ASPECTS OF OPTIMAL MONETARY AND FISCAL POLICIES

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Abstract

This article looks at a detailed approach to optimal government policies, given the implications for monetary policy and fiscal policy. The analysis has three distinct stages. In the first phase of analysis / modeling, we abstain from physical capital, in which case the economy tends towards a steady state, and optimal government policy can be analyzed in static (pure) state. In the second phase, we introduce the capital (variable) notion, thus reintroducing the dynamics into the analysis model. Finally, we will look at some aspects of the timeliness of optimal policies and the continuity of governmental policy (monetary and fiscal) decisions.

Keywords: monetary policies, tax policies, consumption, welfare, macroeconomic Balance

JEL Classification: E52, E62

Introduction

An important role for governments is to implement policies that lead to goals. Assuming governments are willing and guiding their policies towards the well-being of the citizens they represent, then policies can be evaluated through the welfare of representative agents (bringing together both individuals and companies). These issues were introduced in the literature on macroeconomic studies under the title of „Economic Policy Theory” which were originally dealt with by Timberger (1952) and Teil (1958) followed by Turnovsky (1977), Atkinson and Stiglitz (1980), and and others.

In this article, we will try to have a detailed approach to optimal government policies, focusing both on monetary policy and fiscal policy, but also on the compromise between them. Our approach has three distinct stages. Thus, in the first phase, we abstain from physical capital, in which case the dynamics of the economy decreases (degenerates) and the economy is in a steady state. This
approach has the advantage that optimal government policy can be analyzed in a static (pure) state, which allows us to use the traditional approach to public finances by using the Ramsey taxing method. In the second phase, we introduce the physical capital variable, by which we reintroduce the dynamics in the previously considered equilibrium state. Although the analysis becomes more complicated, we can still draw conclusions on some of its main points. Finally, we will consider studying aspects of time consistency of optimal policy and drawing conclusions.

**Literature review**

concerned about the evolution of liquidity and international exchange rates of schib. Gertler and Karadi (2011) develop a model for special monetary policies. Woodford (2013) is concerned with the macroeconomic analysis in the context of excluding the rational expectations hypothesis. Rubio (2011) is concerned with the study of fixed or variable interest mortgage instruments and their correlation with economic cycles and monetary policies.

**Research methodology and data**

- The model on which we rely is a simplified model of the one developed by Turnovsky and Brock (1980), in which for simplicity, on the one hand, we have abstraction from physical capital, and on the other hand we have merged the behavior of companies and individuals in the so-called Representative agent who has the ability to make perfect predictions.

This representative agent has the possibility to decide on its own level of consumption \(c\), access to labor \((l)\), real money availability \((m)\), and possession of government securities \((b)\) so as to maximize intertemporal utility: \(\max \ U(c, l, m, b)\)

The budget constraint of the representative agent is expressed by:

\[ c + m + b = (r - p)b + (1 - \tau)F(l) - pm - \tau T \]  
(1)

On the other hand, the other player in the economy, the government, is considering its own budget constraints, which can be expressed by:

\[ m + b = g + (r - p)b - \tau F(l) - pm - \tau T \]  
(2)

where:
- \(c\) = actual consumption;
- \(g\) = real government spending;
- \(m\) = the real balance of money;
- \(M\) = the nominal balance of money;
- \(b\) = real stock of government securities;
- \(B\) = nominal stock of government securities;
- \(l\) = labor force available;
- \(P\) = price level;
- \(p\) = inflation rate;
- \(r\) = nominal interest rate;
- \(\tau\) = profit tax rate;
- \(T\) = real flat tax.

We assume that for certain given values of \(c, g\) and \(l\), the marginal utility of the balance of money satisfies the condition: \(\text{sgn}(U_m) = \text{sgn}(m' - m)\)

From the above we can conclude that when the agent’s real estate possession is lower than the satiety level, then the marginal utility of the
money possession is positive, and when it is higher, the cost of holding money is higher than the benefits, in which case the net margin of the holding of money becomes negative.

Making the difference between (1) and (2) results in the market condition of the aggregate product:

\[ F(l) = c + g \]  

(3)

In general, the government has at its disposal five instruments (policies): \( M, B, \tau, T \) and \( g \), of which any four variables are independent, but they can vary relatively arbitrarily over time. Given the equilibrium of the model, optimal policies become stationary in time, leading to the idea that we can assume that the government allows for the money supply to increase at fixed rate \( \phi \).

Thus, real monetary growth is expressed by:

\[ \dot{m} = (\phi - \rho)m \]  

(4)

which can be expressed by the accumulation rate of government securities present in the form of:

\[ \dot{b} = g + (r - \rho)b - \tau F(l) - \phi m - T \]  

(5)

- Macroeconomic equilibrium can be described by the equations:

\[ U_c(c, l, m, g) = \lambda \]

\[ U_l(c, l, m, g) = -(l - \tau)F'(l)\lambda \]

\[ U_m(c, l, m, g) = r\lambda \]

\[ \dot{\lambda} = \lambda[\beta - (r - \rho)] \]  

(6)

In the absence of capital accumulation, we have to show that the dynamics of the system decreases (degenerates) and tends towards a steady state of balance. The fact that the dynamics of the macroeconomic equilibrium decreases can be noticed by the fact that the utility function is separable by summing up to \( m \), and solving leads to the situation in which the solution for \( c, l \) is \( \lambda \) that are constant over time.

In this case, we can conclude that the real interest rate can be expressed by \( r - \rho = \beta \), from which it follows that the inflation rate is of the form:

\[ r = \frac{U_m(m)}{\lambda} - \beta \]  

(7)

In the absence of capital accumulation we can show that the dynamics degenerate and the system always reaches a steady state which leads to the solution of the equations of the form:
\[ U_m(\bar{m}) = (\phi + \beta)\bar{\lambda} \]  
(8)

In this situation we can approximate the evolution of the real volume of manes around the balance by:

\[ \dot{\bar{m}} = -\frac{\bar{m}U_{mm}(\bar{m})}{\bar{\lambda}}(m - \bar{m}) \]

Given \( c \) and \( l \) constants, equation (5) can be solved easily and we find the solution in which for the equilibrium situation we need to fulfill the condition:

\[ b + \frac{g - \tau F(l) - \phi m - T}{\beta} = 0 \]  
(9)

Which is, in fact, the government’s budget inter-temporal constraint, stating that the budget deficit in the real interest condition \( \beta \) must be zero.

Thus, the predicted equilibrium can be described by the equations:

\[ U_c(c, l, m, g) = \lambda \]
\[ U_l(c, l, m, g) = -F'(l)\lambda \]
\[ U_m(c, l, m, g) = (\beta + \phi)\lambda \]  
(10)
\[ b_0\beta + g = \tau F(l) + T + \phi m \]
\[ F(l) - c - g = 0 \]

All these equations indicate that if there is no dynamic there is no accumulation of titles or money, and the economy is firmly in balance. In fact, any shock in the system will generate an instantaneous leap in prices that will cause the real money balance to grow to reach a new stage of equilibrium described by (8).

The five equations describing the stationary solution for \( c, l, m, \lambda \) and one of the instruments of politics \( \phi, \tau \) or \( g \). In other words, three of the policy parameters can be chosen arbitrarily, while the remaining one adapts to Satisfies the state of equilibrium.

Macroeconomic equilibrium highlights two key aspects of monetary policy in this rational intertemporal context, namely:

- First, if the utility function is separable in additive, on the one hand in \( c \) and \( l \) and on the other in \( m \), it is clear that the states of equilibrium for consumption, labor and output and can be considered as independent of monetary policies. Thus, monetary policies have an impact on the system only
through the real money balance and the substitution rate between consumption and labor;

- Second, the fact that there must be some form of taxation for a certain level of monetary growth given to maintain steady state, emphasizes that there is a constraint / interdependence between monetary and fiscal policies.

In the following we will analyze the situations where $\phi$ where the monetary growth rate is chosen to optimize the welfare of the population / agent, taking into account two cases: (1) either by matching a flat-rate tax $T$ on the one hand, respectively by the distortion charge $\tau$ pe de altă parte, caz care poate fi considerat ca fiin o măsură de potrivire/ajustare fiscală.

- Let us consider the case in which the economy is stationary and the conditions that characterize the optimal choices of government policies. We suppose the government seeks to find policies that maximize the inter-temporal wealth of the representative agent, subject to balance constraints. Considering that everything is stationary, this optimization can be accomplished by maximizing the instantaneous utility function, which is subject to static constraints.

The problem can be described either (1) on the one hand by expressing the optimized utility of the consumer as an indirect function to the usefulness of the terms of the government policy variables, followed by optimization or (2) by maximizing the following Lagrangean expression:

$$L = U(c, l, m, g) + v_1\lambda - U_i(c, l, m, g) + F'\lambda - \frac{v_2}{\beta}rF(1) - \tau - \phi m + v_3F(I) - \epsilon$$

We can present optimality from two perspectives.

The first option considers the optimality from the point of view of the private sector variables.

$$\frac{\partial L}{\partial c} = U_c - v_1U_{cc} - v_2U_{lc} - v_3U_{mc} = 0$$

$$\frac{\partial L}{\partial l} = U_l - v_1U_{cl} - \frac{F''(1 - \tau)\lambda + U_l}{\beta} - v_4U_{ml} - v_5F' + v_5F' = 0$$

$$\frac{\partial L}{\partial m} = U_m - v_1U_{cm} - v_2U_{lm} - v_3U_{mm} - \phi v_4 = 0$$

$$\frac{\partial L}{\partial \lambda} = v_1 - v_2F'(1 - \tau) + v_3(\phi + \beta) = 0$$

The second approach is associated with government policy variables.
\[
\frac{\partial L}{\partial \phi} \equiv v_3 \lambda - mv_4 = 0 \\
\frac{\partial L}{\partial \phi} \equiv v_4 = 0 \\
\frac{\partial L}{\partial \tau} \equiv v_2 F' \lambda - v_4 F = 0 \\
\frac{\partial L}{\partial g} \equiv U_g - v_1 U_{cg} - v_2 U_{lg} - v_3 U_{mg} + v_4 - v_5 = 0
\]  

- Optimal monetary growth has been analyzed in the literature under various aspects and perspectives. In the beginning, authors such as Bailey (1956) and Friedman (1971) analyzed the phenomenon from the perspective of maximizing government revenue through tax / inflation rates and showed that optimal monetary growth depends on the elasticity of interest rates over money demand. Tobin (1968) has focused on monetary growth from the perspective of maximizing consumption, and has shown that this involves driving the economy under the gold-money golden rule. But the most important is Bailey’s (1956) and Friedman (1969) approach to optimal monetary growth by maximizing utility. Hence the most important proposal known as Friedman’s „total liquidity” rule, which states that the optimal monetary growth rate can be achieved by contracting the provision of money at a rate equal to the consumer’s preferential time.

- To begin with, we consider the case where the government chooses to maintain balance by means of a flat-rate tax with \( T \).

In this case, the optimality condition leads to \( v_3 \lambda = 0 \) și \( v_4 = 0 \). For the first condition we can have either \( \lambda = 0 \) either \( v_3 = 0 \). For simplicity, we consider this latter value and it follows that equilibrium is reduced to the set of equations:

\[
(U_c F' + U_l) - v_1(U_c F' + U_{cl}) - v_2(U_c F' + U_{ll} + F'(1 - \tau) U_c) = 0 \\
v_1 - v_2 F'(1 - \tau) = 0 \\
U_m - v_1 U_{cm} - v_2 U_{lm} = 0 \\
F'(1 - \tau) U_c + U_l = 0 \\
U_m - (\phi + \beta) U_c = 0 \\
g - b_2 \beta - \tau F(l) - T - \phi m = 0 \\
F(l) - c - g = 0
\]  

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Solving the above equations results in the optimal values for \( c, l, m, v_1 s l, v_2 \), and these are the data we can determine the value of the lump sum \( T \) that is needed to keep the balance.

\[
\begin{align*}
v_1 &= \frac{\tau (1 - \tau) (F')^2 U_c}{\Delta} \\
v_2 &= \frac{\tau F' U_c}{\Delta}
\end{align*}
\]  
\[\text{(14)}\]

where:

\[
\Delta \equiv F'(1 - \tau)\left[U_{cc}F' + U_{cl}\right] + U_{cc}F' + U_{ll} + F''(1 - \tau)U_c.
\]

The optimal rate of monetary growth can be described by:

\[
\dot{\phi} = \frac{\tau F' U_c}{\Delta} \frac{\partial}{\partial m}\left(\frac{U_l}{U_c}\right) - \beta
\]

\[\text{(15)}\]

From this relationship we can see that the optimal monetary growth rate is identical to Friedman’s famous liquidity rule: \( \dot{\phi} = -\beta \) or equivalent to \(i = 0\).

Only if one of the conditions is fulfilled:
- if \( \tau = 0 \), or
- if the marginal rate of replacement of labor consumption is independent of the total amount of money.

In the first case, there is no charge that distorts the choice between consumption and relaxation, and in the second case, since the utility function is separable in \( m \) by adding \( <\text{additively}> \), the choice between consumption and relaxation has no effect on the whole Their being constant).

- Next, we will look at the case where the authorities choose to maintain the balance by adjusting the tax rate, \( \tau \), instead of flat-rate taxation. This approach was originally described by Phelps (1973), who showed that there is a tendency towards optimal policies where the distortion created by the tax / tax is offset by the distortion in the inflation rate. In this case, the relationship of optimality is described by:

\[
\begin{align*}
(U_c F' + U_l) - v_1 (U_{cc} F' + U_{cl}) - v_2 (U_{cc} F' + U_{ll} + F'(1 - \tau) U_c) - v_3 (U_{cm} F' + U_{ml}) \\
- v_4 x F' &= 0 \\
U_m - v_1 U_{cm} - v_2 U_{ml} - v_3 U_{mm} - \phi v_4 &= 0 \\
v_1 - v_2 F'(1 - \tau) + v_3 U_{mm} - \phi v_4 &= 0 \\
v_3 \lambda - mv_4 &= 0 \\
v_2 F' \lambda - v_4 F &= 0,
\end{align*}
\]

\[\text{(16)}\]
In order to achieve a compromise / simplification, it is convenient to assume that utility is separable first summation in \( m \), as we considered in the case of flat-rate taxation \( \phi = -\beta \). This approach in which \( U_{cm} = U_{ct} = 0 \) simplifies the modeling and arrives:

\[
U_m - v_3 U_{mm} - \theta v_4 = 0
\]

Thus, by solving the above equations we obtain:

\[
v_3 = \frac{\tau (F')^2 U_c m}{\Omega}
v_4 = \frac{\tau (F')^2 U_c^2}{\Omega} = \frac{U_c v_3}{m}
\]

where:

\[
\Omega \equiv F'[F'(1 - \tau) - m\phi][U_{cc}F' + U_{ct}] + FF'U_{ct} + FU_{it} + [FF''(1 - \tau) + \tau(F')^2]U_c
\]

- So far we have assumed that government spending is constant / fixed. Now let’s assume that the government decides the level of spending, \( g \), in correlation with a monetary growth rate \( \phi \).

Next, suppose that he uses flat tax as an adaptation method to reach steady state of stability (as we have seen before \( v_3 = v_4 = 0 \)) the marginal condition is reduced to:

\[
U_g - v_1 U_{cg} - v_2 U_{lg} - v_5 = 0
\]

Therefore, removing \( v_5 \) from results

\[
U_g - U_c - v_1 (U_{cg} - U_{cc}) - v_2 (U_{lg} - U_{lc}) = 0
\]

Taking defined above, the optimal condition of government budget expenditures results from the form of:

\[
U_g = U_c \left[ 1 + \frac{\tau(1 - \tau)(F')^2}{\Delta}(U_{cg} - U_{cc}) + \frac{\tau F'}{\Delta}(U_{lg} - U_{lc}) \right] \quad (17)
\]

This condition may in fact be interpreted as the marginal utility of government spending being equal to the marginal utility of private consumption. If the utility function is separable by summing in \( m \), then the optimum monetary growth rate is constant, given a level of budget expenditure.

If we consider that the government chooses an optimal tax rate \( \tau \) in combination with a monetary growth rate \( \phi \) and expenditure level \( g \) it follows that policy optimality can be summed up:
\[ \tau = 0 \]
\[ \phi = -\beta \]
\[ U_c F' + U_t = 0 \]
\[ U_m = 0 \]
\[ U_g = U_c \]
\[ F(l) - c - g = 0 \] 

(18)

In this optimal macroeconomic policy, the tax is considered zero, which means eliminating the source that causes distortion, according to the relationship:

\[ T = g + \beta(b_0 + m) \]

- Since it was developed by Friedman (1969), the theory of optimal monetary growth has been the subject of ongoing research. Three significant approaches can be structured in the literature, which were synthesized by Chari and Kehoe (1999).

The first approach introduces the notion of money into the utility function. Chari and Kehoe provide a general characterization of the robustness of Friedman’s rules. They consider the case where preferences are not affected by satiety on the real money balance, but who is still bounded is optimal if the utility condition \[ U(l, c, m) = V(l, w(c, m)), \] where \( w(.) \) is homothetic.

The second approach refers to the cash-in-advance model where there is money for product and credit for product. Chari and Kehoe have shown that the Friedman model is optimal if the utility function is \( U(l, c_1, c_2) = V(l, w(c_1, c_2)) \) where \( w(.) \) is also homothetic.

The third approach has been reviewed by several authors and refers to the well-known „shopping-time” economy. Kimbrough (1986), Faig (1988), Guidotti and Vegh (1993) Correia and Teles (1996) consider money as an intermediary good where \( \phi(c, m) \) is the time to obtain money units, A real \( m \) cash balance. We can see that (i) for a given amount of money it takes longer to get more goods, respectively (ii) the availability of more money reduces the time needed to get a certain amount of goods. Correia and Teles have demonstrated that the function \( \phi(c, m) \) is homogeneous of any degree.

- Capital taxation is a central theme in public finances. Changing characteristics over time of optimal taxation in the inter-temporal macroeconomic context has been studied / deepened by Chamley (1985, 1986), which used a simplified model of the representative agent, focusing on
direct taxation of capital and labor, and has also failed to convert welfare into additional capital.

In this context, the representative agent (individual or firm) is directly interested in optimizing the equation:

\[ k + b = r(1 - \tau_k)(k + b) + w(1 - \tau_w)l - c \]

with the initial conditions \( k(0) = k_0 \) if \( b(0) = b_0 \), and the condition of optimality is described by:

\[
\begin{align*}
U_c(c, l) &= \lambda \\
U_l(c, l) &= -w(l - \tau_w)\lambda \\
\dot{\lambda} &= \lambda[\beta - (1 - \tau_k)r]
\end{align*}
\]

Through transformations we reach the utility function:

\[
U[c(\lambda, w(1 - \tau_w)), l(\lambda, w(1 - \tau_w))] \equiv V[\lambda, w(1 - \tau_w)],
\]

The one that expresses the agent’s optimal utility in terms of marginal utility and salary (after taxes and fees).

The optimal taxation issue should consider maximizing the welfare of the representative agent, which is subject to (i) constraints on the resources of the economy, (ii) government budgetary constraints, and (iii) the optimal conditions of the representative agent.

\[
\begin{align*}
\dot{k} &= F(k, l) - c - g \\
\dot{b} &= g + r(1 - \tau_k)b - \tau_k rk - \tau_w wl
\end{align*}
\]

(20)

We consider the multiplier \( \nu \geq 0 \), which is associated with the non-negativity restriction \( r(1 - \tau_k) \geq 0 \); thus, if the \( \nu \geq 0 \), restriction is fulfilled.

At zero, the marginal value of consumer utility \( \lambda \) has no restrictions, Atkinson and Stern (1974) show that \( \nu(0) \geq 0 \), so \( r(1 - \tau_k) \geq 0 \). Therefore, at first the capital must be taxed at the maximum feasible rate \( \tau_k = 1 \).

It is understood that the condition \( r(1 - \tau_k) \geq 0 \) can not be kept indefinitely, otherwise the marginal utility of consumption would increase indefinitely. Chamley pointed out that the restriction is always fulfilled when \( t > T \) and has concluded two tax regimes for capital - capital must be taxed at the maximum or none.

On the other hand, Lucas (1990) argues that capital taxation should be distributed over time in a similar way to other goods so that the taxation reflects the degree of consumption. Since taxation of new capital has the effect
of taxing future consumption at a higher rate, this leads to the conclusion that taxation is not desirable in the long term. Thus, at first, when capital is acquired, it must be taxed at the maximum rate, but over time the taxation must be reduced, eventually even eliminated.

Another approach to optimal taxation is presented by Judd (1985), which takes into account two types of consumers - workers and investors. If taxation of labor and capital can be made differently, it appears that, in the steady state of balance, capital taxation reaches zero. This is also true in the case of the limit for the two categories of consumers - workers do not have capital and investors do not work.

- Conclusions so far have referred to the fact that in the long run, capital taxation must be zero, it is a very strict condition. In the case of growth / emerging economies, we need to take into account two other issues; the first concerns congestion, and the second is that government spending is limited to the size of the economy.

Let us assume that the representative agent has a production unit that runs contracts with the government benefiting from public goods contracts denoted by $G^s = G(k/k)^{1-\sigma}$, where $\sigma$ is a parameter describing The relative degree of congestion associated with the public good, and $k$ the individual capital of the firm, and $\bar{k}$ the capital agreed in the economy. Congestion occurs when the use of aggregate capital exceeds the use of individual capital.

Thus, for $\sigma = 1$, we have the case in which the services received by the government from the government are constant and are at $G$ level, regardless of the degree of capital use. Good $G$ can be considered to be available to all individuals, and in this case we are not talking about congestion.

At the other extreme we have $\sigma = 0$, in which case only if $G$ increases in proportion to the agreed capital growth rate $\bar{k}$, the services available for that company may remain constant. This case is called relative congestion.

The case $\sigma < 0$, can be considered extreme congestion, where $G$ needs to grow much faster than the economy, so that it can maintain the level of constant services.

The firm equilibrium condition is described by:

$$F_k(k, l, G) = \beta$$

From the above we can conclude that by increasing the capital $k$, assuming that $\bar{k}$ remains constant, the agent expects to be able to benefit from a larger share of public goods. The condition of long-term optimality, where the agent is taxed at rate $\tau$, can be described by:

$$(1 - \tau)\left[F_k + (1 - \tau)\frac{G}{k}F_g\right] = \beta$$  (21)
Thus we can conclude that the case $\sigma = 1$ is reduced to the Camley-Judd presented above, by which long-term capital must not be taxed. On the other hand, relative congestion induces an incentive for the entrepreneur to increase its own capital, which creates congestion for others - an externality, which requires us to consider long-term capital taxation as correct.

We take into account the fact that the government’s expenditures are not economy-related or other, where the economy is growing. This is $G = gF(k, l)$, where $g$ is considered constant. This means that the size of the government increases in proportion to the growth of the economy.

In this situation, the government’s long-term optimal condition can be described by:

$$(1 - g)F(k, l) = \beta$$

However, the representative agent tends not to recognize this link and will continue to increase capital, which will generate an externality. In this case, in order to correct the situation, it is necessary to introduce a tax $\sigma = g$, which means that in the long-term stable equilibrium the capital taxation is not zero.

- Until now, we have been considering whether governments are choosing certain capital-tax policies that are considered optimal at the time of zero, and then respect their decisions. However, in the course of implementing these policies, it is necessary to make adjustments / adaptations to current market conditions. In this case, we can assume that this new decision sets a new zero start time from where a new implementation cycle begins. The new trajectory of implementing optimal capital taxation policies, but with a high probability, differs from the initial one, so we may consider it temporally inconsistent.

Over time, many studies and research have been done on temporal inconsistency, as well as in the development of optimization models focusing on forward-looking agents; Among the first we mention Kydland and Prescott (1977). In general, we can say that an economic policy is inconsistent over time, when decisions taken later, but which are on the optimal trajectory, determine an evolution that can no longer be considered optimal in the future (although the specific conditions have not changed significantly). From the point of view of optimal capital taxation so far, we have shown that taxation is fixed in the short term and variable in the long run, but governments are unwilling to make any changes in this regard.

This problem can be analyzed based on a time pattern with two distant periods, starting from those described by Fischer (1980), and Kydland and Prescott (1977, 1980). We consider two time periods $1$, and $2$ where the
representative agent has the initial capital $k_1$, consumes during these periods $c_1$ and $c_2$, and works / produces only during the 2nd period with $l_2$. In this case, the utility of the agent can be described by:

$$U = \ln c_1 + \delta [\ln c_2 + \alpha \ln (\bar{l} - l_2)] + \gamma \ln g_2 \quad \alpha > 0, \quad \gamma < 0$$ \hspace{1cm} (23)

where $0 < \delta < 1$ is a discount factor, $\bar{l}$ is the labor force available in period 2, and $g_2$ represents government expenditures during period 2 (it is assumed that during the period 1 was not spent).

Thus, we can for the two periods we have:

$$c_1 + k_2 = (1 + b)k_1 \equiv RK_1$$

$$c_2 + g_2 = al_2 + RK_2$$ \hspace{1cm} (24)

where we denote with $a$ the marginal value of labor (constant over time), and with $b$ the marginal value of capital generation (constant).

• Governments are interested in maximizing agent welfare, and to this end they must determine optimal policies by choosing certain variables, in our case they are $c_1, c_2, l_2, k_2$ and $g_2$, and the optimum condition can be described by:

$$c_1 = \frac{a \bar{l}/R + RK_1}{1 + \delta(1 + \alpha + \gamma)}$$

$$c_2 = \gamma R c_1$$

$$\bar{l} - l_2 = \alpha c_2/a$$

$$g_2 = \gamma c_2$$

This solution is consistent over time and is the best possible solution the government has at its disposal at the beginning of the second period of time.

• In the following we take into account the assumption that the government finances its expenses by imposing taxes / taxes so the representative agent will tend towards optimization with the following restrictions / constraints:

$$c_1 + k_2 = RK_1$$

$$c_2 = a(1 - \tau_2)l_2 + R_2 k_2$$ \hspace{1cm} (26)

where $\tau_2$ is the labor tax rate, and $R_2$ is the rate of tax on capital return.

In this situation, the government’s problem is reduced to the choice of $\sigma_2, R_2$ and the expenses $g_2$ so as to maximize the welfare of the agent, but taking into account the budgetary limitations.
\[(R - R_2)k_2 + \tau_2a_l_2 = g_2\]  

(27)

In order to achieve this optimization, the government needs to find out what the agent’s expectations are, and in the ideal case, we can assume that the agent can always accurately predict the behavior of the government. Thus, optimization leads us to:

\[g_2 = \gamma c_2\]  

(28)

which means that for the government, the optimal means to bring to the same level the marginal utility of government expenditures with those of private spending.  

• We can say that the tax rate is not consistent over time because in the second period, the tax level chosen as optimal in the first period is no longer an optimal one. Thus, the agent chooses other values \(c_2, l_2\) to maximize its usefulness:

\[U = \ln c_2 + \alpha \ln (\bar{I} - l_2) + \gamma \ln g_2\]

Considering that in this case that \(\tau_2, R_2\) and \(g_2\) are given, the condition of optimality can be described as:

\[\tau_2 = 0\]

\[R_2 = \frac{R(1 - \alpha) - a\bar{l}/k_2}{1 + \alpha + \gamma}\]

Hence, we can conclude that the government needs to increase its revenue by increasing capital taxes, leaving undeclared work to counteract the side-effects due to the reduction in the balance of the labor force. Temporary inconsistency arises from the fact that governments have no non-dictatorial taxes / dues.

• The consistent solution over time can be obtained by using the optimal principles of Dynamic Programming, which means that modeling begins in the period 2 and resolves backwards over the period 1. Both the government and the agent tend to get the best in The second period leads us to the solution similar to the one previously obtained, that work should not be taxed, and the capital does. If we look at period 1, knowing the behaviors of period 2, the agent will calculate the optimum for consumption and saving.

\[U_2 = (1 + a) \ln[a\bar{l}(1 - \tau_2) + R_2k_2] - (1 + a) \ln(1 + a) + a \ln(a/a(1 - \tau_2)) + \gamma \ln g_2\]  

Thus, the agent tends to maximize utility during period 1 through:

\[U_1 = \ln c_1 + \delta U_2\]

where \(\tau_2, R_2, g_2\) are given.
Conclusion

In this study, the authors sought to analyze the main monetary policies as well as optimal fiscal measures.

The Fischer example with the two periods presents the easiest way to deal with the problem of temporal inconsistency. Other instances of temporary inconsistency can be studied, given that, on the one hand, the governments do not respect the decisions taken by their predecessors or, on the other hand, do not act in the interest of the representative agent. The problem of temporal inconsistency has generated a challenge for those who study optimal policies and has been the subject of many research since 1980. One of the research directions focuses on the commitment solution and assumes that the current government can convince those who are to follow their decision. Actually, in fact, this is not the case, and the Agent knows from experience that a new government will be overwhelmed and will tend to modify the decisions taken by the current government, makes the decisions from the Agent’s point of view not to be credible. On the other hand, the government knows that repeated breaches of promises will erode the government’s reputation and generate additional costs, and the agent’s response is an important issue for the government. The problem of reputation balance has been studied by several authors, among whom we can mention Lucas and Stokey (1983), Persson, Persson and Svensson (1987). Other authors have extended the dynamic programming solution used in Fischer’s example with the two-period intervals to infinity, in which case the backwards solutions do not correctly reflect the phenomenon.

References