)$$

- Temporary extreme values represent a shock followed by a gradual return to the general trend of the series. Such values are frequently encountered in phenomena such as strikes, for example, because after a strike, the return to the initial activity is not spontaneous, but is achieved gradually after several periods of time.

$$e_t = \alpha_t + \omega_T/(1 - \eta B) I_{t_0}(t) \quad (20)$$

- Level changes. These changes in the general trend can generally be caused by changes in country or product nomenclatures.

$$e_t = \alpha_t + \omega_T/(1 - B) I_{t_0}(t) \quad (21)$$

• **Decomposition of time series using the ARIMA model**

Unlike the structural approach, starting with the specification of the models directly for the time series components, the ARIMA-based approach initially identifies a model for the observed series and then, on its basis, the models for each component are obtained.

Starting from the ARIMA model for the observed series, the decomposition in the factors of the polynomial of the autoregressive part produces autoregressive polynomials for the series components. If the spectrum for all components is non-negative then the decomposition is acceptable. For an ARIMA model identified for the observed series, there is generally no single decomposition. In principle, decomposition variants differ by how „noise" is allocated to the components. By entirely assigning the noise of the random component a unique decomposition is obtained which represents a canonical decomposition.

In the actual decomposition phase of the observed SEATS series, we will start from the assumption that the observed time series was initially linearized by the TRAMO procedure.

The ARIMA based time series decomposition approach starts from the assumption that a observed process X_t consists of several unnoticed processes such as the seasonal component St the trend T_t, the cyclic component Ct and the random residual component R_t.

It is assumed that the observed process is related to its unobserved components through an additive relationship of the relationship type (2). If the initial relationship is not additive, it can be brought to this form through a series of operations. For example, if we have a series of multiplicative links, of the relationship type (1), by logarithm it can be brought to the additive form.

Trend can be seen as a deterministic equilibrium relation:

$T_t = a + bt$ where t represents the variable time

which implies $\Delta T_t = \boldsymbol{b}$.

The equation above is the equation of a straight with a slope or, economically, a steady increase. Such developments do not meet in practice in economic life, so in a more realistic approach we can introduce a certain disturbance in the conflict. This disruption assumes a zero mean and a constant variant, thus obtaining a stochastic trend of the form:

$$\Delta T_t = b + a_{pt} \text{ where: } \quad a_{pt} \sim niid(0, \sigma_p^2) \tag{22}$$

which implies:

$$\Delta^2 T_t = a_{pt}$$

and, more generally, a stochastic trend can be described by an IMA (2,1) model:

$$\Delta^2 T_t = (1 + \theta_p B) a_{pt} \tag{23}$$

A more general class of patterns for the trending component is represented by:

$$\Phi_p(B) \Delta^d T_t = \theta_p(B) a_{pt} \tag{24}$$

where Φ_p and Φ_{ns} are polynomials of relative order, and the roots of the Φ_p are all real, positive and stable and d = 1, 2, in general, and very rarely d = 3.

As for the seasonal component, it can be modeled from the deterministic point of view, depending on an alternative variable. If we have the seasonal component expressed monthly, then we can write:

$$S_t = \sum_{i=1}^{12} \beta_i \, d_{it} \tag{25}$$

where d_{it} = 1 for i=t and d_{it} = 1 in rest. The parameters β, have the property as:

$$\beta_1, \beta_2, \dots, \beta_{12} = 0$$

from which it follows that the sum of the values of 12 consecutive months is 0:

S_1 , S_2 ....S_{12} =0 or more generally

$$\sum_{i=0}^{11} S_{t-1} = 0 \tag{26}$$

Because in reality this equilibrium is disturbed by a certain shock that we will consider a moderate stationary process we can write:

$$\sum^{11} S_{t-1} = w_t \tag{27}$$

To cover a larger class of models, we can write the more general form:

$\Phi_{ns}(B)S_t = \theta_{ns}(B)a_{ns}$ where:

$$a_{ns} \sim niid(0, \sigma_p^2) \tag{28}$$

The roots of polynomial Φ_{ns} (B) are associated with seasonal frequencies.

The random component is assumed to be „white noise". In addition to the fact that there is a stationary transient component, this is represented by an ARMA model of the form:

$$\Phi_c(B)C_t = \theta_c(B)a_c \tag{29}$$

The roots of the Φ_{ns} component are sometimes associated with the fixed periods of a cyclic component. In economics, the term cycle is often used to designate the deseasonalized time series and no trending component.

What is relevant so far is that trend and seasonality, which are persistent and regular over time, are associated with the concept of non-stationarity, and the transient or cyclical component and random component are associated with the concept of stationarity.

If we generalize and assume that there are K components in the time series the model will be represented by the following set of equations:

$$x_t = x_{1t} + x_{2t} + \ldots + x_{kt}$$

$$\Phi_i(B)x_{it} = \theta_i(B)a_{it}{}_{i=1,\ldots,k} \qquad (30)$$

where Φ_i (B) *and* θ_i (B) are polynomials finite in B of order p_i and q_i respectively, do not have roots common and all roots are on or off the circle unity and

$a_{it}, \sim niid(0, \sigma_i^2)$ is a „white noise".

Since ARIMA models are aggregated, all ARIMA models are obtained by x_i, it will also be expressed by an ARIMA model:

$$\Phi(B)x_t = \theta(B)a_t \qquad (31)$$

where

$a_t \sim niid(0, \sigma_a^2)$ is a „white noise"..

Starting from the previous relationships it can be shown that the polynomial of the autoregressive component of the model (32) for the observed series xt satisfies the relation:

$$\Phi(B) = \Phi_1(B)\Phi_2(B) \ldots \Phi_k(B) \qquad (32)$$

and the polynomial of the moving average can be obtained from the relationship

$$\theta(B)a_t = \sum_{i=1}^{k} \Phi_{nt}(B)\theta_i(B)a_{it} \qquad (33)$$

where Φ_{ni} (B) is the product of all polynomials Φ_j (B), $j=1,\ldots,k$ excluding the polynomial Φ_i (B).

• **Elements of a qualitative nature that underlie the model**

Because not always all the unnoticed components are of interest, we will still consider a relationship with a higher degree of aggregation and a more practical character made up of signal and non-signal. The signal represents the unobserved component that is expected to be estimated and the non-signal represents the remaining part of the series.

$$x_t = s_t + n_t \qquad (34)$$

where: s_t - the signal

n_t – the non signal.

In a seasonal adjustment problem, ie if the seasonal component is to be removed, the signal will be represented by the non-seasonal part, ie the signal will be

$$s_t = T_t + C_t + R_t \quad (35)$$

In addition, it is necessary to mention the premises of Box, Hillmer and Tiao (1978), Burman (1980), and Hillmer and Tiao (1982), which laid the foundation for the decomposition method based on ARIMA models.

Premise no. 1

Unobserved components are unrelated. It is an assumption that has an explanation based on the fact that it is intuitively normal for different components of the observed series to be generated by different forces. Seasonal fluctuations, for example, are caused by seasonal events such as seasons, holidays, holidays, etc. while the evolution of the trend is caused by factors such as technological progress, productivity gains, etc.

Premise no. 2

The unobserved components can be described by ARIMA models of the form:

$$\Phi_s(B)s_t = \theta_s(B)a_{st} \text{ for the signal} \quad (36)$$

$$\Phi_n(B)n_t = \theta_n(B)a_{nt} \text{ for the non signal} \quad (37)$$

Where a_{st} and a_{nt} is the „white noise" with the ν_s variance respectively ν_n Models are considered irreducible.

Premise no. 3

The polynomials of the autoregressive component (AR), Φ_s (B) and Φ_n (B) and Φn (B) do not have common roots, which means that the spectrum for the unobserved components does not get high values at the same frequency.

Premise no. 4

The model for the observed series is known, ie the polynomials Φx (B), θ_x (B) and the variance of innovation, ν_a , are known.

From the above assumptions it follows that the observed series model, xt is an ARIMA model of the form:

$$\Phi_x(B)x_t = \theta_x(B)a_t \quad (38)$$

where,

$$\Phi_x(B) = \Phi_n(B)\Phi_s(B)$$

$$\theta_x(B)a_t = \Phi_s(B)\theta_n(B)a_{nt} + \Phi_n(B)\theta_s(B)a_{st} \quad (39)$$

and a_t is a „white noise", normally distributed with the variance ν_a.

• **Admissible decomposition**

Because of the fact that the autoregressive polynomials of the components, the signal and the non-signal are determined by factoring the polynomial Φ_x (B) the unknowns of the component model are the coefficients of the polynomials Φ_s (B) and Φ_n =3 and the variants of the innovations ν_s and ν_n. In the model (38), the information about the stochastic structure of the components is provided by the observed data series and the general relation (39). This relationship involves a system with

$$max(p_s + q_n, p_n + q_s) + 1 \quad (40)$$

equations of covariance while the number of unknowns is $q_n + q_s + 2$. In the situation where:

$$max(p_s + q_n, p_n + q_s) + 1 < q_n + q_s + 2 \quad (41)$$

in the absence of additional conditions, there are an infinite number of solutions for the system of covariance equations, and so there is an infinite number of variants for the decomposition of the xt series. Any decomposition that checks the relationships of the general model and the spectrum of the components is not negative is an acceptable decomposition. All admissible decompositions are equivalent from the point of view of the observed series.

Due to the fact that there are smaller possible structures that generate the same series observed the time series components are generally unidentifiable. For example, if we want to extract the seasonal component from a series of times, we start from the hypothesis that seasonality is represented by fluctuations with a one-year period that correspond to spectral amplitudes located at seasonal frequencies. This definition is not quite restrictive. Large spectral amplitudes are generated by large values of the autoregressive polynomial, but due to the fact that no condition is imposed for the mobile media and for the variant of innovation, a multitude of possibilities for the model is allowed.

The conclusion is that, in order to identify the components of the time series, some arbitrary assumptions are required in addition to the four premises presented in the previous paragraph. One of the benefits of decomposition based on ARIMA models is that assumptions can be explicitly made while, in the case of empirical methods, these assumptions are not known.

• **Canonical decomposition**

The canonical decomposition was first proposed by Box, Hillmer and Tiao (1978) and then by Pierce (1978). The canonical decomposition treats the problem of identifying the model as a problem of noise distribution. The way it is distributed is an a priori decision and therefore another condition is explicitly stated.

The canonical decomposition consists in specifying the component that is to be estimated, ie the signal, so as to be as free as possible from the noise. Noise is mainly distributed to the random component. The noise emitted signal is called a canonical signal and shows a zero value in its spectrum, which corresponds to a unit root in the moving average polynomial. Therefore, the canonical signal is non-invertible.

The hypothesis of component independence involves the following relationship:

$$g_x(\omega) = g_s(\omega) + g_n(\omega) \quad (42)$$

If we did:

$$\varepsilon_s = min_{\omega} g_s(\omega)$$ and
$$\varepsilon_n = min_{\omega} g_n(\omega)$$

then the quantity $\varepsilon_{s} + \varepsilon_{n}$ can be seen as the variant of the noise included in the spectrum of the observed series and which can be arbitrarily assigned. It is clear that the problem of identifying the model is due to the fact that a fraction of ε_n and εs can be extracted from the spectrum of any component and attributed to the other component. If we eliminate the noise as permitted by the st signal and assign it to the non-signal n_t, then we obtain $g_n^0(\omega) = g_s(\omega) + \varepsilon_s$ the spectrum of the canonic signal, non invertible and $g_n^0(\omega) = g_n(\omega) + \varepsilon_s$ spectrum non-signal in which all the noise is concentrated..

An important property of canonical decomposition is that admissible signal models can always be represented as the sum of the canonical signal plus an orthogonal „white noise". Moreover, Hillmer and Tiao (1982) have shown that canonical decomposition minimizes the signal innovation variant and the random component innovation is maximized when the other components are considered canonical.

• **Component Estimation**

The ultimate object in the decomposition of a time series is the estimation of the S_t, signal, that is, of the component that is expected to be estimated in the time series structure.

If we start from a certain X_t realization of the observed process and if we note with $\hat{S}_t$, the signal estimator, in order to determine an optimal estimator, we will make the difference between the signal and its estimator minimal, ie the estimation error is minimal . Which means we will determine $\hat{S}_t$ so that:

$$E\left[(St - \hat{S}t)^2 \mid Xt\right]$$

be minimal. Under normal distribution conditions, the estimator $\hat{S}_t$ is equal to the expected conditional value $E(st \mid Xt)$ e it is a linear function of X_t elements.

The actual decomposition procedure consists in determining the estimator $\hat{S}_t$, in the context of the assumptions about the structure of the model made in the previous paragraphs.

a. For a complete accomplishment of the process $X_t = \{x_{-\infty} \dots x_t \dots x_{+\infty}\}$

The estimator $\hat{S}_t$, is determined, as Whitle (1963) has shown, by the following symmetric filter known as the Wiener-Kolmogorov (WK) filter:

$$V(B,F) = \frac{ACGF(St)}{ACGF(Xt)} = \frac{Vs\dfrac{\theta s(B)\theta s(F)}{\phi s(B)\phi s(F)}}{Va\dfrac{\theta(B)\theta(F)}{\phi(B)\phi(F)}} \tag{43}$$

The estimator $\hat{S}_t$ is determined by the relationship:

$$\hat{S}_t = V(B,F)Xt \tag{44}$$

where the filter $V(B,F)$ is the ratio of the generating functions of the autocorporation for the signal and for the observed series.

An important feature of the WK filter is that it is only necessary to specify the signal model and the observed series to determine the estimated signal values.

From relations (43) and (44) we obtain:

$$\hat{S}_t = ks\,\frac{\theta s(B)\phi n(B)}{\theta(B)}\,\frac{\theta s(F)\phi n(F)}{\theta(F)} \tag{45}$$

where $k_s = V_s / V_a$. From the previous relation, we notice that the filter is the generating function of the autocoverse, ACGF, for the next stationary model:

$$\theta(B)z_t = \theta s(B)\phi n(B)b_t \tag{46}$$

where b_t is „white noise” with Vs / Va. So we infer that the filter is convergent, centered and symmetric.

If we pass into the frequency space we can have a representation of the filter of this form:

$$\hat{V}(B,F) = g_s(\omega)/\ g_s(\omega) \tag{47}$$

which is known as the benefit of the filter.

The spectrum of signal estimator $\hat{S}t$, will take the shape:

$$g_{\hat{s}}(\varpi) = \left[\frac{g_s(\varpi)}{g_x(\varpi)}\right]^2 g_x(\varpi) = [\hat{V}(B,F)\,]^2 g_x(\varpi) \tag{48}$$

The square of the filter benefit shows how the variant of the series contributes to the signal version. When for a certain frequency, the signal dominates the noise, *V(BA* approaches 1 and when the noise dominates the signal, $\hat{V}(B,F)$ it approaches 1.

b. For a finalized process $X_t = \{x_1, x_2 .... x_t\}$

The hypothesis of an infinite series was necessary because the WK filter in relation (43) goes through the real right -∞ to +∞. In practical conditions, however, the observation is finite. Because the filter is convergent, it can be interrupted smoothly at a certain point. For periods very close to both ends of the series it is impossible to apply a WK filter that is symmetrical. At the beginning and at the end of the observed series, the estimator's calculation involves the knowledge of past and future values that are not known at the time of decomposition. It is therefore necessary to extend the time series with estimated values and predicted values.

The Burman-Wilson algorithm (Burman, 1980) allows the filtering to be performed efficiently with a minimum number of predicted and predicted values.

Estimation errors can be broken down into two types of errors: the final estimation error and the revised error.

The final estimation error calculated as St - þt is obtained in the hypothesis of a complete series achievement. In practice, however, when the number of achievements is high enough, the average estimate error refers to the values in the center of the series.

The revised error is related to the impossibility of achieving infinite achievements and refers to the calculated estimator near the extremes of the time series. The independence of the two types of errors was demonstrated by Pierce (1980).

Conclusions

From the authors' study and presented in this article it is concluded that the spectral model can be successfully used in chronological series analysis. The considered mathematical relations constitute a practical possibility of analyzing the empirical data in order to determine the parameters and trend of the analyzed phenomenon. It follows that the decomposition of time series can be achieved by using the ARIMA model, which is based on qualitative elements. We can consider admissible or canonical decomposition. The canonical decomposition consists in specifying the components (factor) that are meant to be expressed so that it is as close as possible to reality. The ultimate goal in decomposing a time series is to estimate the signal, that is, the component to be estimated. Component Estimation aims at completing the process under review and finalizing this process. The use of the Burman-Wilson algorithm allows for efficient filtering even in the case of a small number of estimated and predicted values.

References

1. Anghelache, C., Anghel, M.G. (2016). *Bazele statisticii economice. Concepte teoretice şi studii de caz*, Editura Economică, Bucureşti
2. Anghelache, C., Anghelache, G.V. (2009). Utilization Of The Chronological Series Within The Stochastic Processes. *Metalurgia Internaţional, XIV* (4) Special Issue, Editura Ştiinţifică F.M.R., 154-156
3. Anghelache, C. (2008). *Tratat de statistică teoretică şi economică*, Editura Economică, Bucureşti
4. Bardsen, G. si colaboratorii (2005). *The Econometrics of Macroeconomic Modelling,* Oxford University Press
5. Benjamin, C., Herrard, N., Houée-Bigot, M., Tavéra, C. (2010). *Forecasting with an Econometric Model*, Springer
6. Ghysels, E., Osborn, D. (2001). *The Econometric Anallysis of Seasonal Time Series*, Cambridge University Press, United Kingdom
7. Müller, U.K. (2007). A Theory of Robust Long-Run Variance Estimation. *Journal of Econometrics, 141*, 1331-1352
8. Newbold, P., Karlson, L.W., Thorne, B. (2010). *Statistics for Business and Economics*, 7th ed., Pearson Global Edition, Columbia, U.S
9. Phillips, P.C.B., Sun, Y., Jin, S. (2006). Spectral Density Estimation and Robust Hypothesis Testing using Steep Origin Kernels without Truncation. *International Economic Review, 47*, 837-894.