ANALYSIS OF SOME ASPECTS OF THE LUCAS CONCEPT

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Abstract

In this article, the authors sought to highlight some definitive aspects of the Lucas model compared to its Philips model. The autoregressive nature of the dVAR model has become an important tool in the macroeconomic analysis known as Box-Jenkins chronological series and ARIMA models. A number of economists have sought to explain how predictions can be made, concluding that the dVAR model provides robust forecasts for unstable chronological series that are subject to intermittent changes over time. Starting from some of Lucas’s criticisms of the Philips curve, the authors sought to critically highlight some aspects of the Lucas curve. In this regard, it starts from the fact that this Lucas curve turns the causality of the conventional Philips curve into a different optics than the natural one, because the Philips curve is commonly used for inflation analysis. Lucas also states that conventional Philips curve models include structural discontinuities that can lead to economic changes. It is argued that changes in economic policy can lead to evolutionary changes. Of course, practical life, but especially the theoretical concept, have some divergences that we have sought to exemplify in this article. Conditional econometric models are theoretically subject to instability and sometimes to failure if unmodulated probabilities, not taken into account, change. Either in this article we tried to demonstrate mathematically what this possibility is. The article starts from the fact that Lucas’s critique is aimed at behavioral equations of systems of simultaneous equations that need to be analyzed in a certain context, in a certain macroeconomic situation. The authors also emphasized the comparison of the probabilities of the results obtained with the data base model, ie the classic models that respond to ideas and the Philips model submitted to the Lucas critics, which lead to a more just possibility to analyze the situation under analysis. An example is given in the article with two agents A and B about their prognoses and mathematical explanation, we come
to the conclusion that the criticisms expressed by Lucas are limited in many circumstances. While it is logically possible for conventional Philips curves to actually be reversed Lucas functions, it can be verified only for specific models. It has been concluded that Philips curve is stable over periods of time that included regime changes and structural changes that invoke and change the probability of realization. In essence, the authors conclude that this Lucas critique is a possibility theorem and not a specific one for hypotheses that have verifiable implications.

Keywords: macroeconomics, chronological series, probability, Lucas curve, critical analysis.

JEL Classification: C61, E03

Introduction
dVAR autoregressions have become tools of tradition in macroeconomics, in the form of Box-Jenkins chronological series and ARIMA models. The differential vector autoregressive vector dVAR tends to win in the forecast competitions. The explanation is based on the above forecasting comparison: dVAR provides solid prognoses for unstable chronological series that are subject to intermittent regime changes. In order to compete with them, the user of a macroeconomic model must regularly apply segment corrections and corrections of other opinions. An important contribution to verifying Lucas’s critique by dedicated rational methods belongs to Engle and Hendry (1993). Surveys of Lucas’s empirical demonstration are treated by Ericsson and Irons (1995) and Stanley (2000). Although very different in methodology, the two studies conclude in a similar fashion that there is little evidence to support the applicability of Lucas’s critique to the Norwegian Philipps curve. As an alternative to rational probabilities, we note the possibility for agents to establish probabilities based on the observed data properties. Interestingly, there is a close relationship between data-based forecasting rules that agencies can use and chronological series models that have been successful in macroeconomic forecasting.

Literature review
studies that highlight their assumptions. Martin (2013) studies the situation of asset prices behavior in the case of an endowment economy. Aghion, Howitt and Murtin (2011) emphasize the correlation between health and growth, which is weaker than in the 1960s, and the growth in productivity is influenced by mortality decrease associated to the below 40 age group. Ferri (2000) develops on the dynamics of wages in the context of the Phillips curve. Chang and Kim (2013) analyze the heterogeneity of the labor market. Einav et al (2016) evaluate the economic content of risk scores, considering the expansion of statistical and data analysis instruments in the economic studies. Stanley (2000) comments on the Lucas critique, on basis of empirical evidence, and explains how Lucas critique can be contradicted by applying proper models. Anghelache and Anghel (2016) have presented the statistical instruments applicable for economic analyses, from both theoretical and practical viewpoints. Carrell, Sacerdote and West (2013) demonstrate that peer group’s manipulation in the purpose of achieving a desired result can be influenced by social relationships in the group. Gruen, Pagan and Thompson (1999) have analyzed the behavior of the Phillips curve in the Australian economy for the reference interval of forty years, considering both prices and cost of unit labor. Gonçalves and Vogelsang (2011) have developed on the case of HAC robust tests. Müller and Watson (2008) have considered the testing for models characterized by low-frequency variability.

Research methodology, data, results and discussions

The reversal of dependent and independent variables is a continuing controversy in the literature on inflation modeling. The way in which the Lucas curve turns the causality of the conventional Phillips curve was previously shown. Lucas’s critique also states that the conventional Phillips Curve models will experience structural discontinuities whenever economic prospects change (for example, following a change in economic policy).

- In a hypothesis of superexogenicity, the results for a conventional econometric model, such as the conventional Phillips curve, are not invariant to a renormalization according to the formula:

\[
\hat{\beta} : \hat{\beta}^* = r_{yx}^2
\]

where:
\[ r_{yx} = \text{coefficient of correlation}; \]
\[ \hat{\beta} = \text{estimated regression coefficient when } y \text{ is the dependent variable} \]
\[ \text{and } x \text{ is the regression factor}; \]
\[ \hat{\beta}^* = \text{estimated coefficient in inverse regression}. \]

The regime changes cause alterations of correlation structures, so changes of \( r_{yx} \). If, owing to superexogenicity, \( \hat{\beta} \) it is still constant, then \( \hat{\beta}^* \) it can not be constant.
The working equation generally applies to $r_{yx}$ interpreted as a partial correlation coefficient. So if, for example, the Phillips curve is estimated by OLS, then finding that $\hat{\beta}_{w1}$ is stable entails that $\beta^*$ for the re-normalized equation (at the unemployment rate) is unstable. Finding that a stable Phillips curve on a survey period that contains changes in partial correlations denies any claim that the model has an interpretation of Lucas’s demand curve. This simple procedure also applies to estimation by instrumental variables (due to endogenity $\Delta q_t$ and / or $\Delta p_t$), provided the number of instrumental variables is lower than the number of endogenous variables in the Phillips curve.

- Conditional econometric models are prone to instability and failure whenever undefined probabilities change. Lucas’s criticism can be confirmed or rejected empirically.

Generally, it is considered to be a single chronological (random) $y_t$ variable that can be divided into an explanation part $y_t^P$ and an unexplained independent part $\varepsilon_{y,t}$:

$$y_t = y_t^P + \varepsilon_{y,t}$$  \hspace{1cm} (2)

We consider $y_t^P$ as a plan attributable to the agents and $\varepsilon_{y,t}$ - the difference between the planned and the actual $y_t$ results. So:

$$E[y_t | y_t^P] = y_t^P$$  \hspace{1cm} (3)

$\varepsilon_{y,t}$ is an innovation related to the plan, so:

$$E[\varepsilon_{y,t} | y_t^P] = 0$$  \hspace{1cm} (4)

We further assume that agents use a lot of information $I_{t-1}$ for rational probabilities for the $x_t$ variable, ie:

$$x_t^P = E[x_t | I_{t-1}]$$  \hspace{1cm} (5)

and probabilities are connected to:

$$y_t^P = \beta x_t^P$$  \hspace{1cm} (6)

which is motivated by economic theory.

From the formula assuming $\varepsilon_{y,t}$ of (2) is an innovation, we obtain:

$$E[\varepsilon_{y,t} | I_{t-1}] = 0$$  \hspace{1cm} (7)

and therefore:

$$E[y_t | I_{t-1}] = y_t^P$$  \hspace{1cm} (8)

Initially, $x_t^P$ is assumed to follow a first-order AR process (inertia / instability is still considered):

$$x_t^P = E[x_t | I_{t-1}] = \alpha_1 x_{t-1}, \hspace{1cm} |\alpha_1| < 1$$  \hspace{1cm} (9)

So,

$$x_t = x_t^P + \varepsilon_x, \text{ or}$$  

$$x_t = \alpha_1 x_{t-1} + \varepsilon_{x,t} \text{ s.t. } E[\varepsilon_{x,t} | x_{t-1}] = 0$$  \hspace{1cm} (10)

For simplicity, we assume that $\varepsilon_{y,t}$ and $\varepsilon_{x,t}$ are independent.

We will still assume that the only parameter of interest is $\beta$. The reduced form of $y_t$ is obtained from equations (2), (6) and (9):

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$y_t = \alpha_1 \beta x_{t-1} + \epsilon_{x,t}$  
(11)

where $x_t$ is weakly exogenous for $\xi = \alpha_1 \beta$, but the parameter of interest $\beta$ is unidentifiable only from (11). In addition, the reduced form equation (11), while allowing estimation of $\xi$ in a state of nature characterized by stability, is susceptible to Lucas’s criticism because $\xi$ is not invariant to changes in the autoregressive parameter of the marginal model (9).

Basically, Lucas’s critique aims at „behavioral equations” in systems of simultaneous equations, for example,

$y_t = \beta x_t + \eta_t$  
(12)

with the term deviation:

$\eta_t = \epsilon_{y,t} - \epsilon_{x,t} \beta$  
(13)

It can be directly shown that the estimation (12) by OLS on a sample $t = 1, 2, ..., T$, leads to:

$\lim_{T \to \infty} \beta_{OLS} = \alpha_1^2 \beta$  
(14)

establishing that the regression $y_t$ to $x_t$ is not the opposite $y_t^P = x_t^P \beta$ of (6).

Specifically, instead of $\beta$, estimating $\alpha_1^2 \beta$ changing the $\alpha_1$ parameter affects the stability of estimates, thus confirming Lucas’s criticism.

However, the applicability of the critique is based on the initial assumptions. For example, if the hypothesis is changed from $|\alpha_1| < 1$ to $\alpha_1 = 1$, so that $x_t$ has a single root but is cointegrated with $y_t$, Lucas criticism does not apply: in cointegration, $\lim_{T \to \infty} \beta_{OLS} = \beta$ since the cointegration parameter is unique and can be estimated consistently by OLS.

As another example of the importance of the exact set of hypotheses made, it is considered to be a replacement (6) with another economic theory, namely the contingent plan:

$y_t^P = \beta x_t$  
(15)

Equations (15) and (2) lead to:

$y_t = \beta x_t + \epsilon_{v,t}$  
(16)

where $E[\epsilon_{y,t} | x_t] = 0 \Rightarrow \text{cov}(\epsilon_{y,t}, x_t) = 0$ and $\beta$ can be estimated by OLS in the case of $|\alpha_1| < 1$.

- Comparing the probabilities of model-based and data-based results

  Apparently, it is often forgotten that the formulation of the „classic” regression (16) corresponds to the idea: „behavior is driven by probabilities, but not by model-based probabilities or rational probabilities with unknown parameters that need to be estimated.”

  To interpret the probabilities in (16), the relationship (5) is replaced by:
\[ y_{t+1}^P = \beta x_{t+1}^e \] and supposedly resolves \( \Delta x_{t+1}^e = 0 \) to get \( x_{t+1}^e \). Substituting \( x_{t+1}^e = x_t \) and using (2) leads to \( y_{t+1}^P \) (16).

\( \Delta x_{t+1}^e = 0 \) is an example of a non-parametric invariant prediction rule based on data properties or data-based probabilities. In realistic terms, it is possible to choose the use of data-based predictors due to the cost of collecting and processing information associated with model-based predictors.

It is true that approaches based on \( \Delta x_{t+1}^e = 0 \) use a specified wrong model of the \( x \) (10) process, and so their forecasts will not reach the predicted average predictive error. So, under the conditions of stability, there are gains from the estimate \( \alpha_1 \) of (10). However, there is virtually no guarantee that the parameters of the \( x \) process remain constant until the forecast horizon and the unstable state of a model-based forecast can not be considered as better than the prognosis derived from the simple rule \( \Delta x_{t+1}^e = 0 \). In fact, depending on the date of the change in the production-forecasting regime, the data-based forecast will be better than the model-based forecast in terms of interference.

We introduce a growth term in (10), ie:
\[ x_t = \alpha_0 + \alpha_1 x_{t-1} + \xi_{x,t} \quad \mathbb{E}[\xi_{x,t}|x_{t-1}] = 0 \] (18)
and assume there is a change in \( \alpha_0 \) during the \( T+1 \) period.

We consider two agents, A and B, who forecast \( x_{T+1} \). Agent A collects \( x_t \) for \( t = 1, 2, 3, \ldots, T \) and can find real values of \( \{\alpha_0, \alpha_1\} \) or that period. However, due to the unpredictable change in the \( T+1 \) period, the predictive error of A will be:
\[ e_{A,T+1} = \alpha_0^* - \alpha_0 + \xi_{x,T+1} \] (19)
Agent B, using the \( x_{T+1} = x_T \) data forecast, will encounter a forecast error:
\[ e_{B,T+1} = \alpha_0^* - (1 - \alpha_1) x_T + \xi_{x,T+1} \]
which can be expressed as
\[ e_{B,T+1} = \alpha_0^* - \alpha_0 + (1 - \alpha_1) (x_T^0 - x_T) + \xi_{x,T+1} \] (20)
where \( x_T^0 \) is average \( x_T \) (ie, for the segment before the change \( \phi_0 \), \( x_T^0 = \alpha_0/(1 - \alpha_1) \)). The comparison between relations (19) and (20) shows that the only difference between the two forecast errors is the term \( (1 - \alpha_1)(x_T - x_T^0) \). Thus, both forecasts are affected by the regime change that occurs after the forecast is made. Conditioned media and dispersions (distributions) of the two errors are:
\[ \mathbb{E}[e_{A,T+1}|T] = \alpha_0^* - \alpha_0 \] (21)
\[ \mathbb{E}[e_{B,T+1}|T] = \alpha_0^* - \alpha_0 + (1 - \alpha_1)(x_T^0 - x_T) \] (22)
\[ \mathbb{V}[e_{A,T+1}|T] = \mathbb{V}[e_{B,T+1}|T] \] (23)

Establishing that in this example (considering a post-prognosis change), there is no correlation between the two forecasting methods for the
first two moments of the forecast error. Conditional forecast error dispersions (distributions) are identical and model-based prognosis influences are not lower than those of the data-based predictor; Suppose, for example, that $\alpha_0^* > \alpha_0$ at the same time $x_T < x^*_T$, data-based interference may be less than model-based interference. In addition, unconditionally, the two predictors have the same influence and dispersion:

$$E[e_{a,T+1}] = E[e_{x,T+1}] = \alpha_0^* - \alpha_0$$
$$\text{Var}[e_{a,T+1}] = \text{Var}[e_{x,T+1}]$$

We continue to consider the forecasts for the $T + 2$ period, provided $T + 1$ as an example of an ante-prognosis regime change ($\alpha_0 \rightarrow \alpha_0^* \rightarrow \alpha_0^*$ in the T+1 period). Unless A discovers the change in $\alpha_0$ and successfully corrects the forecast, the impact of the error will be again:

$$E[e_{a,T+2}] = E[e_{x,T+2}] = \alpha_0^* - \alpha_0$$

The influence of forecasting error on agent B on the other hand becomes

$$E[e_{b,T+2}] = (1 - \alpha_1)(x^*_x - x_T),$$

where $x^*_x$ is the unconditional medium change of post-regime change $x$, i.e. $x^*_x = \alpha_0^*/(1 - \alpha_1)$.

Clearly, the data-based predictor interference may be less than the predictive model-based error (but the opposite may be true). However,

$$E[e_{b,T+2}] = CE[e_{b,T+2}] = C$$

Unconditional forecast errors are always lower for predictions based on data in the case of ante-progression mode change.

The analysis generalizes the case of a unit root in the $x$ process, in fact it is remarked from previous ones that the data-based forecast errors have better properties for the case $\alpha_1 = 1$, for example $E[e_{b,T+2}] = 0$ of (16). If $x_T$ is $\{d\}$, then resolving $\Delta^d x_T = 0$ to obtain $x_T^{(d)}$ will result in a prognosis with the same regime change stability as illustrated in the previous example. This predictor class belongs to forecasting patterns that are spread in terms of differences in original data, thus, vector-differentiated autoregressions, called dVAR.

- **Verifying the significance of Lucas’s critique**

While it is logically possible for conventional Phillips curves to actually be reversed Lucas functions, it can be checked for specific models. The opinion that the Phillips curve is stable over sample periods that included regime changes and changes in correlation structures is sufficient to reject the inversion. At the same time, Lucas’s critique is a theorem of possibility, not a truism, and its assumptions have verifiable implications.

For example, Lucas’s criticism implies: $\hat{\beta}_{\text{OLS}}$ is a non-constant while $\alpha_1$ changes (within a single cycle), and determinants $\alpha_1$ (identifiable in
practice) should affect $\beta_{OLS}$ if they were included in the $y_t$ conditional model $\gamma_t$.

Instead, Lucas’s critique is inconsistent with the common view of stable conditional relationships and the change of regime that takes place in the process leading to an explanatory variable. Based on this logic, rational methods for verifying Lucas’s critique have been developed.

**Conclusion**

The author’s study of the Lucas concept aims to establish the concrete framework that can ensure the use of both models (curves) in macroeconomic analyzes. From the way this analysis was carried out, some conclusions regarding how to use the two models can be applied and can be useful in the macroeconomic analysis. A first conclusion is that an alternative to rational probabilities is the possibility for agents to set the evolution perspective based on the data series recorded in earlier periods. There is a close relationship between data-based forecasting rules that agencies can use and chronological series models can provide a macroeconomic forecast and, as a consequence, a structural forecast of macroeconomic aggregates. Conditional econometric models are prone to instability whenever unmodified probabilities change. In this context, Lucas’s viewpoint can be confirmed or rejected based on empirical database analysis. The article concludes that the dVAR and EqCM models should also be analyzed from the point of view of Lucas, especially when it comes to macroeconomic analysis.

**Selective references**