MODEL REGARDING THE DYNAMIC MANAGEMENT OF SHARES PORTFOLIO

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Abstract

The treatment of dynamic management of share portfolio is part of a theoretical approach and research that were started at the end of the 60s by Merton and Samuelson. They explored and defined the dynamic portfolio within a continuum economy with HARA utility functions. In 1968, Mossin demonstrated that HARA functions are the only one functions for which myopic approach is optimal when there are no serial correlations for the profits. In e 90s, Deaton and Carroll examined the effect of liquidities constraints on the optimal saving behaviour. Later, in 2000, Barberis estimated the significance of return predictability on the American exchange market. In 1999 and 2000, Campbell, Viciera and Barberis estimated this hedging demand numerically. The effect of profit predictability on the optimal structure of the initial portfolio became surprisingly important for an agent with risk aversion equal to 10 and a time strategy developed on ten- year time horizon. The optimal investment in shares represents 40% of the current wealth without predictability. This will climb up to 100% when mean-reversion is considered. But, still in 1986, Detemple already examined the asset demand problem under incomplete information and learning.

Key Words: *Dynamic management, share portfolio, HARA functions, time horizons, predictability, Utility functions*

1. INTRODUCTION

We always ask if our investment decisions might be modeled mathematically and statistically in a way that allows the setting of a dynamic strategy. In other words, we will try to evaluate the effect of the investment horizon of an investor with regard to the risks related to his portfolio.

The usual treatments applied to this subject suggests that the time horizon on short term will lead to excessively conservative strategies. In 1989, Samuelson and others, addressed the following question: "As you grow older and your investment horizon shortens, should you cut down your exposure to lucrative but risky equities?" Conventional wisdom will find an affirmative answer to this question. It will say that the investors with a long-horizon may tolerate a higher risk because they have much more time available to recuperate the transitory loses. The scientific theory does not support this argument. Especially Samuelson, in his papers of 1963 and 1989, considers that this „time diversification" is based of an interpretation fallacy with regard to The Law of the Large Numbers. By repeating an investment pattern during many periods of time does not generate any risk, even beyond long periods of time. This error or fallacy is illustrated by the following problem formulated by Samuelson in 1963. If $\tilde{x}_1, \tilde{x}_2, \ldots, \tilde{x}_n$ are variable expressing wealth independently, identically and they are distributed at random, then $\tilde{x}_1+\tilde{x}_2+\ldots+\tilde{x}_n$ is characterized by a variation *n* times bigger of each of these risks. The following mathematical expression sustains the Law of Large Numbers

$$\frac{1}{n}\sum_{i=1}^{n}\tilde{x}_i$$

(not $\sum_{i=1}^{n}\tilde{x}_i$) tends to $E\tilde{x}_1$ almost surely as *n* tends to infinity. By subdividing, and not by adding risks, by diversification risks can be made disappeared.

2. SOME ASPECTS REGARDING DECISION DIVERSITY

The problems of dynamic decision need a special understanding of the "backward induction" method. Let us suppose that you have to make a sequence of two decisions α_0 in period 0 and α_1 in period 1. Decision α_0 regards exposure to risk with profit z (α_0, *x*) depending upon realization of *x* of a random variable $\tilde{x}$. It is important to notice that *x* is appears after the selecting of α_0, but before you take the decision α_1. The objective *ex ante* is to maximize the expectation of a function *U*a for (α_0, α_1, $\tilde{x}$):

$$\max_{\alpha_0,\alpha_1} EU(z(\alpha_0,\tilde{x}),\alpha_1).$$

"Backward induction" firstly consists in solving the problem of the second period. This set of results is completely resumed by the payoff *z* obtained during the first period. The optimal strategy α_1* for the second period will depend generally upon *z* called the dynamic program state variable. This second period problem contingent to „state *x"* is rewritten as

$$v(z) = \max_{\alpha_1} U(z,\alpha_1)$$

The optimal value of the given objective z is written *(z)*. Function v is called value function or Bellman function. The problem of the first period can be solved by selecting the risk exposure $_0$ which maximizes the expectation of value function $Ev(z(\alpha_0, \tilde{x}))$. By doing so, the decision maker internalizes the effect of his future contingent strategy upon wealth *U,* given the definition *v.* the decision maker is called „dynamically consistent". This technique will transform any dynamic problem into a sequence of static problems by using the value function.

3. MAIN ASPECTS REGARDING INVESTMENT DYNAMICS

Let's consider the following problem. An investor characterized by wealth $_0$ lives for two periods. At the beginning of each period, he has the opportunity to assume a certain risk that will be realized and visible at the end of the first period. It is important to notice that the investor will become aware of loss or gain generated by the risk he assumed in the first period before he becomes aware of the size of the risk he will take in the second period. This situation will give a intrinsic dynamic nature of the problem and will introduce a certain flexibility essential for any dynamic management of the risk. In order to illustrate this, the investors will take less risk in the second period in case they had major portfolio losses in the first period.

To be more precise, let us suppose that the second period problem represents a decision Arrow-Debreu portfolio. There are S possible states of nature $s=0, \dots, S-1$. The dominant uncertainty of the second period is defined by the probability vector $(p0, \dots, p_{s-1})$. Π_s is the unit price of the Arrow-Debreu security associated to state s. Let us suppose that the risk-free rate is zero. this means that a claim paying one Leu for each state of nature should itself cost 1 Leu: In other words, if the investor takes the risk in the second period, he will end by having the same final wealth as head in the first period. Given z as the cumulated wealth at the end of the first period, the investor will select a portfolio $(c0, \dots, c_{s-1})$ that will maximize EU of his wealth at the end of the first period subject to his budget constraints:

$$v(z) = \max_{c_0,\dots,c_{S-1}} \sum_{s=0}^{S-1} p_s u(c_s) \quad \text{subject to} \sum_{s=0}^{S-1} \Pi_s c_s = z$$

During period zero, the investor should make a decision α_0 implying a payoff $z(\alpha_0, x)$, that depends of the realization of x of a random variable $\tilde{x}$. Particularly, this could be another problem of portfolio choice. The optimal exposure to the risk in the period 0 is obtained by solving the following program:

$$\alpha_0^* \in \arg\max_{\alpha_0} Ev(z(\alpha_0, \tilde{x}))$$

We intend to estimate the impact of the opportunity of assuming risk in the second period on the exposure to risk in the first period. In order to attain this, we compare solution α_0* obtained in the dynamic program with the optimal exposure to risk in the first period, when there is no optimal of assuming the risk in the second period. The investor with a short span of life, as well as the myopic investor might select the level α_0 which would maximize of EU of $z(\alpha_0, \tilde{x})$:

$$\hat{\alpha}_0 \in \arg\max_{\alpha_0} Eu(z(\alpha_0, \tilde{x}))$$

The absolute risk tolerance degree of *v* is characterized by the following proposition. *The value function for the Arrow-Debreu portfolio problem has a degree of absolute tolerance given by*

$$T_v(z) = -\frac{v'(z)}{v''(z)} = \sum_{s=0}^{S-1} \Pi_s T(c_s^*)$$

Where *c* is the optimal solution to the problem and T (·)= -u'(·)/u"(·) represents the absolute tolerance to risk for the final consumption.*

Proof. The optimal solution to the problem discussed here is denoted by *c*(z)*. It satisfies the following first order condition:

$$u'(c_s^*(z)) = \xi(z)\pi_s, \quad s = 0, \ldots, S-1$$

where $_s=\Pi_s/p_s$ represents the price per unit of probability, and ξ(z) represents Lagrange multiplier associated for a particular *z.*

4. ELEMENTS REGARDING TIME DIVERSIFICATION

Let us consider a simpler model where the investor has the chance to assume risk at moment *t*1. More detailed, we assume that the payoff for the game of assuming the initial risk is $z(\alpha_0, \tilde{x})$, where α_0 represents a decision variable, and $\tilde{x}$ is some random variable. Further, the investor has certain consumption during remained periods *n*, numbered $t = 0, \ldots, n - 1$. We assume that the investor may save or borrow at a zero interest rate, and he has no opportunity to take risks starting with the time $t = 0$.

Much more, in each period he earns a labor income *y* from work. This problem indicates the same dynamic structure presented previously. In order to determine the optimal exposure to risk in the first period, it is needed to solve first consumption-saving problem as soon as the risky effect is indicated.

As we already indicated, the structure of the problem consumption-saving is essentially the same as the one of portfolio Arrow-Debreu in the previous proposition. The main difference stays in the fact that we don't have state prices, so we have to suppose for all *s*. As to the consumption-saving problem with certainty for dates $t = 0, n - 1$ with zero interest rate. With a zero interest rate, the degree of tolerance to risk on initial wealth equals the sum of the absolute tolerances to risk on consumption over the lifetime of a consumer. For a small initial risk(z), The absolute risk tolerance relative to wealth is proportional to the lifetime of the investor. In this way, an investor who waits to live two times longer than another investor with the same annual income, may invest approximately two times more in stocks than the other one at date $t = 1$. This is the real meaning of „time diversification".

Of course, we assume that there is only one moment in time when consumers assume risk. In the real world, consumers may have stocks and may assume risks any moment. This more realistic assuming will not change the previous result HARA case. By using the backward induction and adding opportunity of assuming future risks might not change concavity of the value function at a random date when HARA is assumed. In this case, investors are myopic regarding the future risks.

5. ELEMENTS REGRADING PREDICTIBILITY IN PORTFOLIO MANAGEMENT

The existence of mean-reversion in stocks returns is a reality recently accepted. Thus, a high return in the risky portfolio today might imply a smaller portfolio return tomorrow. Good news of today might bring bad news for the future opportunity. We take into account this type of predictability of the optimal dynamic portfolio. Evidently, investors will follow a flexible strategy with an optimal exposure to risk conditioned by opportunity organizing. Still, investors will try to anticipate any possible shock with the opportunity setting. They may take into account the possibility of hedging against bad news about their opportunity set. This is quite simple to do if exchanges are statistic correlated with the current returns. The demands for stocks due to this anticipation is called „hedging demand for stocks". Because stocks are considered safer in the long run than in the short run, intuitively it is suggested that an investor with a longer planning horizon will take more risks when younger.

To simplify, we will limit our analysis to a relatively constant risk aversion with a time horizon with two periods. The relatively constant risk aversion implies myopia relative to time horizon when predictability is absent. We assume economy has economy has one risk-free asset with zero return, and one risky asset whose return in period t is denoted by $\tilde{x}_t$, $t =$

0,1. The opportunity set in the second period is completely described by $\tilde{x}_1$. The predictibility comes from the assumption that the distribution of $\tilde{x}_1$ is correlated with $\tilde{x}_0$. We assume that E$\tilde{x}$0 > Investors invest only with the view to their retirement at the end of the second period with no intermediary consumption. In order to determine the optimal demand for the risky asset in the first period, and particularly the hedging component, it is necessary to follow the method presented in section 2.

When returns are somewhat predictable, the hedging demand is defined as ${}_0^*-{}_0^m$. This hedging demand will be positive if derivative H evaluated at ${}_0^m$ is positive. In other words:

$$H'(\alpha_0^m) = E[\tilde{x}_0 h(\tilde{x}_0)(w_0 + \alpha_0^m \tilde{x}_0)^{-\gamma}] \geq 0$$

anytime $E[\tilde{x}_0(w_0+{}_0^m \tilde{x}_0)^{1-\gamma}] = 0$.

When the risk aversion is constant and larger than unity, a longer time horizon should induce investors' wish to take more risks. The opposite is true if the risk aversion is smaller than unity. We note that when investors have a logarithmic utility function (γ =1,) myopia is still optimal in the presence of predictability.

Choosing a initial risk portfolio is dictated by the fall of the marginal value of wealth at the end of initial period. This marginal value of wealth depends on the future opportunities. In case predictability reduces the wealth marginal value in state of abundance, making it grow in states it is small, then predictability has the same effect as a risk aversion reduction: it will raise the optimum level risk in the portfolio. As a consequence, we can see that the central objective of this analysis will be the determination of the effect produced by us by deteriorating the FDS in the return generated by a risky asset on the marginal value of wealth. In the special case of means reversing, we can see two different effects of an increase in x_0. The first effect is the wealth effect: because the return expected for the second period becomes smaller, so becomes wealth which becomes smaller too. This event grows the marginal value of wealth since v is concave in z. The second effect is a precautionary effect: The investors will invest less in the risky asset thus reducing the risk exposure. Under prudence, the investor will reduce the marginal value of wealth. The global effect of an increase in x_0 of the marginal value is ambiguous. When risk aversion is constant and larger than unity (and this happens if and only if the absolute prudence is smaller than two times the absolute risk aversion, that explains why this condition implies the fact that precaution effect is dominated by wealth effect), The wealth effect will always dominate precaution effect, and the hedging demand is positive. When the relative risk aversion is smaller than unity, the wealth effect is dominated by precaution effect.

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