ESSENTIAL ELEMENTS ON MODELING AND COMBINATION OF SUBSYSTEMS

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Abstract

In practical work, it is a problem for the built models to be based on the combination of subsystems that are in line with the structure of the phenomenon under analysis. In the development of large-scale macroeconomic models, the traditional approach is to express through an equation the possibility of determining the outcome. By adopting maximum probability methods with limited information, the parameters of an equation can be obtained, while the parameters of other equations remain restricted. After building the relevant submodels by marginalization and conditioning, reduced models of subsectors can be aggregated into a broad macroeconomic model. Macroeconomic models are used as the basis for establishing economic strategies, so it is important to model the coefficients of all explanatory variables. Therefore, obtaining long-term properties of subsystems is of particular importance. Thus, we can speak of analysis based on the inflation model and the Phillips curve. Following the discovery by Phillips of an empirical regularity between unemployment and wage growth, this was a novelty taken over by all of the most prominent researchers in the period ahead.

Keywords: *subsystem, Phillips curve, Lucas critique, coefficient, equation*

JEL Classification: C44, C61, E31

Introduction

In this article, based on the authors' study, it is intended to identify the essential elements used in the modeling and combining of the subsystems. Methods used by various researchers that led to the development of macroeconomic models based on which macroeconomic evolution strategies can be established are presented. Next, a comprehensive analysis of the inflation model and the Phillips curve is being used. Also, the aspects that depend on the Phillips curve application are highlighted. The relationship between wage growth and economic activity has been analyzed at macroeconomic level since the neo-classical period. An empirical version of the Phillips curve, called triangular inflation model, has also been used in current conditions, of course, with adjustments being made. The Phillips Salary Curve system in an open economy is a full specification of the inflation dynamics model.

Literature review

Anghelache, Anghel et al (2017) highlighted a series of elements regarding the use of the Philips curve in macroeconomic analyzes. Anghelache and Anghel (2014) and Anghelache (2012) analyzed the main aspects of economic modeling. Bjerkholt (2005) studied concepts and planning. Dickens (2008) examined methods of estimating the time variation. Ewing and Seyfried (2003) examined concepts of modeling the Phillips curve. Johansen (2002) studied hypothesis tests on cointegration vectors. Karanassou and Snower (2007) addressed aspects of the Philips Curve Review. Lee and Aaker (2006) presented a Monte Carlo study of growth regressions. Levy (2004) studied elements of frequency cointegration. Mandel and Tomšík (2003) presented the main aspects of the consumer function.

Research methodology, data, results and discussions

• In the development of macroeconomic models on a large scale, the traditional approach was to estimate an equation (or submodel) once and to establish the results simultaneously. Often, this was done without checking the suitability of the procedures. The approach could be justified in terms of estimates. By adopting the maximum probability methods with limited information, the parameters of an equation can be estimated, while the parameters of other equations remain unrestricted, as shown by Anderson and Rubin (1949) and Koopmans and Hood (1953).

It was argued that methods with limited information were stronger than the erroneous equations specified in the system, in situations where there were better theories or reliable information about a subset of variables. Limited information methods have been adopted for practical reasons to avoid the complexity of calculations specific to full-information methods - such as full-scale probability based on complete information.

After the construction of the relevant submodels by marginalization and conditioning, small models of subsectors are aggregated into a broad macroeconomic model.

A general theory for the three steps will invariably contain criteria and conditions that are formulated for the entire system, which raises the question if there are ways to solve when the complete model is too complex for a simultaneous modeling.

One solution would be to use models with a high degree of aggregation that are small enough to be analyzed as a complete system. In such an approach, several economic mechanisms, important and relevant to the description of the economy are omitted.

The general approach can be seen as a gradualism - following the establishment of the structure in the submodels.

The first two conditions in the definition of the partial structure do not require full knowledge of the model. Excluding important explanatory variables from the model leads to the invalidity of the submodels, which is detected in the submodel when the correlation between the included variables and those that were omitted is changed.

For the latter, it is possible, at least in principle, to develop a model as the last extension of the information, thus establishing that the structure or partial structure is a way to solve Johansen's problem. Basically, it is known that the complete model is inaccessible. However, conditional consumption is constant when the survey interval is extended by nine years of quarterly observations; remains unaltered over the period of financial deregulation and supports the simultaneous modeling of private consumption, household disposable income, population wealth, and real estate prices. Thus, the inductive proof can be validated that the conditional consumption function will represent a partial structure. The simultaneous model is hardly an ideal substitute for a better model for "bid" effects that operate through the labor market, yet it provides protection against alternative views. For example, income is in fact the correction of balance, not consumption.

Macroeconomic models are used as the basis for economic policy making. From this point of view, it is important to model the coefficients of all relevant explanatory variables by conditioning all knowledge (relevant and applicable) on institutional conditions in the studied society. We rely on higher aggregation specifications where gross coefficients create combined effects of the included variables, and correlated omitted variables can lead to wrong policy recommendations. Such interference is more damaging to the governors than the simultaneity interference that can occur by combining the submodels. Whether it remains valid or not an interesting issue that can be explored through Monte Carlo simulations on particular model specs.

It follows that obtaining the long-term properties of the submodels is of special importance. Once a cointegration equation is determined, it is invariant to extensions of the set of information. On the other hand, this is a property that needs to be determined in each case. The concepts of separation in cointegrated systems are described by Granger and Haldrup (1997). The idea is to break down each variable into a persistent component and a transient component. In a vector equilibrium correction model, two subsystems are considered, where the variables of a subsystem do not include cointegrating equations of the other subsystem (cointegration separation). However, short-term effects of variables in a subsystem on the other variables may occur and co-integrative equations of a system may affect the short-term development of the other's variables. The absence of both types of interaction is called complete separation, whereas if only one of them is present, it becomes partial separation. These concepts are closely related to the exogenity of the variables in a subsystem on the parameters of the other. Both submodels, partially or completely separated, are verifiable assumptions that need to be tested as part of cointegration analysis. Hecq et al. (2002) expand the results of Granger and Haldrup (1997). The conclusion of Hecq et al. (2002) is, however, that verification of separation requires the whole system to be known, which corresponds to Johansen's observation.

• The inflation model and the Phillips curve originated in the same era of macroeconomics. But while Aukrust's model departed from academia, literature on the Phillips curve developed in the 1960s and made a tremendous impact over the next four decades. Below are some of the significant steps in Phillips curve developments. In the 1970s, the Phillips curve and the Aukrust model were considered as alternatives, representing the "demand" and "supply" inflation model (Frisch 1977). However, as Aukrust (1977) points out, the difference between considering the labor market as an important source of inflation and paying particular attention to the product market Phillips curve is more a matter of principle than the principle, the two mechanisms can work together. Next, it shows how the two approaches formally can be combined, giving the Phillips curve the short-term relationship of nominal wage growth, while the main thesis is valid in the long run.

Within the chapter are addressed key issues for the application of the Phillips curve in the current context, its representation in a system of cointegrating variables; consistency and changes in the unemployment rate; the uncertainty of the Phillips curve estimated NAIRU (Non-Accelerating Inflation Rate of Unemployment); and the inversed Phillips curve status, respectively Lucas's supply curve.

• Following the discovery by Phillips of an empirical regularity between unemployment and wage growth in the UK, the Phillips curve was integrated into macroeconomics through a series of works in the 1960s. Samuelson and Solow (1960) interpreted it as a compromise faced by the governors, and Lipsey (1960) was the first to estimate the Phillips curves with multivariate regression techniques. Lipsey interpreted the relationship from the perspective of classical price dynamics with the unemployment rate acting as an intermediary between excess demand and labor market friction. Importantly, Lipsey has included rising consumer prices as an explanatory variable in its regressions and so formulated what became known as the Phillips curve of probabilities. Subsequent developments include the distinction between the short-term Phillips curve where inflation deviates from projected inflation and the long-term Phillips curve where inflation forecasts are met. Finally, the concept of a natural rate of unemployment was defined as the stable unemployment rate, corresponding to a long-term vertical curve (Phelps 1968 and Friedman 1968).

The relationship between wage growth and economic activity also appears in neo-classical macroeconomics (Lucas and Rapping, 1969 and 1970 and Lucas 1972). However, in the neo-classical economy, the causal relationship of Phillips's original model was overthrown: if there is a correlation between inflation and unemployment in absolute terms, causality goes from inflation to activity and unemployment. The inversion of Lucas and Rapping is based on the appreciation that the price level is anchored in the relationship between quantitative theory and the autonomous stock of money. The rise in prices and wages is determined outside the Phillips curve, so the correct wording is that on the one hand there is the unemployment rate and wage increases (and / or inflation) on the other.

Lucas (1972) launches another known approach, based on rational probabilities about relatively uncertain product prices. If the probabilities are met (on average), the aggregate offer is unchanged over the last period. However, if there are surprise prices, there is a distance from the average level of long-term production. This establishes a “surprise only” bidding relationship.

Lucas's bidding function is the opposite of the long-term Phillips vertical curve, augmented by Lipsay's predictions, but derived from microeconomic theory and the rational probability hypothesis.

In addition, for the conventional aggregate demand specification (Romer 1996), the model involves a positive association between output and inflation or a negative relationship between unemployment and inflation. Thus, there is a neo-classic correspondence for the short-term Phillips curve. However, Lucas's demand curve, when applied to data and is estimated by the least squares (OLS) method, is not a causal relationship that can be exploited by the governors. On the contrary, it will change when, for example, money supply has increased to stimulate production in a way that leaves politics without the possibility of influencing real output or unemployment.

This concept is called Lucas's critique (Lucas 1976), which was formulated as a critique of the inflation-unemployment compromise of the Phillips curve, which was dealt with in the academic literature as well as in the 1970's macroeconomic models (Wallis 1995). The force of criticism, however, comes from its generality: it is a potential hazard for all conditioned econometric models.

The issue of causality is manifested in connection with the latest versions of the Phillips Curve - the Neo-Keynesian version.

In the US, an empirical version of the Phillips curve, called the "Triangular Inflation Model," spread despite Lucas's critique (contributions to this effect are found in Gordon 1983 and 1997, Staiger et al., 2001). As we will continue to argue, an explanation of the validity of the American Phillips curve is that unemployment rate shocks were generally of less magnitude than in European countries.

As we have shown before, there are many ways in which a Phillips curve for an open economy can be deduced from economic theory. Our appreciation of the Phillips curve is based on Calmfors (1977), which reconciled the Phillips curve with the Scandinavian inflation model. We intend to take a step forward, however, and incorporate the Phillips curve into a framework that takes into account wage and price data series. Reconstructing the model in terms of cointegration and causality reveals that the Phillips curve version of the master model imposes a mechanism to correct the balance on the system. Thus, while consistent with Aukrust's main theory, the Phillips curve is also a special model because it includes only one of the wage setting mechanisms discussed by Aukrust.

Without departing from generality, we will focus on salaries in the Phillips curve and recall that, in accordance with Aukrust's theory, it is assumed that:

1. $(w_{\varepsilon,t} - q_{\varepsilon,t} - a_{\varepsilon,t}) \sim I(0)\ si\ (u_t) \sim I(0)$, possibly after removal of deterministic changes; and

2. the causal structure is „a path" represented by $H4_{mc}$ and $H5_{mc}$.

Consistency with assumed cointegration and causality requires a balance correction model for the nominal wage rate in the exposed sector. Assuming a first order dynamic for simplification, a Phillips curve system is defined by the following two equations:

$$\begin{aligned} &\bullet\ \Delta w_t = \beta_{w0} - \beta_{w1} u_t + \beta_{w2} \Delta a_t + \beta_{w3} \Delta q_t + \varepsilon_{w,t} \\ &0 \leq \beta_{w1}, 0 < \beta_{w2} < 1, 0 < \beta_{w3} < 1 \\ &\bullet\ \Delta u_t = \beta_{u0} - \beta_{u1} u_{t-1} + \beta_{u2} (w - q - a)_{t-1} + \beta_{u3} z_{u,t} + \varepsilon_{u,t} \\ &0 < \beta_{u1} < 1,\ \beta_{u2} > 0, \beta_{u3} \geq 0 \end{aligned} \tag{1}$$

where (notation is simplified by renunciation to „e").

ε_{wt} and ε_{ut} are innovations regarding information available during t-1, marked I_{t-1}. Equation (1) is the basic idea that profitability (in sector e) is a factor explaining the change in the unemployment rate.

Z_{ut} represents other $I(0)$ variables (and deterministic terms) which, if the other factors do not change, will lead to the decrease of the unemployment rate. The factor will typically include a measure of the rate of growth of the economy, and other factors linked to the supply of labor.

To determine the main rate of equilibrium unemployment, equation (1) is rewritten as a form:

$$\Delta w_t = -\beta_{w1}(u_t - \breve{u}) + \beta_{w2}\Delta a_t + \beta_{w3}\Delta q_t + \varepsilon_{w,t} \tag{2}$$

where

$\breve{u} = \frac{\beta_{w0}}{\beta_{w1}}$ is the unemployment rate that does not affect wage growth.

Using unconditional environments, denoted by E, we get on both sides of (3):

$$E[\Delta w_t] - g_f - g_a = -\beta_{w1}E[u_t - \breve{u}] + (\beta_{w2} - 1)g_a + (\beta_{w3} - 1)g_f \tag{3}$$

Using the assumption of a fixed wage weight, the left is zero. Thus, using g_a and g_f as notations of steady growth rates of productivity and external prices, we obtain:

$$E[u_t] \equiv u^{phil} = (\breve{u} + \frac{\beta_{w2}-1}{\beta_{w1}} g_a + \frac{\beta_{w3}-1}{\beta_{w1}} g_f) \tag{4}$$

as a solution for the equilibrium unemployment rate, noted u^{phil}. The long-term average of the salary weight is accordingly:

$$E[w_t - q_t - a_t] \equiv wsh^{phil} = -\frac{\beta_{u0}}{\beta_{u2}} + \frac{\beta_{u1}}{\beta_{u2}} u^{phil} + \frac{\beta_{u3}}{\beta_{u2}} E[z_{u,t}] \tag{5}$$

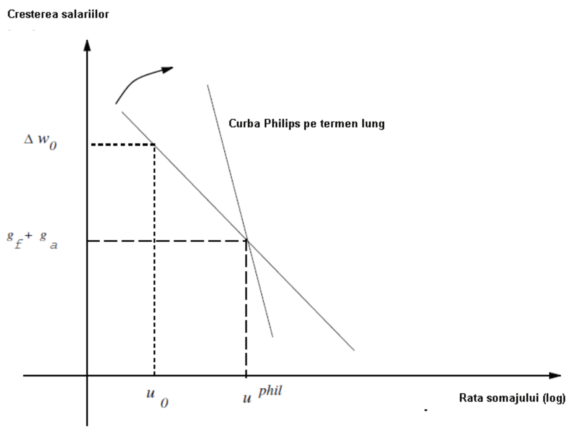
In addition, u^{phil} and wsh^{phil} represent the stable and unique state of the corresponding pair of deterministic difference equations.

The known shape of the Phillips curve is illustrated in Figure 1. It is assumed that the economy initially exhibits a low level of unemployment, ie uo in the figure. The short-term Phillips curve determines the wage growth rate Δw_0. The share of wages according to equation (1) is above the long-term equilibrium value, implying that unemployment is rising and wage growth decreases along the Phillips curve. The steep shape of the Phillips curve is defined for the case $\Delta w_t = \Delta q_t + \Delta a_t$.

The slope of the curve is given by $-\beta_{w1}/(1- \beta_{w3})$, being referred to as the Phillips curve in the literature. The stable balance is reached when the salary increase equals constant steady growth, ie, g_f+g_a and the unemployment level is given by the u^{phil}.

Dynamics and balance of the Phillips curve in an open economy

Figure 1



The problem of the long-term Phillips curve is seen as depending on the β_{w3} coefficient, the elasticity of wage growth determined without regard to the price of the products. In the figure, the long-term curve has a decreasing tendency, ie β_{w3} <1, which conventionally refers to a dynamic inhomogeneity in wage setting. Conversely, dynamic homogeneity implies β_{w3} = 1 and a vertical Phillips curve. Given the dynamic homogeneity, the u^{phil} equilibrium rate is independent of global inflation g_f.

The long-term slope of the Phillips curve was one of the most debated problems in macroeconomics in the 1970s and 1980s. One argument in favor of the long-term vertical curve Phillips is that it has clearly been noticed that workers are able to obtain full compensation for inflation. It follows that β_{w3} = 1 is a normal restriction on the Phillips curve, at least if Δq_t is interpreted as a variable of probabilities. The downward slope of the Phillips curve in the long run has been challenged on the grounds that it offers a picture too optimistic about economic policy: that the government can permanently reduce unemployment levels below the natural rate by „fixing" a high level of inflation.

In an open economy, this discussion appears to be somewhat exaggerated because the long-term compromise between inflation and unemployment does not come from the premise of a long-term downward curve. In contrast, according to Figure 1, the stable level of unemployment is determined by the imported inflation rate g_f and the increase in exogenous productivity g_a. These two indicators are not considered as means (or intermediate targets) of economic policy.

In the real economy, cost-of-living constraints play a significant role in wage setting. Thus, applied econometric research usually includes current and delayed inflation, reflecting the emphasis on cost-of-living considerations in wage negotiations. To represent this possibility, the following system is considered:

$$\Delta w_t = \acute{\beta}_{w0} - \acute{\beta}_{w1} u_t + \acute{\beta}_{w2} \Delta a_t + \acute{\beta}_{w3} \Delta q_t + \acute{\beta}_{w4} \Delta p_t + \acute{\varepsilon}_{w,t} \quad (6)$$

$$\Delta u_t = \beta_{u0} - \beta_{u1} u_{t-1} + \beta_{u2} (w - q - a)_{t-1} + \beta_{u3} z_t + \varepsilon_{u,t} \quad (7)$$

$$\Delta p_t = \beta_{p1} (\Delta w_t - \Delta a_t) + \beta_{p2} \Delta q_t + \varepsilon_{p,t} \quad (8)$$

In order to achieve a formal distinction, a distinctive sign is used above the other coefficients (and above the term deviation). The second equation is identical to the unemployment equation (1). The last equation of stochastic prices combines the definition of consumer prices with the identical assumption of the stability of the wage weight in the covered sector and the increase in wages in the exposed sector.

Using (8) to remove Δp_t in (6), we return to the equation with coefficients, with coefficients and ε_{wt} redefined accordingly. It is useful to express uphil in terms of coefficients of an extended system (6) - (8):

$$u^{phil} = \breve{u} + \frac{\acute{\beta}_{w2} - 1}{\acute{\beta}_{w1}} g_a + \frac{\acute{\beta}_{w3} + \acute{\beta}_{w4} (\beta_{p1} + \beta_{p2}) - 1}{\acute{\beta}_{w1}} g_f \quad (9)$$

because there are two homogeneous restrictions required for the long-term Phillips curve, namely: $\acute{\beta}_{w3} + \acute{\beta}_{w4}$=1 and $\beta_{p1} + \beta_{p2} = 1$.

The Phillips salary system in an open economy is a complete specification of the dynamics of the inflation model. Clearly, the dynamic properties of the model apply to other versions of the Phillips curve. In particular, all Phillips curve systems assume that the natural rate of unemployment (NAIRU) is a stable solution. As a single equation, the *equation* of the Phillips curve is unstable for a given rate of unemployment. The dynamic stability of wage and unemployment rates depends on the integrated equalization mechanism in the unemployment rate equation. In this respect, a definition of wage formation based on the Phillips curve can not be adapted to an economic policy that targets the unemployment rate, since only the natural rate of unemployment corresponds to a stable wage weight. Any other level (targeted) leads to a continuous increase or decrease of the salary weight.

Dynamic stability of the natural rate is a subject of interest and can not be addressed in an incomplete Phillips curve system, that is, by estimating a Phillips curve model with a single equation. But there are practical approaches where estimating the natural rate of unemployment is based on such incomplete systems. Staiger et al. (1997) is an important study example following the tradition of estimating only the Phillips curve, considering the implicit balancing mechanism (1). For other countries, especially European ones, where the instability of the unemployment rate is stronger, the problem of the correspondence between estimated rates and stability is a topical issue.

Conclusion

Following the study, a wide-ranging analysis of the work of large researchers in the field of modeling and macroeconomic analyzes has produced relevant elements that can be used in current analysis models. From the Phillips curve, aspects such as Phillips curve dynamics and balance are emerging in an open economy governed by free market laws. Dynamic determination of the natural rate is a topic of interest and must therefore be estimated by a Phillips model with a single equation. Although the analysis of the natural rate of inflation is based on incomplete systems, it is still possible to identify the evolutionary trend of the unemployment rate. It is concluded that in the real economy models resulting from the cohabitation of the subsystems lead to obtaining parameters that can be used in macroeconomic analyzes.

References

1. Anghelache, C., Anghel, M.G. et al (2017). *Using the Philips Curve in Macroeconomic Analysis*, Romanian Statistical Review, Supplement, no. 7, pp. 3-15 / 16-28,
2. Anghelache, C., Anghel, M.G. (2014). *Modelare economică. Concepte, teorie şi studii de caz*, Editura Economică, Bucureşti
3. Anghelache, C. (2012). *Elemente de modelare economică*, Editura Artifex, Bucureşti
4. Bjerkholt, O. (2005). *Markets, models and planning: the Norwegian experience,* Oslo University, Department of Economics in series Memorandum
5. Dickens, W.T. (2008). *A New Method to Estimate Time Variation in the NAIRU*, Federal Reserve Bank of Boston in Conference Series ; [Proceedings]
6. Ewing, B.T., Seyfried, W.L. (2003). Modeling the Philips Curve: A Time-Varying Volatility Approach. *Applied Econometrics and International Development*, vol 2-3, 7-24
7. Johansen, S. (2002). A small sample correction for tests of hypotheses on the cointegrating vectors. *Journal of Econometrics*, Elsevier, 111(2), 195-221
8. Karanassou, M., Snower, D. (2007). Inflation Persistence and the Philips Curve Revisited, Kiel Institute for the World Economy in Kiel Working Papers series
9. Lee, A.Y., Aaker, J.L. (2006). *A Monte Carlo Study of Growth Regressions*, Stanford University, Graduate School of Business in Research Papers series
10. Levy, D. (2004). *Cointegration in Frequency Domain*, EconWPA in its series Econometric Haldane, A., Quah, D. (2000) – "UK Philips Curves and Monetary Policy", Centre for Economic Performance, London School of Economics in CEP Discussion Papers
11. Mandel, M., Tomšík, V. (2003). *The Consumption Function and Ricardian Equivalence in a Small Open Economy*, University of Economics, Prague in Politická ekonomie.