TIME DIVERSIFICATION IN CONSUMPTION AND SAVING

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Abstract

The concept of dynamic portfolio was imposed in literature with the expansion of the time horizon analysis and of the investment behavior modeling. Thus, coming out of scope definition that static analysis has imposed to the agent election outcome, the time horizon expansion and its involvement as an active dynamic factor into the decision of investment processes increased the chances of a thorough analysis as of a more accurate modeling of psychological processes related to individual decision.

Key words: *diversification, risk, consume, tolerance, prediction*
JEL clasification: *G10, G11*

Introduction

In the investment context, it is specified a unique moment of consumption. This implies that all risks of lifetime were generated at that moment. In most cases, investors will want to use their own portfolios to finance their consumption throughout life. Obviously, this model is much simplified. For example, actual tax incentives may work against this possibility. This situation presents a major advantage for the allocation of current risks relating to the risks lesser welfare on a wider horizon of consumption. This produces an important diversification effect over time, which leads those with a broader horizon planning to take more risks.

Literature review

If consumers can not borrow money when there is a temporary negative impact on their income they will be more willing to accumulate *ex ante* wealth. This buffer deposit leads to another reason to save (Deaton 1991; Carroll 1997). Anghelache, Anghel and Manole (2015) describe the tools for economic modeling. Anghelache, Anghel and Popovici (2015) apply multiple regression analysis for private and public consumption, Anghelache, Sun and Popovici (2015) evaluated the influence of final consumption on GDP evolution in Romania, an analysis of the same macroeconomic indicator is performed by Anghelache, Manole and Anghel (2015). Anghelache, (2008), Anghelache, Mitruţ and Voineagu (2013) are concerned with the macroeconomic indicators measurement, Anghelache and Voineagu (2009), Anghelache, Anghelache and Bădulescu (2008), Ghysels and Osborn (2001) studied the time series. Hribar and McInnis (2012) examin the forecasting errors of benefits associated to investment. Kalaman and Zhalinska (2012) describe the role of investment funds in financing innovation. Ravi et.al. (2014) is concerned with the correlation between volatility and asset prices.

General apects regarding time diversification

To explain this, we will consider a simpler model where the agent has the opportunity to take a risk on the moment $t = -1$. In detail, we assume that the profit of the initial game risk assumption $z(\alpha_0, \tilde{x})$, where α_0 represents a decision variable and $\tilde{x}$ an ordinary variable. Next, the agent uses in the remaining periods *n,* numbered $t = 0, \ldots, n - 1$. Assuming that the agent can save or borrow with a zero interest rate, and that he has no opportunity to take risks starting with $t = 0$. Moreover, every time he earns from employment an income *y.*

This issue shows the same dynamic structure presented in Section 7.1. To determine optimal exposure to risk in the first period, we need to solve firstly the consum-saving issue after after the risky effect is shown. For a given wealth *z* accumulated prior to t = 0, we can note

$$v(b) = \max_{c} \sum_{t=0}^{n-1} p_t u(c_t) \text{ subject to restrictions } \sum_{t=0}^{n-1} c_t = b + ny$$

where p_t is a reduction factor associated to the moment *t,* and $z + ny$ it is is the welfare of the entire lives. With this function of value *v* , it can be determined the optimal level of assumption of risk tolerance relative to the original value function *v.*

As we noted above, the structure of consumption – saving problem is essentially the same as the Arrow – Debreu problem of Proposition & .1. The main difference is that prices do not have a status, so we must assume that $\prod_s = 1$ for all s. From the above sentence, we deduce:

$$Tv(b) = \sum_{t=0}^{n-1} T(c_t^*)$$

where c_t* is the optimal solution to the problem. In the problem of certain relation between consumption and saving for $t = 0, n - 1$ with zero interest rate, the degree of tolerance to risk the at the initial welfare equals the sum of absolute tolerances to risk in consumption over the life of the consumer.

We will further examine the effect of an increase in *n* over $T_v(z)$. For simplicity, we assume that all consumers are not anxious, so p_t = 1 for all *t*. Thus, it is best to completely calm consumption: $c_t^* = y + (z/n)$ at every moment *t*.

In this arrangement, all gains and losses to the risk assumed are equally allocated over the *n* consumption periodes left. The property can be writtten as:

$$Tv(b) = nT\left(y + \frac{n}{b}\right)$$

For a smaller initial risk (small *z*), the absolute risk tolerance relative to the welfare is proportional to the life of the agent. Thus, an agent who expects to live twice as much than another agent with the same annual income can invest approximately twice more in actions than the other one at the moment *t* = 1. This is the real meaning of the „time diversification".

Of course, we suppose that there is only a single point in time when consumers assume the risk. In the real world, consumers may own shares and may assume risks at any time. This more realistic assumption would not change the result obtained previously for HARA. Indeed, using reverse induction, adding the risk-taking solution in the future would not change the concavity function value at some time when HARA is assumed. Agents are absent against future risks in this case, and still it supports the previous property.

Further evaluation in a realistic background is the existence of restrictions of cash. Time diversification operates well only if consumers can borrow money at an acceptable interest rate when faced with an adverse shock to income and own money when their reserves are exhausted. This is an unrealistic assumption. Agents that do not have cash, can not cope with negative revenue shock over borrowing money from their bank. They cannot do a complete time diversification. Such financial restrictions involve the situation when the agent should have a higher aversion to risk. This is an additional argument in favor of decreasing absolute risk aversion.

Some aspects of portfolio management with predictable revenue

We examined a portfolio decision problem where the investment opportunity set was invariable in time. In real world, it often happens that opportunity to make stochastic setting and some changes status to be predictable. Predictability may occur, for example, from the existence of serial correlation of returns relative to shares. The existence of a reversible means for receipts relating to shares has been accepted recently: A large collection of a risky portfolio today implies a lower expected collection tomorrow. Good news coming today involve bad news in the future about opportunities

In this section, we consider the effect of such predictability of dynamic optimal portfolio. Obviously, investors will follow a flexible strategy the optimal exposure to risk is conditional upon the holding of opportunity. But investors will try to anticipate any impact of the framework of opportunities. More specifically, they may consider the possibility to cover against any bad news related to the context of their future opportunities. Of course, this is done relatively simply, if exchanges are statistically correlated to actual profits. Demand for shares due to this anticipation is called „demand coverage" for actions. Since actions are thought to be safer in the long term than short term, intuition suggests that an investor with a longer planning horizon will take risks earlier in life.

For sake of simplicity, we limit the analysis to the case of relatively constant aversion to risk γ with a horizon of two time periods. Aversion relatively constant to risk involves myopia to the relative timeframes in the absence of predictability. Supposing that the economy has an un-risky asset with zero receipts and a risky one, whose profit for the period t is denoted by $\tilde{x}_t$, t = 0,1. The framework opportunity in the second period is completely described by $\tilde{x}_1$. Predictability comes from the assumption that the distribution of $\tilde{x}_1$ is correlated with $\tilde{x}_0$. We assume that $E\tilde{x}_0 > 0$. Investors only invest for retirement at the end of the second period, so there isn't an intermediate consumption.

To determine the optimal demand for risky asset in the first period, and in particular the hedging component, it is necessary to follow the above method. Let's start solutioning the problem that investors faced in the second period for each possible situation. What is new here is described not only the welfare z accumulated in that period, but also the profit of the risky asset x_0 in the first period. Specifically, the value function v is defined by:

$$v(b, x_0) = \max_{\alpha} E\left[\frac{(b + a\tilde{x}_1)^{1-\gamma}}{1-\gamma} | x_0\right]$$

We noticed that the optimal solution for this program is a separate function $\alpha_1(z,x_0) = a(x_0)z$.

Ths in turn implies that the value function is separable, with $v(z,x_0) = h(x_0)z^{1-\gamma}/(1-\gamma)$, where $h(x_0) = E[(1 + a(x_0)\tilde{x}_1)^{1-\gamma} \mid x_0]$.

Now, we get back to the decision problem of the first period. This can be written as:

$$\alpha_0^* = \arg\max_{\alpha} H(\alpha) = E\left[h(\tilde{x}_0)\frac{(\omega_0 + \alpha\tilde{x}_0)^{1-\gamma}}{1-\gamma}\right]$$

To determine the hedging component for risky asset demand, we compare α_0* to demand for risky assets when there is no predictability, for example when $\tilde{x}_1$ is independent of $\tilde{x}_0$. In this case, we know that myopia is optimal. Thus, in the absence of predictability investors reach the version:

$$\alpha_0^m = \arg\max_{\alpha} E\left[\frac{(\omega_0 + \alpha\tilde{x}_0)^{1-\gamma}}{1-\gamma}\right]$$

When profits are somehow predictable, the hedging demand is defined as α_0*- α_0^m. This request for hedging it will be positive if its derivative H evaluated at α_0^m is positive. In other words, the relationship becomes

$$\mathbf{H'(\alpha_0^m)=E[\tilde{x}_0 h(\tilde{x}_0)(\omega_0+\alpha_0^m \tilde{x}_0)^{-\Upsilon}]\geq 0}$$

Everytime $E[\tilde{x}_0(w_0+\alpha_0^m \tilde{x}_0)^{1-\gamma}]=0$

To evaluate a specific type of predictability, we will examine if an increase in x_0 will deteriorate $\tilde{x}_1$ distribution in the sense of first order stochastic dominance (FSD). A special case is when the stochastic process $(\tilde{x}_0, \tilde{x}_1)$ indicates a reversal of means. Supposing that the conditional distribution of $\tilde{x}_1$ can be written as $\tilde{x}_1 | \tilde{x}_0 = -kx_0 + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is assumed as independent of $\tilde{x}_0$ and where k it is a positive scalar. Since any change of FSD in $\tilde{x}_1$ reduces EU of final welfare, this assumption implies $\partial v/\partial x_0$ negative. Since $v(z,x_0)$ is equal to $h(x_0)z^{1-\gamma}/(1-\gamma)$, that h' must be negative when $\gamma<1$ and h' when $\gamma>1$.
Let's suppose that the relative risk aversion of γ is greater than unity. Since h' should be positive in this case, it follows that for all x_0,

$$x_0 \mathbf{h}(x_0)(w_0+\alpha_0^m x_0)^{-\Upsilon} \geq x_0 \mathbf{h}(0)(w_0+\alpha_0^m x_0)^{-\Upsilon}$$

For the second situation

$$H'(\alpha_0^m) \geq \mathbf{h(0)E}[\tilde{x}_0(w_0+\alpha\,\alpha_0^m \tilde{x}_0)^{1-\gamma}]=0$$

Thus, the demand for hedging is positive when the aversion relative to risk is greater than unity. If we have a relative risk aversion less than unity, $\gamma<1$, then h' is negative and the above inequality is reversed. This result is shown in the following sentence.

Supposing that a profit growth from the first period is deteriorating profit distribution profit of the second period for the purposes of the first order stochastic dominance. Then, the hedging demand for risky assets is positive (or negative) if risk aversion is relative constant to larger (respectively smaller) than unity.

Another way to interpret this result is the following: when relative risk aversion is constant and greater than unity, a longer time horizon should induce to investors a more risk-taking desire. The opposite is true if relative risk aversion is lower than unity. Let us note that when investors have a logarithmic utility function (*γ =1)* myopia is still optimal in the presence of predictability. Choosing an initial risk portfolio is dictated by the collapse of marginal value of wealth at the end of the initial period. This marginal value of wealth depends on the opportunities frame. If predictability reduces the marginal value of welfare in its abundance states, and makes it grow where it is low, then predictability has the same effect as a reduction of risk aversion: it raises the optimum level of risk in the portfolio. As a result, we observe that the central step of the analysis is to determine the effect of the change produced by us that deteriorates FSD in profits generated by a risky asset will have the effect it will have on the marginal value of welfare. In the particular case of reversal means, we can see two different effects of the increase in x_0. The first effect is the effect of welfare: as expected profit in the second period becomes smaller, the same happens with welfare, it becomes smaller. This event increases the marginal value of wealth, since v is concave z. The second effect is an effect of caution: investors will invest less in risky assets thus reducing risk exposure. Prudently, it reduces the marginal value of wealth. The global effect of an increase in x_0 of the marginal value of wealth is ambiguous. When relative aversion to risk is constant and greater than unity (and this happens if and only if prudence absolute is less than twice aversion absolute towards risk, which explains why this condition implies that the effect of precaution is dominated by wealth effect), the wealth effect always dominates the precaution effect, and demand for hedging is positive. When the relative aversion to risk is less than unity, the wealth effect is dominated by the caution effect.

Let's assume that like in the process of learning Bayes, an increase in profit in the first period profit improves distribution in the second period for the purposes of the first order stochastic dominance. Then, the hedging request for risky assets is negative (or positive) if relative constant aversion to risk is larger (or smaller) than unity. Suppose the relative risk aversion is constant and equal to $\gamma = 2$. Let's solve the problem of portofolio choice in the secod period conditional on each of the two possible observations made in the first period. I we notice a big profit in the first period, direct calculation shows that the investor should invest $a(2) = 40{,}22\%$ of his wealth in risky assets in the second period of his life. The value of function $v(z,2)$ is equal to $0{,}76u(z)$ in this case. On the other hand, if we notice a low profit in the first period, optimal investment in the second period would involve $a(-1) = -12{,}67\%$ of wealth invested in the risky asset, and the value function $v(z,-1)$ is equal to $0{,}97u(z)$.

Returning to the problem of the first period, the investor resolves the next problem:

$\max_{\alpha} \frac{1}{2} 0{,}76u(w_0+2\alpha+\frac{1}{2}0{,}97u(w_0- \alpha)$.

This generates an optimal request for the risky asset from the first period of α_0^*=7,66% from fis initial wealth w_0. It is easy to verify that the agent who suffers from myopia or acts myopic would invest α_0^m=12,13% from his wealth in the risky asset. That's why, the hedging request $\alpha_0^*- \alpha_0^m$ is negative, thus showing that the learning process tends to induce a prudent investment behavior in the early stages of learning.

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