THEORETICAL ELEMENTS REGARDING THE MODELLING THROUGH THE SUB-SYSTEMS COMBINING

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Abstract

The traditional approach of the macroeconomic models development took into consideration the estimation of an equation one time and the simultaneous setting up of the results. This aspect is valid for sub-models as well. The operations were not always accompanied by the adequate checking of the procedures. The parameters of an equation may be estimated by utilizing the methods of maximum probability with limited information but the parameters of other equations would remain un-restricted. It is well-known the fact that the methods with limited information have been stronger as against the equations wrongly-specified from the system. This fact was verified mainly in the situations when there have been better information or trustful information concerning a sub-multitude of variables. Adopting the methods regarding the limited information was based on certain practical considerations, the goal consisting of the calculations simplification by eliminating the complexity typical to the methods grounded on complete information (maximum probability based on complete information).

Key words: *parameters*, *equation, unemployment, sub-model, sub-system*

1. Introduction

The theoretical approaches generalized for the submitted stages will include criteria and conditions, formulated for the entire system. However, there is the following question mark: are there or not settlement ways in case the complete model is too complex for applying the simultaneous modelling?

The utilization of the models of high degree of aggregation represents a recommendable solution in this respect. These models are simple enough

to facilitate their analysis as a complete system. A series of economic mechanisms, considered as important and relevant for the steps meant to describe adequately the economy, are not taken into account in case of such an approach.

Taking into consideration the fact that we are aiming to set up the structure within sub-models, the characteristics of the general approach allow us to appreciate this as being a gradualism one.

In the context of the two conditions included in the definition of the partial structure, knowing the complete model is not automatically implied. The sub-models may become invalid if the significant explicative variables are excluded from the model. If the correlation model is modified between the included variables and those being excluded, we can see the sub-model lack of validity.

Analysing the last conditions, we can allege that, as ultimate extension of the information, it is possible to develop a model, appraising thus that the structure or the partial structure is representing a modality to solve the Johansen problem previously submitted.

The substantiation of the macroeconomic policies is based on the utilization of the macroeconomic modelling. Thus, the modelling of all the relevant explicative variables through conditioning all the knowledge concerning the institutional conditions in the studied society, acquires a particular significance. Specifications of high degree of aggregation are utilized, characterized by the fact that the groups of gross coefficients are generating combined effects of the included variable. Also, excluding the correlated variables leads to wrong political recommendations. The simultaneity interference, which may occur through the sub-models combination, is a minor issue for the political decision-factors in comparison with these inferences. The Monte Carlo simulations, applied on particular model specifications, allow a profound study on this matter.

We can conclude that the setting up of the long-term properties of the sub-models is of a particular significance. The setting up of a co-integrating equation grants it an invariable character as against the extensions of the mass of information. The property must be established in an adequate way for each case.

On the other hand, this is a property requiring to be set up in every case. The applied principle implies the decomposition of the variables in persistent component and transitory component. We consider that model of equilibrium correction with vectors would include two sub-systems, with the property that the variables associated with a subsystem do not include co-integrating equations relating to the second subsystem. We notice the possibility of occurrence of short-term effects of the variables of a system

on the variables of the second one, as well as the fact that the co-integrating equations of a system may have a certain impact on the development of the other one variables. If none of the interaction types is present, we can talk about a complete separation, the presence of at least one of them leads to a partial separation, concepts correlated with the erogeneity of the variables within one of the subsystems concerning the parameters of the other one. In the frame of the co-integration analysis, it is necessary that both types of sub-models are tested, considered as verifiable hypotheses. .

2. The utilization of the Philips Curve model in macroeconomic analyses

In the frame of the analysis, we treated the essential aspects regarding to the application of the Phillips curve in the present macroeconomic context, the representation of the curve into a co-integrated variable system; the consistency and modifications of the unemployment rate; the uncertainty of the Phillips curve NAIRU estimated; and the reversed Phillips curve status, respectively the Lucas offer curve. .

The significance of Phillips curve in the macroeconomic analyses has been grasped since the years '60. Phillips synthetized an empirical correlation between two macroeconomic indicators of extreme significance, respectively the unemployment rate and the wage increase. The contributions of Samuelson and Solow (1960) aimed the characteristics of the curve as a compromise which the governors are confronted with. The utilization of the regression multi-varied techniques in the estimation of the Phillips curve has been introduced by Lipsey (1960), his interpretation aiming the correlation from the point of view of the classical evolution of the prices, the unemployment rate acting as intermediary between the demand in excess and the fluctuating evolutions on the labour force market. Meantime, the Lipsey regressions take into consideration also the consumption prices, as explicative variable. The Lipsey's approach is known mainly as the Phillips curve of the probabilities. The subsequent researches treated the issue of the difference between the short-term Phillips curve ond the long-term Phillips curve.

Phelps (1968) and Friedman (1968) have studied the natural unemployment rate, corresponding to a long-term vertical curve while Lucas (1972) applied rational probabilities about the relatively uncertain prices of the goods. The achievement of the probabilities leads to the situation where the aggregated offer does not change as against the last period. The existence of some surprise-prices generates a larger gap as from the average level of the long-term production.

Lucas obtained his offer function starting from the reversal of the

long term Phillips curve, developed on the basis of the forecasts achieved by Lipsay, derived through the methods of the microeconomic theory and the characteristics of the rational probabilities.

The approach of Romer (1966), based on the conventional specifications of the aggregated demand, emphasizes a positive correlation between production and inflation as well as a reverse correlation between the unemployment rate and the inflation rate, considerations describing a short-term Phillips curve with neoclassic tint.

Lucas (1976) underlines that for the political deciding factors, the Phillips curve does not represent a decisional model when utilized for the analysis of the concrete data and estimated through the least squares method. The attempt to stimulate the production by augmenting the money offer will lead to the modification of the curve and the deciding factors will be not in the position to influence neither the unemployment, nor the actual production. This is representing the so-called Lucas's criticism.

Considered as a theoretical criticism of the compromise described by the Phillips curve between inflation and unemployment, the Lucas's criticism has been largely developed by the academic literature from the macroeconomic modelling domain. In this respect, the studies of Wallis (1995) have to be remarked. The strong point of the criticism consists of its general character, the criticism being capable to alter any conditioned econometric model. The versions of neo-Keynesian type of the curve are characterized by the occurrence of the causality issue.

The American acceptance on the Phillips curve, synthetized in the triangular model of the inflation, has been formulated and developed by Gordon (1983 and 1997), Staiger and others (2001). These contributions did not pay a big significance of Lucas's criticism, maybe due the fact that the unemployment rate recorded less intense fluctuations within the American economy as comparatively with Europe.

There are more modalities to develop the Phillips curve for an open economy on the basis of the economic theories. A major contribution in this respect belongs to Calmfors (1977), namely the reconciliation between the Phillips curve and the Scandinavian model of the inflation. The Phillips curve can be integrated into a system which takes into consideration the wages and prices through the intermediary of a data series. The application of the co-integration and causality in redefying the model is reflecting the compulsoriness of correcting mechanism of the system equilibrium in the context of the Phillips curve for the main model.

Although observing the main theory of Aukrust, the Phillips curve is a special model because it includes only one of the mechanism of setting up

the wage submitted by Aukrust.

We undertake to focus on the wages of the Phillips curve. According to the concepts developed by Aukrust, we assume that: $(w_{s,t} - q_{s,t} - a_{s,t}) \sim |(0)$ si $(u_t) \sim |(0)$, possible after removing the determinist changes, as well as that the causal structure is "a way" represented by $H4_{mc}$ and $H5_{mc}$.

The principle of the consistency with the assumed co-integration and causality imposes the existence, in the frame of the exposed sector, of a model type EqCM for the nominal salaries rate.

In order to simplify, we assume a first rank dynamics. Thus, the Phillips curve is defined on the basis of the following equations:

$$\begin{aligned} &\bullet\ \Delta w_t = \beta_{w0} - \beta_{w1} u_t + \beta_{w2} \Delta a_t + \beta_{w3} \Delta q_t + \varepsilon_{w,t} \\ &0 \le \beta_{w1}, 0 < \beta_{w2} < 1, 0 < \beta_{w3} < 1 \\ &0 < \beta_{u1} < 1,\ \beta_{u2} > 0, \beta_{u3} \ge 0 \\ &\bullet\ \Delta u_t = \beta_{u0} - \beta_{u1} u_{t-1} + \beta_{u2} (w - q - a)_{t-1} + \beta_{u3} z_{u,t} + \varepsilon_{u,t} \end{aligned} \quad (1)$$

The significance of the utilized notations is that ε_{wt} and ε_{ut} are innovations concerning available information during the period t-1, noted I_{t-1}. The first equation describes the short-term Phillips curve. The interpretation of the second equation is that the profitableness of the sector explains the evolution of the unemployment rate. The term Z_{ut} designates other variables which will lead to the decrease of the unemployment rate, if the other factorial variables do not change. In the structure of the factor z_{ut} there will be a variable which emphasize the economy growing rate, as well as other factorial variables which characterize the labour offer. If the second equation is introduced into the first one, an explicit model for wages is resulting.

If we set up the unemployment main rate, it is necessary to re-write the pervious equation through the relation:

$$\Delta w_t = -\beta_{w1} (u_t - \breve{u}) + \beta_{w2} \Delta a_t + \beta_{w3} \Delta q_t + \varepsilon_{w,t} \quad (2)$$

where

$\breve{u} = \frac{\beta_{w0}}{\beta_{w1}}$ is representing the unemployment rate which does not influence the salaries increase. By utilizing the non-conditioned environment E on both sides of the equality from the previous formula, we shall get the relation:

$$E[\Delta w_t] - g_f - g_a = -\beta_{w1} E[u_t - \breve{u}] + (\beta_{w2} - 1) g_a + (\beta_{w3} - 1) g_f \quad (3)$$

The left side of the equality is zero, in the context of a stationary weight of the salaries. Taking into account the constant increase rate of the

productivity and the constant rate of the external prices increase, we shall get the solution for the equilibrium unemployment rate, in the form of the relation:

$$E[u_t] \equiv u^{phil} = \left(\bar{u} + \frac{\beta_{w2} - 1}{\beta_{w1}} g_a + \frac{\beta_{w3} - 1}{\beta_{w1}} g_f\right) \quad (4)$$

where

u^{phil} represents the equilibrium unemployment rate,

g_a represents the constant rate of the productivity increase,

g_f represents the constant rate of the external prices increase

Consequently, the long-term average of the salaries weight is described by the formula:

$$E[w_t - q_t - a_t] \equiv ws^{phil} = -\frac{\beta_{u0}}{\beta_{u2}} + \frac{\beta_{u1}}{\beta_{u2}} u^{phil} + \frac{\beta_{u3}}{\beta_{u2}} E[z_{u,t}] \quad (5.9)$$

We shall assume that initially the unemployment has a low value. The rate of the salaries increase Δw_0 is established by the short-term Phillips curve. In conformity with the previous equation, the salaries weight is bigger than the long-term equilibrium value, which leads to the increase of the unemployment level and to the diminishing of the salaries increase in the Phillips curve.

The relation $\Delta w_t = \Delta q_t + \Delta a_t$ describes the abrupt form of the curve. In order to set up the curve slope, we apply the relation: $-\beta_{w1}/(1- \beta_{w3})$.

When the salaries increase equals the constant increase in equilibrium condition (g_f+g_a and the unemployment level is given by u^{phil}), it is considered that the stable equilibrium is reached.

The coefficient β_{w3} describes the elasticity of the salaries increase, established without considering the goods prices. This coefficient is significantly influencing the slope of the long-term Phillips curve.

Out of the figure study, we notice a down-warding tendency of the long-term curve, characterized by a sub-unity value of the coefficient β_{w3}. Conventionally, this situation is interpreted as a dynamic inhomogeneity in the salaries setting up.

In the situation when the coefficient equals 1, the interpretation will refer to the dynamic homogeneity while Phillips curve will be vertical. Also, there is an interdependence to be seen between the equilibrium rate u^{phil} and the world inflation g_f.

Numerous macroeconomic studies have been issued ad published on the long-term Phillips curve subject. The fact that employees get full compensation for the inflation rate is a justification of the vertical slope of the long-term Phillips curve.

A normal restriction on the Phillips curve is the relation $\beta_{w3} = 1$, at least in the situation when Δq_t is interpreted as a variable of the probabilities.

The criticisms as to the down-warding slope of the long-term Phillips curve pointed out the fact that its presentation offers a too optimistic picture for the economic policy; it is considered that political decisions makers may bring, permanently, the unemployment rate below the nature rate, by favouring a high level for the inflation rate.

In the context of the real economy, a significant role in configuring the salaries is plaid by the elements resulting out of the analysis of the life cost. In this respect, the present inflation and the delayed inflation are included in the applied researches in the frame of the econometrics domain, as an expression of the importance granted to the life cost in the context of the salaries negotiations.

These grounds are expressed by the below relations:

$$\Delta w_t = \hat{\beta}_{w0} - \hat{\beta}_{w1} u_t + \hat{\beta}_{w2} \Delta a_t + \hat{\beta}_{w3} \Delta q_t + \hat{\beta}_{w4} \Delta p_t + \hat{\varepsilon}_{w,t} \quad (5)$$

$$\Delta u_t = \beta_{u0} - \beta_{u1} u_{t-1} + \beta_{u2} (w - q - a)_{t-1} + \beta_{u3} z_t + \varepsilon_{u,t} \quad (6)$$

$$\Delta p_t = \beta_{p1} (\Delta w_t - \Delta a_t) + \beta_{p2} \Delta q_t + \varepsilon_{p,t} \quad (7)$$

To notice that in the first equation of the system, the modification of the consumption prices Δp_t, is added to the previous relation, by the coefficient $0 \leq \hat{\beta}_{w4} \leq 1$.

Thus, the dynamics of the inflation model takes the advantage, in the context of an open economy, of a complete specification, respectively the system of the Phillips curve. The dynamic properties of the model are accordingly applied also for other instruments of the Phillips curve type. An unitary, common characteristic, of the systems of the kind is the assumption that the natural rate of the unemployment is a stable solution[1].

The equation of the Phillips curve is considered as unstable for a given unemployment rate, since it is singular. In the dynamics of the salaries weights and unemployment rate, the stability is decisively influenced by the balancing mechanism integrated in the equation describing the unemployment rate. Hence, we conclude that the salaries forming based on the Phillips curve cannot be adapted, for a policy aiming to influence unemployment rate; this is grounded also through the idea that only the natural rate of the unemployment corresponds to a stable salaries weight and any other goal concerning the unemployment rate leads invariably to continuous modifications of the salaries weight.

An incomplete system of the Phillips curve kind, cannot be utilized for a realistic and sustainable approach of the stability in the dynamics of the unemployment rate , in other terms, it is not recommended to appeal to the

1. NAIRU - Non-Accelerating Inflation Rate of Unemployment, rata naturală a unemployment

estimation of a curve model with only one equation.

Significant contributions in this respect are to be found in Staiger and others (1997), which is an example of practical study of estimating the natural rate of the unemployment on the basis of such an incomplete system. The authors of the study have taken into consideration the implicit balancing mechanism in the frame of their approach concerning the Phillips curve. The problem appears through totally different facets in Europe, since the unemployment rate is much more instable so that the correlation between the estimated rates and the stability is a reason of present interest for the theoretical and practical studies.

We shall focus on three approaches which may be utilized for estimating a confidence domain of the Phillips curve, of the natural unemployment rate kind .

Since the natural unemployment rate is a non-linear function of a number of regression coefficients, there are no confidence intervals in most of the analysis connected to its value.

Defining the confidence intervals for the natural unemployment rate can be achieved through three methods:

- Wald method;
- Fieller method (approach);
- Method of the probability coefficients.

The last two methods are to be preferred as they are characterized by the finite properties of the studied sample.

The ground of the Wald method, which is considered as the most intuitive one, is the associated standard error and the proportion t related to the estimated coefficients.

Thus, if for the estimation of the Phillips curve for salaries the previously indicated formula of reference is utilized, if we consider a total transition of the productivity increases to the salaries and exclude "the money illusion", the value of the coefficient u_{phil} from the Phillips curve of the type of natural unemployment rate is established by the relation

β_{w0}/ β_{w1}. The estimated value of the coefficient is accordingly set up, through the formula $\frac{\widehat{\beta}_{w0}}{\widehat{\beta}_{w1}}$.

We have already underlined that the equation of the Phillips curve can be re-drawn according to the relation:

$$\Delta w_t - \Delta a_t - \Delta q_t = -\beta_{w1}\left(u_t - u^{phil}\right) + \varepsilon_{wt} \quad (8)$$

Where : the coefficient u^{phil} can be directly estimated through the non-linear least squares method.

Meantime, the configuration of the confidence interval takes into

account the non-restricted estimation of the coefficient u^{phil}, which is set up through the relation $\frac{\widehat{\beta}_{w0}}{\widehat{\beta}_{w1}}$ and a certain domain around that value. Applying the heuristic technique, we shall consider that each value of the coefficient which does not observe the below hypothesis will be included in the confidence interval:

$$H_W: \frac{\beta_{w0}}{\beta_{w1}} = u_0^{phil} \tag{9}$$

If we assume that $Fw\left(u_0^{phil}\right)$ is the F statistic of Wald type for the checking of the parameter H_W, while Pr(·) is representing the probability of its argument, we can allege that the interval $\left[u_{low}^{phil}, u_{high}^{phil}\right]$ defined by $\Pr\left(F_w\left(u_0^{phil}\right)\right) \leq 1-\alpha$, for $u_0^{phil} \in \left[u_{low}^{phil}, u_{high}^{phil}\right]$, is representing a confidence interval of (1-α)%.

We consider that we successfully applied the Wald method when achieving the precise estimation of the elasticity of unemployment rate in the Phillips curve. Un-doubtfully, the accuracy of this estimation is negatively influenced by limited samplings. However, the volume of the information obtained through observation, is establishing the dimension of the sample. A situation where the Wald approach may lead to significant errors is that when β_{w1} is vaguely estimated, which is not statistically significant. It is considered that Wald method does not take into account the mode in which $\frac{\widehat{\beta}_{w0}}{\widehat{\beta}_{w1}}$ is acting when the value of $\widehat{\beta}_{w1}$ is relatively close to zero, the term of relative pointing out the uncertainty in the estimation of β_{w1}.

In the analyses on the basis of the Phillips curve on the European economies, the estimations of βw1 are typically non-significant from statistical point of view and this concern is connected with the calculation of the natural rates through the Phillips curb for Europe. This situation appears, as previously described, due to the fact that $\widehat{u}_n - \widehat{u}_n$ is a non-linear function of the normal distributed estimators ($\widehat{\beta}_{w0}, \widehat{\beta}_{w1}, \widehat{\beta}_{w0}, \widehat{\beta}_{w1}$).

The Fieller method avoids this situation by transforming the non-linear hypothesis into a linear one, namely:

$$H_F: \beta_{w0} - \beta_{w1} u_0^{phil} = 0 \tag{10}$$

The method of the calculation of the probability coefficients implies the setting up of the confidence interval for the considered hypothesis, through the utilization of the statistical coefficient of credibility.

The method is the numerical equivalent with a solution based on

credibility in the situation when the original model is a linear one in its parameters, offering a previous generic justification.

Conclusions

When the estimated Phillips curve does not submit dynamic homogeneity, the value of the relation $\frac{\widehat{\beta}_{w0}}{\widehat{\beta}_{w1}}$ is only a component of the estimation of the natural unemployment rate, in accordance with the basic theory. In this situation, complex, additional calculations are necessary for the natural unemployment rate.

First of all, it is compulsory to take into calculation the co-dispersion of the terms such as $\frac{\widehat{\beta}_{w0}}{\widehat{\beta}_{w1}}$ and $\frac{\widehat{\beta'}_{w3} + \widehat{\beta'}_{w4} - 1}{\widehat{\beta'}_{w1}}$. On the other side, excepting the situation when the gap as against the homogeneity has a high numerical value, the $\left[u^{phil}_{low}, u^{phil}_{high}\right]$ may be representative for the uncertainty associated with the natural rate estimated for the equation of the Phillips curve. Another worthy situation to be mentioned is the index of the monetary conditions, which is also acting within the economy.

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