THEORETICAL ASPECTS OF THE DYNAMIC PORTFOLIO MANAGEMENT

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Abstract

Items as reverse diversification, investment dynamics are subject to this article. Investors are characterized by a longer timeframe vision concerning investing activity. Corporate managers decisions focused on early gains allows the estimation that they are strongly concerned of short-term strategy which is considered as safe, based on sub-investment in research and development risky projects. Taking risks on long-term investment policy might generate significant advantages and benefits for investors, while mutual funds are specifically oriented towards obtaining relevant short-term benefits, focusing on long-term expectations.

Keywords: *portfolio, management, dynamic decision diversification*

Classification JEL: *G 10, G 11*

Introduction

Most often, investors have a long-term view on their investments. Many times, investment decisions are not reversible and we might conclude that investment management has an obvious dynamic nature. It raises the question whether investment decisions can be inferred from a static mathematical model that can be used to define a dynamic strategy. In other words, the focus is on determining the effect of investment horizon of an agent on his portfolio related risks. The usual treatment suggests that short-term time horizon often leads to excessively conservative strategy. It is well known that private companies provide a substantial benefit from the ability they have to focus on long-term projects. It is considered that mutual fund managers focus on strategies that ensure satisfactory short-term gains, without sacrificing long-term expectations.

Literature review

The first to solve the problem dynamic portfolio of in a continuous time economy with HARA utility functions were Merton and Samuelson in 1969. Mossin (1968) proved that only the HARA functions consider myopia optimal when there is serial correlation of profits. Deaton and Carroll in the '90s examined the effect of liquidity constraints on the behavior of optimal savings. In 2000, Barberis estimated the predictability of significant earnings produced by the US stock market. Kim and Omberg (1996) showed that the standard deviations of ten-year earnings were 23.7% lower than 45.2% of the value implied by the standard deviation of monthly earnings, which, they said, indicated that that happens when risk relative constant aversion is to larger than unity. In 1999 and 2000, Campbell, Viciera and Barberis estimated that demand for numerical coverage. The effect on profit predictability on initial portfolio optimal structure is surprisingly important. For an agent with relative risk aversion equal to 10 and a ten years horizon, optimal investment in shares represent 40% of current welfare without predictability. It rises to 100% when the reversal of means is considered. In 1986, Detemple is the first to examine the matter of active demand under incomplete information and knowledges.

Ameur and Prigent (2010) analyzes the risky behavior in the context of portfolio structured management. Anghel (2013) study portfolios management and analysis using some dedicated models. Bhalla (2008) envisages investment management. Anghel (2013b) is concerned of identifying key role of financial instruments in portfolios management. Anghelache and Anghel (2015, 2016) prepare reference paper regarding statistical and econometric tools, Anghelache and Anghel (2014) described the utility model regression analysis and portfolio management. Anghelache, Anghel and Popovici (2016) studied the role specialized modelling in dynamic management of the shares portfolio. Anghelache, Anghel and Popovici (2016) presented a number of significant features of the evolution of investment. Anghelache and Anghel (2013) are concerned with identifying patterns in the analysis and portfolio management. Bade, Frahm and Jaekel (2009) addressed Bayesian optimization to financial instruments portfolio. Eeckhoudt, Gollier and Schlesinger (2005) analyzed the decision behavior under risk. Hafner and Wallmeier (2008) studied volatility and its impact on optimal investment characteristics. Malcolm, Taliaferro and Wurgler (2006) considered the variable investment income possibility by forecasting managerial decision. Phillips and Sul (2003) are concerned by homogeneity testing and dynamic estimation under certain conditions.

In the field of specialized literature, the issue of risk horizon and received the highest interest by providing portfolio strategies according to agent appropriate age. Thus, Samuelson in 1989 and still other authors addressed the question: “as you age and your investment horizon narrows, you will limit your initiatives regarding lucrative but risky share?” Conventional wisdom answers “yes!” to this question, adding that investors with a higher horizon of time can tolerate a higher risk because they have more time available to recover transitional losses. This argument was not supported by scientific theory however. As particularly Samuelson (1963, 1989) considered, the argument of “temporal diversification” is based on a misinterpretation of the Law of Large Numbers. Thus, repeating a pattern investment over several periods of time does not generate risk or even beyond long periods of time. This error is illustrated by the following concerns of Samuelson on this topic in 1963.

“I proposed some buddies to bet every 200 Euro to 100 Euro that the coin will not fall as they bet at the first pitch. A distinguished colleague replied: “I will not bet because I feel that is 100 Euro lost is more than $ 200 won. But I bet if you promise to let me bet this 100 times. “

This story suggests that independent risks are complementary. Samuelson goes on asking why it would be best to accept separately the 100 undesirable bets. The distinguished colleague replies:

“One throw is not enough to demonstrate that the law of arithmetic mean will return in my favor. But a hundred throws of the coin, the law of large numbers will turn everything in my favor. “

Obviously, this colleague of Samuelson misinterprets the Law of Large Numbers! Accepting a second chance will not reduce the risk associated with the first pitch. If v $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$ are variables of wealth distributed independently and identically and randomly, then $\tilde{x}_1+\tilde{x}_2+\ldots+\tilde{x}_n$ is characterized by n times greater of each of these risks. This is what supports the Law of Large Numbers

$$\frac{1}{n}\sum_{=1}^{n}\bar{x}_i$$

almost certainly tends to $E\tilde{x}_1$ as n tends to infinity. By lowering, and adding risk, they are disappearing through diversification.

2. The concept of reversing diversification

Problems solving dynamic decision involve a good understanding of the method generally known as “reverse induction”. Suppose you took a sequence of two decisions α_0 in the interval 0, and α_1 in the interval 1. Decision α_0 concerns exposure to risk whose profit z(α_0, x) is subject to the completion of a variable x some $\tilde{x}$. It is important to note that x occurs after selection of α_0, before the decision α_1 to be made. The objective ex ante points to maximizing the expectation of a function Ua of (α_0, α_1, $\tilde{x}$):

$$\max_{\alpha 0,\alpha 1} EU(z(\alpha_0,\widetilde{x}),\alpha_1)$$

Reverse induction consists primarily in solving the problem of the second period for each possible outcome that would be prevalent at the beginning of that period. This set of results is complete summary of profit *z* obtained in the first period. The optimal strategy α_1* for the second period will generally depend on *z*, which is called dynamic variable status. This issue of the second period contingent with "*z* status" is written as follows:

$$v(z) = \max_{\alpha 1} U(z,\alpha_1)$$

The optimum value of *z* is noted by *v(z)*. Function *v* is called value function, or Bellman function. The problem the first period 0 can solved by selecting risck exposure α_0 that maximizes the expectation of function $Ev(z(\alpha_0, \tilde{x}))$. In doing so, the decision-maker internalizes the effect its future contingent strategy on welfare *U*, by *v* definition. He is "dynamic consistent". This technique turns any dynamic problem into a sequence of static problems by the value function.

3. The significance of dynamic investment

We will further examine the effect of future risk-taking opportunity on the will to assume risk on short term. In other words, an investor with a long-term planning horizon will want to invest a larger size of well-being in risky actions in opposition to safer bonds? Suppose the investor has the goal of maximizing the welfare of EU accumulated at a particular date. For example, this is the case when an investor is preparing his retirement. This money is not used for intermediate consumption. In standard terminology this is called an investment problem. Next, we introduce intermediate consumption, as we assume here the idea that while the risks are independent, a condition that will relax further. We can illustrate the problem examined in this section as follows. Starting from Samuelson's question, suppose you are asked to bet whether a currency will fall „head" (C) or „tails" (P). Win three times your stake when the coin falls „head" (C) and loose in the opposite situation. Suppose that, given your aversion to risk, you want to bet on this single bet. Now, suppose you are told that you will allow yourself to bet sequentially in two independent throws of the coin. How will affect your bet the initial coin toss? This question is equivalent to the time horizon of the investor's optimal portfolio composition.

Consider the following more general problem. An investor with wealth experiences two periods. At the beginning of each period, he has the opportunity to take some risk whose accomplishment will be visible at the end of that period. It is important to observe that the investor will become aware of the loss or gain arising from the risk assumed in the first period before deciding to measure the risk he assumed in the second period. This will make the problem inherently dynamic and it will some flexibility which is essential for dynamic risk management. To illustrate, under DARA conditions, investors will assume less risk in the second period in case they suffered major portfolio losses in the first period. To be precise, suppose that the problem of the two periods is a decision Arrow-Debreu portfolio. There are *s* of the nature state *s=0, ... , S-1* . The uncertainty prevailing in the second period is characterized by the vector of probabilities ($p0, \ldots, p_{s-1}$). Πs is the unit price of Arrow-Debreu security associated to state *s*. Let us uppose the un-risky rate is zero. This implies that a claim has a value of 1 leu for each natural state must itself cost 1 leu; ΣsΠs = 1. In other words if an investor assumes the risk in the second period he will end up with the same final welfare as in the first period.

Given welfare *b* being accumulated at the end of the first period, the investor selects a portfolio ($c0, \ldots, c_{s-1}$) that maximizes the EU's welfare at the end of which is subject to restriction:

$$v(z) = \max_{c_0,\ldots,c_{s-1}} \sum_{s=0}^{S-1} p_s u(c_s) \text{ is subject to restrictions } \sum_{s=0}^{S-1} \Pi_s c_s = b$$

This is equivalent to the problem indicated by the equation, and

$$U(z,\alpha_1) = p_0 u\left(\frac{b - \sum_{s=1}^{S-1} \Pi_s c_s}{\Pi_0}\right) + \sum_{s=1}^{S-1} p_s u(c_s)$$

During zero period of time, investor must take a decision involving a profit $z(_0, x)$ depending on the attainment of a variable *x*. In particular, this could be another problem of portfolio choice. The optimum exposure to risk during the period 0 is obtained by accomplishing the following schedule:

$$\alpha_0^* \in \arg\max_{\alpha 0} Ev(b(\alpha_0, \tilde{x}))$$

We aim to determine the impact of risk-taking opportunity in the second period in connection with the exposure in the first period. To achieve this, we compare the obtained solution α_0* within a dynamic program with optimal exposure to risk in the first period, when there is no option to take risks in the second period. Investor with short life, like the shortsighted would select the level that would maximize EU for $z(\alpha_0, \tilde{x})$:

$$\hat{\alpha}_0 \in \arg\max_{\alpha 0} Eu(b(\alpha_0, \widetilde{x})) .$$

We see that the only difference between programs consists in the fact that utility function *u* in the first program replaces the function of the second value v. This can be expressed by saying that the future effect is completely captured by the characteristics of the value function. In the present context, risk taking opportunity in the future increases the willingness to take risks today if *v is* less concave than *u* as Arrow-Platt assesses.

The optimum exposure to risk in the first period is greater than the myopic exposure operated if the value function *v* defined as less concave than the original utility function *u*, for instance *v* is more risk tolerant than *u.* The degree of absolute risk tolerance *v* is characterized by the news of his sentence.

Value function for the problem of portfolio Arrow Platt (& .3) has a high absolute risk tolerance given in equation

$$Tv(b) = \frac{v'(b)}{v''(b)} = \sum_{s=0}^{S-1} \Pi_s T(c_s^{\ *})$$

*where c * is the optimal solution to the problem and T (·)= -u'(·)/u''(·) represents the absolute risk tolerance for final consumption.*

Proof. The optimal solution is denoted by *c * (z).* It satisfies the following first order condition:

$$u'(c_s * (b)) = \xi(b)\pi_s \text{, s = 0, …, S-1}$$

where $_s = \Pi_s / p_S$ is the state price per unit of probability. From the condition of complete differentiation on *z* and eliminating$_s$, we have

$$(c_s *'(b)) = \frac{\xi'(b)}{\xi(b)} T(c_s * (b))$$

A full differentiated limitation on the budget brings

$$\sum_{s=0}^{S-1} \Pi_s c_s^{\ *}(b) = 1$$

Replacing c_s**'(z)* the expression implies

$$-\frac{\xi'(b)}{\xi(b)} = \left[\sum_{s=0}^{S-1} \Pi_s T(c_s^*(b)) \right]^{-1}$$

Finally, the fully differentiating *v(z)*, which by definition equalizes $_s u(c_s*(z))$, involves

$$v'(z) = \sum_{s=0}^{S-1} p_s u'(c_s{*}(z)) c_s{*}'(z) \xi(z) \sum_{s=0}^{S-1} \prod_s c_s{*}'(z) = \xi(z).$$

A second equality above stems from the condition of first order. This result confirms that the classic Lagrage multiplier associated with budget restriction equals the shadow price of welfare. From this result we can see that $\xi'(z)$ and $T_v(z) = \xi(z)/\xi'(z)$.

Let us remember that we accept that the rate of un-risky second period is zero, following that $\sum_s \prod_s = 1$. This has removed the assumption of a potential effect of welfare for those who are allowed to invest in the second period. Then, this property states that absolute risk tolerance against value function is an average degree of final consumption risk tolerance, which, in the language of finances is a martingale theory, ie doubling the bet every loss. This property enables us to compare the degree of concavity of *u* and *v*. Suppose, for example, that *u* shows „ absolute hyperbolic aversion to risk" (HARA) that is that T is linear in c. This implies that $Tv(z) = \sum_s \prod_s (c_s{*}) = T(\sum_s \prod_s (c_s{*}) = T(z)$. Thus, when *u* is HARA, the value function has the same degree of concavity like $u{:}v(\cdot) = Ku(\cdot)$. This has the con sequence that the two programs have exactly the same solution. In other words, under HARA preferences, the option of risk taking in the future has no effect on risk exposure today: in this case, myopia is optimum.*Ceteris paribus*, young and less young investors should select the same composition of the portfolio.

Alternatively, assume that the utility function *u* expresses a convex absolute tolerance for risk. Applying the Jensen inequality, it follows that $Tv(z) = \sum_s \prod_s (c_s{*}) = \quad T(\sum_s \prod_s (c_s{*}) = T(z)$: risk taking opportunity in the future increases tolerance against current risks. Assuming that *T*" is non-negative is compatible with the intuition that a wider time horizon would induce additional risk taking that is found in the sign of the fourth derivative of the utility function. On the other hand, if *T'* is non-positive, a longer time horizon for investment might suppose a more conservative investment in the short term.

Assuming a zero non-risk rate, concerning Arrow-Debreu portfolio dynamics with independent serial profits, a longer time horizon raises or reduces the optimal exposure to risk in the short term if the absolute risk tolerance $T(\cdot) = -u'(\cdot)/u''(\cdot)$ is convex or concave. Under HARA, the time horizon has no effect on the optimum portfolio.

Conclusions

When long-term investment is targeted for consumption at a given date, if the investor should amend its exposure to risk as the time horizon narrows, it becomes an empirical issue based on convexity, linearity and concavity tolerance absolute to risk. Of course, none of these conditions of absolute risk tolerance should be applied to all levels of welfare. It may be a risk tolerance that is sometimes convex, sometimes concave. For such an individual will not be able to predict the effect of a longer planning horizon regarding investment strategy. Depending on the circumstances, this individual will sometimes invest more in shares and in other periods will invest more in securities that will be invested in myopic investment conditions.

We might appreciate convexity/concavity of absolute risk tolerance by introspection. Let us remember that that equation in euro optimal amount invested in shares is approximately proportional to T. Under DARA, we have welfare gains. The question is whether we are dealing with an increase at an increasing rate with the wealth increasing. If so, it would be an argument for *T* convex and a positive effect of opening time horizon in risk taking. Most theoretical models use financial utility functions HARA. In these models, investment myopia is optimal, which greatly simplifies the analysis. One might suspect, however, that this assumption is made to simplify things, than for a realist approach. HARA preferences econometric tests are extremely scanty literature resources.

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