SENSITIVITY ANALYSIS METHODS IN UNCERTAINTY ENVIRONMENT

Prof. Alexandru MANOLE PhD.
„ARTIFEX" University of Bucharest
Prof. Constantin ANGHELACHE PhD.
„ARTIFEX" University of Bucharest, Bucharest University of Economic Studies
Assoc. prof. Mădălina Gabriela ANGHEL PhD.
„ARTIFEX" University of Bucharest
Andreea MARINESCU PhD. Student
Bucharest University of Economic Studies

Abstract

The realization of the sensitivity analysis in the practice of investments is a determinant element in the choice of optimum variance. In practice, the variables and data taken into consideration when determining the investment happen to modify. There must be interpreted the variables considered also from the viewpoint of each one's sensitivity. For the control of the investment process, it is mandatory to realize feasibility studies, by taking into account the complexity of the business environment, the change of the influences of some factos or even the occurrence of some that have not been initially considered. The model used in the elaboration of the feasibility study must outline the factorial variables of sensitivity, specifical to uncertainty.

Among the models used for sensitivity analysis, we have emphasized the Monte Carlo simulation model, because it takes into account all posibilities to combine the influence factors on the Net Value Added (VAN). The steps in using the Monte Carlo simulation model are thoroughly presented and explained by precise case studies. Also, we have presented the decision tree method and others, likewise effective.

Key words: *scenario, strategy, estimation, input-output, profitability, sensitivity*

Introduction

When achieving the sensitivity analysis or in the frame of the analysis of various scenarios the effects on the activity should be aimed also in the case that in the economic reality changes of larger amplitude than the forecasted pessimistic estimations take place.

In the managerial language, this is translated to: how much the sales may decrease before the project develops a negative resulting-value. In order to solve this dilemma, the profitableness threshold is calculated from the point of view of two aspects: Financial profitableness threshold: VAN = 0 and Accounting profitableness threshold: net profit = PN = 0.

Literature review

Anghelache (2016) presents the econometric instruments used in economic analyses. Anghelache and Anghel (2014) approach the economic-financial modeling. Anghelache, Manole and Anghel (2016) focus on the asymptotical normality for single equation estimators. Anghelache, Manole and Anghel (2015) apply the regression method for the analysis of influence that some macroeconomic indicators put on the Gross Domestic Product.

Anghelache et. al. (2015) analyze the economic growth from the viewpoint of investments and consumption influence. Anica-Popa and Manole (2008) realize an analzsis of sensitivity for environment investment projects. Bloom (2009) takes into account the impact of uncertainty, similar preoccupations are outlined by Bolton et al (2014), Grenadier and Wang (2007). Hafner and Wallmeier (2008) study the optimum and volatility, Ravi et al. (2014) approach volatility at the macroeconomic level. Saman (2009) studies uncertainty at the macroeconomic level.

Profitability threshold

There are several methods to set up the financial threshold, with the same result at the end of the day. In order to illustrate the calculation, we have chosen the method based on the present values of the cash inflows and outflows.

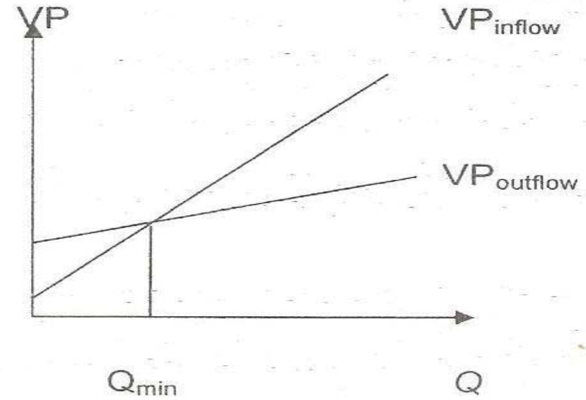
The method is grounded on the following table (for which columns we have allocated values in order to exemplify the calculation):

Q (no. of sold units)	Inflow	Outflow				The present value of the inflows	The present value of the outflows	Net present value =(1)-(2)
	Sales in the years 1-10	Year 0 - investment	Years 1-10					
			CV	F	Tax			
0	0	200	0	50	-35	0	275.27	-275.27
200	400	200	270	50	30	2010.2	1956.3	50.9
220	440	200	297	50	36.5	2207.92	2124	83.92

*) the up-dating is made at the rate of 15%;

**) CFD t are constant over the period n of the investment exploitation which allows the utilization of the factor „a" of annuity: a = (1-1/1.1510)/0.15 = 5.018.

In a graphical interpretation, the profitableness threshold is given by the intersection of the up-dated incomes curve with that of the relating expenses:



Q<Qmin => **VAN**<0

Q>Qmin => **VAN**>0

The Qmin must be established so that the project brings VAN > 0. As much as Q is exceeding the minimum admitted limit, the project is viable.

The calculation formula for the financial profitableness threshold is the following:

QPR = [F+(I0/1-τ)(1/a- τ/n)]/(p-v).

Substituting with our data, we get:

QPR = [50,000 + (200,000/1-0.5)(1/5,018 – 0.5/10)]/(2,000 – 1,350) = 168,769 thousand units.

Consequently, in order that the company records, at this project, VAN >= 0, it must sell at least 168,769 pieces of the product A.

b) In practice, the most utilized is the accounting profitableness threshold, due to the fact that the accounting data are the most available while the calculating relation is a simple one:

Q'PR = (F+I0/n)/(p-v)

The same hypotheses apply, namely: CFDt = constant; linear amortization.

In these conditions, the sales level for which the accounting profit is zero is the following: Q'PR = (50,000 + 200,000/10)/(2,000 – 1,350) = 107.69 thousand pieces

The accounting profitableness threshold is significantly lower as a result of the fact that the opportunity cost of the initial investment of de 200,000,000 lei is not taken into calculation. In the linear amortization the accounting department is recording annually a provision for depreciation of

20,000,000 lei, while the present value of the cash-flows take into account also the possibility to re-invest this amount at the up-dating rate of 15%.

Thus, we state out that the company will fix a profitableness threshold at an accounting level (of 107.69 thousand pieces in our example) and will record a negative VAN for its investment project, as result of losing the re-investment possibility of the cash-flows. Hence, it is a major error to consider the accounting profitableness threshold as limit of the sales, when an investment project is evaluated.

The financial profitableness threshold is providing other valuable information for the company. Thus, the economic elasticity coefficient (operational gear) can be calculated, which provides a measurement of the economic risk (operational, of exploitation):

c= Q0/(Q0-QPR), where Q0 = the sales volume in the initial (basic) situation. In our case c is 200/(200-168.769) = 6.4.

In other words, if the sales volume changes by one percent, the investment project VAN changes by 6.4%. Our project is considered as risky which in fact confirms its vulnerability at the modifications of the selling unit price or at those of the market segment. As closer to the estimated sales volume (under normal conditions) the profitableness program is, the project is riskier.

The Monte Carlo simulation – sensitivity analysis model

The sensitivity analysis is providing us with the effects of the unilateral modification of the essential variables; following up the net up-dated value of the project under more alternative scenarios, we can calculate the effects of a limited number of modifications.

To apply the Monte Carlo simulation, *several stages* are required.

- The model construction

CFDt = [Qt (p-v) – F – I0/n](1- τ)+(I0/n)*T - (Qt – Qt - 1) *p * DACRnet/360.

It is necessary to simplify the model, in the sense that the variables Q, p, v and F of the cash-flows are considered as being independent from each one, the amortization is linear, the tax on profit is in unique quota (T) and the net circulating assets are in perfect positive correlation with the sales volume.

- The identification of the inter-correlations between the factors and introducing them into the model

The interdependence of the sales volume with the cash-flows size and their present value may be measured through the operational lever, as submitted previously. For the five determinant factors of the cash-flow a similar model must be found out, a model which may be in the form of a function like: Y = a+b*x (simple linear correlation).

Y=a+b1x1 + b2x2 + ... + bnxn (multiple linear correlation)

Or, it can be linear of the form :

Y = a*xb (power), y = a*bx (exponential); y = x1a. x2b (Cobb-Douglas) and so on.

The correlation of each factor in the year is dependent in various degrees on the estimations made in the previous years; the coefficient „a" of adjustment may be an error factor statistically recorded between the forecasts and the achievements over a previous period.

This stage has a particular informative value for substantiating the investment decision, this is the reason for applying, as a rule, to the assistance of speciality from experts.

- The association of the probabilities for each simulated dimension of the factors

Similarly to the previous stages, we apply to a set of hypotheses:

a) the normal distribution of the frequency of the different sizes of the analysed factor;

b) the domain of the plausible sizes of the factor = (-2σ;+2σ);

c) a specified size of the square mean deviation of the residual factor σε concerning the deviations of the estimates as against the mean (with μ(ε) = 0) etc.

Under these hypotheses, the computer is associating alleatory numbers for different sizes of the analysed factor and then establishes the possible frequency of the occurrence of the respective size.

- The cash-flows simulation

It is done by computer; this one is selecting a value at random for each essential value (for instance, at the first execution of the model, the chosen value refers to the number of the sold units); the value selected for each variable, together with the values relating to the factors considered as constant, such as the expenses with the tax on profit and the amortization, are utilized in the frame of the model in order to establish the cash-flows of each year; these steps are repeated several times , for instance 500 times, leading to as many cash-flows with a distribution of specific frequency for each year and which, naturally, get closer to a normal distribution[1]. The form of this distribution is outlines a more accurate idea on the implicit risk of the investment project.

1. These cash-flows can be then utilized for establishing the VAN of the project, first at the first execution of the simulation and then at all the 500 values, obtaining thus a probability distribution of VAN. But, the targeted goal in the Monte Carlo simulation consists of obtaining a probable distribution of the cash-flows not of the net present values. [6] Thus, an individual project might have yields with a high degree of uncertainty is it is evaluated as an independent project, but if its profitableness is not correlated with the profitableness of the other assets of the company, the project might be not very risky from the point of view of the risk of company or of market.

We get thus, for each year, a specific distribution of possible cash-flows out of which we calculate the mathematical expectation and dispersion for identifying the expected annual cash-flow and the total risk associated to it. VAN will be calculated through the up-dating of the expected annual cash-flows at the up-dating rate corresponding to the systematic risk (not the total one) of the analysed investment project.

E(VAN) = [E(CF)t/(1+k)t + E(VR)n/(1+k)n] – I0

It is considered that VAN obtained through up-dating the future cash-flows at the of riskless interest (see the foot-note 3) does not include but even avoids to take into account of the risk. In these conditions, the risk is given by the dispersion of VAN. If a project has a certain number of VAN-s possibly to achieve, then it makes no sense to associate the present value with the price term of the investors as to the diversification of the investments portfolio; most of the times it is to be preferred to a singular project due to the additive property of VAN (meaning that if two independent projects get combined, the portfolio VAN is the VAN sum for the projects individually [6]).

It is difficult to estimate the distribution of VAN as much as the riskless interest rate does not coincide with the opportunity cost of the capital; additionally, the discounting at the riskless interest rate is inadequate at the annual cash-flows with a certain dispersion; hence, there is no economic reasoning which might justify the up-dating at this rate.

Meantime, is it known that VAN is obtained by comparing the present value (V0) of the investment with the dimension of the invested capital (I0). Hence, V0 is expressing the present price of the capacity of the project to emit future cash-flows while the distribution of present values with the reference prices of the market against the opportunities of similar investments is irrelevant. The arising conclusion is that what is actually counting is the expected VAN not the dispersed one.

The disadvantage of the Monte Carlo simulation consists of the fact that, besides the time and money expense, it is extremely difficult to estimate the dependences between the studied parameters; meantime, as a result of such analysis, there is no precise rule as to undertaking a decision, as no mechanism is provided in order to show whether the expected yield of the project, measured by the estimated value of VAN, is sufficient to compensate the risk of the project.

The decision tree in the field of investments

The variants of evolution for the variables which influence the value of the financial flows, accompanied by the occurrence probabilities are placed on the arbour branches.

The method of the decision tree is structuring the decisional process carried on by several time moments, providing a sequential, rigorous approach of the different possible situations. Thus, the decisional process concerning the carrying on of the investing process is decomposed in a succession of sequential decisions linked up in the form of an arborescent structure. There are **two types of knots** on the arbour:

- the decisional knots - a decision is taken (for investing or disinvesting), hence, the branch to be followed is chosen (the one of the higher value would be chosen); the value in a decisional knot is equal with the value of the best option ("the direction") which go from the respective knot; they are represented by a square.
- Knots of « event » type – the decision does not belong to the investor; "the fate" is deciding which one from the possible variants, forecasted by the decision maker will be achieved in reality; they are represented by small circles and the value in these knots is an average of the forecasted flows, weighted with the occurrence probabilities (the sum of the probabilities equals 1). Remark: at the STOP points the project becomes non-profitable.

The principle put at the basis of the analysis through the decision tree is expressed by Brealey & Myers as follows: „If the today decision is affecting what you can do tomorrow it must be analysed before acting rationally today".

The arbour construction is made applying the following rule: a knot of event/uncertainty type is preceding a decisional knot if and only if the uncertainty situation submitted in the first knot is solved or eliminated before taking the decision in the second knot. The representation keeps on going on till the entire decisional process is completely described and the calculation of the expected values is started.

All these possible modifications are added to the value of the initial project.

VAN =VAN(CFD expected) + The value of the real option

Thus, the investor will be more motivated to allocate capital knowing, since the very beginning, what is the estimated value of the option for extending the project, as well as whether the abandonment option bears, at this turn, a positive value, under conditions which would prove to be unfavourable.

Example:

An entrepreneur may invest the amount of 100,000$ in a technological line for producing the equipment necessary for winter sports practice. Is the weather is suitable over the entire season time (heavy snow) and the competition strong, incomes amounting 120,000$ can be obtained while in case he is facing a weak competition, the gain may count for 160,000. In the case of an unfavourable evolution of the situation, the business would be abandoned, getting the amount of 40,000$ through disinvestment. If the weather is unfavourable, there is also the option of the readjustment of the technology, for producing also other types of sportive equipment, which requires an additional investment amounting 30,000$; the potential gain obtained in this case amounts 110,000$.

The probability that the weather is favourable to the winter sports practice counts for 75%; in case of favourable development of the weather, the competition for the equipment needed by the practice of these sports would be strong, with a probability of 80%.

The approach of the matter begins with the construction of the possible options, in the form of a decision tree.

The following stage consists of the setting up of the profiles obtained in the final points of each branch of the arbour; excepting the case where there is no investment and, obviously, the gain is zero, the other values obtained in the case of the investment keep on being further on calculated:

Favourable weather Strong competition Incomei 120.000 Investment -100.000 Gain 20.000	Unfavourable weather Weak competition Income 160.000 Investment -100.000 Gain 60.000
Abandon Income from disinvesting 40.000 Investment -100.000 Gain -60.000	Unfavourable weather Modification of the technological process Income 110.000 Investment -100.000 Additional investment -30.000 Gain -20.000

Then, the values of each direction to follow are calculated, from the final knots to the roots, for each knot separately. Now, the arbour would look as follows:

Considering the target of the value maximizing, in the case of the favourable weather, any rational investor would take the decision to continue the business; the value within this decisional mode is:

20,000*80% + 60,000 * 20% = 28,000$

If the weather is nevertheless unfavourable, he would decide to limit his lost at 20,000$ by achieving an additional investment leading to the modification of the technological process. The final step consists of the interpretation of the outcomes of the decision tree. The decision concerning the investment achievement has an average value of 16,000$ as against a null gain obtained in the case when nothing is undertaken. Hence, in the case of a non-aversion to risk, there would be investment. We should not ignore the fact that 16.000$ (which amount is an average of the possible gains) are never to be gained; the amount which might be gained counts for 20.000 or 60.000 or 20.000, respectively 60.000$ might be lost. This implies that the decision tree is based on the hypothesis of the non-aversion to risk, namely these individual gains or losses do not affect the investor. This fact may be true if the investments portfolio is diversified enough and of a big value.

The outcomes are very sensitive also to the considered probabilities. These are subjective estimations. The investor from the previous example anterior considered that there are 75% chances for weather favourable to the winter sports practice. Another person may have estimation, based on different arguments, as regards this model. The probabilities associated to an even are representing, in this case, the confidence had in the forecasts concerning the incomes, merely than the probability that these incomes are achieved.

This is not jeopardizing the utilization of the decision trees; there is no other method being able to eliminate the subjectivism in choosing the probabilities. The decision tree is a way only to get the optimum variant depending on the available information.

In this sense, we can calculate the value of the perfect information, namely the value of the fact to know today which will be the development of a future event, let's assume regarding our example, that a certain information may be obtained that the weather will be favourable to the winter sports practice (for instance, the competent prognosis of a weather forecast institute). We must find out which is the maximum amount which the investor would be disposed to pay for this information.

In the situation of holding this "perfect information", the decision tree is restructured as comparatively with the initial case. The variant of the unfavourable weather will go out of the calculation. The variant of the achievement of the investment would count for 28,000$, irrespectively the competition level. At the moment of buying this information, it is not known what it will reveal but it has been established that the investor is confident in proportion of 75% that this will be favourable for his business. Hence, the decision value (at the arbour root) count now for 21,000$, which makes that the value of the privileged information counts for 21,000 – 16,000 = 5,000$

It is useful that the hypotheses of the model are tested by a decision undertaking. In this respect, we can achieve an analysis of the equilibrium point, namely we set up which is the minimum probability (concerning the favourable evolution of the weather)– p – which may be accepted so that the value of the branch corresponding to the investment achievement is zero, moment when the choice of refraining from investing is preferred. The probability that he weather evolution is unfavourable to the business is 1-p.

Out of the calculations, it is resulting that a confidence of at least 42% in the favourable evolution of the weather must exist in order to consider the investment achievement as worth wise[1]; otherwise, the decision to not invest becomes optimum. The same technique may be utilized also in order to set up an equilibrium point (a minimum probability accepted) concerning the evolution of the competition.

An as much pertinent as possible analysis implies that as many variants as possible should be taken into consideration. What happens if the competition is neither strong, nor weak but medium? What happens if the abandon decision is postponed? What happens if the other variables become uncertain (for instance, the price which might be obtained in the moment of the technology sale)?

Trying to answer these questions, the decision tree becomes extremely complicated. This is the main disadvantage of the method since the massive flows of data (as they are in an ordinary company) require a rapid reaction to the modification of the parameters, but the arbour is not providing any information regarding the cause of their modification. Many times it is necessary only to re-dimension some parameters such as the price or he production level and the non-satisfactory would reduce it probability of occurrence. But the economic reality is by itself complex and a decision tree would not be in the position to cover all the uncertain aspects, irrespectively how complex it is. The analysis through the decision tree will not eliminate the uncertainty through simulation, laborious calculations and probabilities but it would help us in perceiving the mode in which a project is functioning, where and how could we interfere so that the things go better towards the desired direction.

Conclusions

From the analysis made regarding the impact studies on the effects of investments, theoretical conclusions arise, which have been explained through particular studies, using reference data.

Through the profitableness threshold we can emphasize the size of the fix expenses (F) as being the main factor of sensitivity. Consequently, the projects with significant fix expenses in the total expenses would have a bigger profitableness threshold and a higher elasticity coefficient.

The Monte Carlo simulation takes into consideration all the possibilities of combining the influencing factors on VAN, providing a larger picture on the distribution of the cash-flows generated by the studied project; it is achieved with the assistance of the computer, similarly to the sensitivity analysis.

More realistic the utilized model is, more truthful the simulation process is but a too complicated model makes the simulation become slow.

1. This result has been obtained by solving the equation: 28,000*p – 20,000*(p-1) = 0.

The main advantage of the simulation is given by the fact that it submits the entire range of possible outcomes and their probabilities being not a mere punctual estimation. In fact, the Monte Carlo simulation is usually achieved by consulting companies and, many times, with an informative purpose, the person in charge with the analysis is not relying on it.

A present decision of investments is not singular, as it depends on the future evolution of the economy, on other subsequent decisions concerning the extension or the development of the business or, contrary, the abandonment, if the outcomes are not the expected ones.

The decision tree provides us with the possibility to analyse VAN by taking into account the temporal inter-correlation of the cash-flows as well as the real options to extend the investment project, to abandon it or the stand-by option.

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