EQUILIBRIUM AND AUTO REGRESSION MODELS USED FOR MACROECONOMIC PROGNOSIS

Assoc.prof. Mădălina-Gabriela ANGHEL PhD
Assoc.prof. Aurelian DIACONU PhD
„ARTIFEX" University of Bucharest

Abstract

The development of econometric models had as prime effect the decrease of critics brought over time on some other types of instruments. In this paper, the authors propose to outline some relevant aspects regarding the making of forecasts through the application of equilibrium and auto-regression models. The prognoses concerning certain classes of the modifications of the parameters out an auto-regressive model are more solid. Hence, in the practical studies one may reach outcomes in which the prognoses of correction type are less correct than those obtained through autoregressive models, which prevent us to assume that a model can function with the same accuracy as to the economic and econometric interpretation as well as to the prognoses accomplishment..

Key words: *model, autoregressive, parameter, prognosis, equation*

Introduction

During the last two decades of the past century, researches and studies have been accomplished on the econometric models which let to their development and diminished the visibility to potential critics being associated with the previous models generations. This approach has been favoured also by the study on the dynamic specifications and the models evaluation, reducing thus the possibility that the models ignoring on a large scale the dynamics and the temporary properties of the data lead to sub-optimal prognoses.

The models evaluated in accordance with the changes occurring within the economic environment, a notable example in this respect being the more detailed modelling of the offer factors, along with the transmission mechanism between the real and financial sectors of the economy. These researches allow us to appreciate that the modern models of the kind of the equilibrium correction might forecast better than the models which are using differentiated data, such as the auto-regressive models.

Models used in prognosis

If we consider that there are constant parameters over the prognosis period, we shall notice that the auto-regressive model is wrongly-specified in comparison with a correctly specified model of equilibrium correction type.

Consequently, the prognosis of the first model will be a weaker one. But, if the parameters change after the prognosis is issued, the second model will be also exposed, being wrongly-specified over the prognosis period. On the other hand, any autoregressive model can be considered as a particular case of the correction model, on the ground that it imposes restrictions of additional unitary root within the system. The change, during the prognosis period, of the parameters associated to the level variables excluded from the model dVAR leads to the erroneous specification of the model EqCM. Thus, the determinant factor is the generating mechanism which is prevailing.

The structural discontinuities are acting differently on the two types of models as far as the prognosis errors are concerned. We take into consideration the fact that the practical models of prognosis are open systems, with exogenous variables. The properties of the studied models are useful for interpreting the prognosis errors of the large systems.

For the beginning, we shall consider that we can treat the macroeconomic chronological series as integral of rank one |(1), which includes determinist terms allowing a linear tendency.

As an example, we can use the simple bi-dimensional system of rank one described by the equations bellow:

$$y_t = k + \lambda_1 y_{t-1} + \lambda_2 x_{t-1} + e_{y,t} \qquad (1)$$

$$x_t = \varphi + x_{t-1} + e_{x,t} \qquad (2)$$

where:

deviations $e_{y,t}$ and $e_{x.t}$ have a normal distribution,

their dispersions are σ_y^2 and σ_x^2 respectively,

$\rho_{y,x}$ is representing the correlations coefficient.

The factor x_t, which is exogenous, describes the opening of the practical models of prognosis. Further on, we shall assume that the above submitted model is a small co-integrated (y_t is also a |(1), but co-integrated with x_t). This assumption will lead us to the inequalities $0 < \lambda_1 < 1$ and $\lambda_2 \neq 0$. On the basis of the system, we can define o conditioned model of correction – equilibrium for y_t, which is simultaneous and marginal model for x_t.
We shall define two parameters, μ and η, through the equations $E[y_t - \beta x_t] = \mu$ and $E[\Delta y_t] = \eta$. If analysing the probabilities from the previous model, we get $E[\Delta xt] = \varphi$ and, meantime, we shall reach the bellow relation between the two parameters:

$$\beta\varphi = \alpha(\varsigma - \mu) \qquad (3)$$

The extraction of the parameter μ from the equation leads to:

$$\mu = \varsigma - \frac{\beta\varphi}{\alpha} = \frac{k - \beta\varphi}{\alpha}\mu = \varsigma - \frac{\beta\varphi}{\alpha} = \frac{k - \beta\varphi}{\alpha} \quad (4)$$

For those achieving prognoses, the linear determinist tendency for several variables of interest is relevant. As an example, we can mention the indicators of the demand and the external prices and the average labour productivity. By their position, other variables, such as, the crude oil prices and the monetary policy instruments, namely the interest rates and the foreign exchange rates are closer to the hypothesis of tendency zero. The representation of the impact of the modifications of the parameters on the prognoses achieved through the two models is important.

The model of equilibrium correction type has been previously defined. The first equation of the model is the conditioned model of y_t. The model has been criticised by the prognoses-make theoreticians and practitioners, it takes into account the impact of the econometric methodology and co-integration theory. The second equation of the model plays the role of marginal equation for the explicative variable x_t. The autoregressive model is defined by the relations:

$$\Delta y_t = \gamma + \pi\Delta x_t + \epsilon_{y,t}\,\Delta y_t = \gamma + \pi\Delta x_t + \epsilon_{y,t} \quad (5)$$

$$\Delta x_t = \varphi + e_{x,t}\,\Delta x_t = \varphi + e_{x,t} \quad (6)$$

where the restriction α = 0 is considered.

The equation $\epsilon_1(y,t)\ (= \varepsilon_1(y,t) - \alpha[y_1(t-1) - [\![\beta x]\!]_1(t-1) - \varsigma]$ is defining the error process in the autoregressive model. We consider that this model will be auto-correlated if an auto-correlation in terms of neglected unbalance exists.

Then, we shall consider that the parameters are known; in prognoses $\mathbf{\Delta x_{T+j}} = \varphi$ (j = 1, ..., h), while the prognoses for the periods T+1, T+1,... T+M are achieved for the period T.

The hypothesis of the known parameters gets abstracted out of the small polling interferences within the correction model. Through the second hypothesis, one of the failure source of the prognosis is invalidated, namely the fact that the non-modelled or exogenous variables are wrongly forecasted but such an idea is practically significant. The prognosis systemic errors in Δx_{T+j} or the change in φ are equivalent in the context of our approach.

We have to consider that the most relevant coefficients in our study are α, β and ζ, namely those coefficients present in the correction model but not in the autoregressive model, although all the other coefficients may change during the prognosis period. We shall study mainly the situation of the coefficients α and ζ, starting from the idea that β is the partial structure, a parameter of co-integration for the analysis of the significance and possibility to detect the changes.

Further on, we shall set up the interferences for the prognoses related to the two models, assuming that both models are wrongly-specified over the prognosis period.

We shall consider two situations:

Firstly, we shall consider that the parameter ζ from the equation of the first mod is evaluating at a new level, namely $\zeta \rightarrow \zeta^*$ subsequent to the prognosis achieved in the period T. Since we keep the constant α is kept in the analysis, we conclude that the modification ζ is the result of a modification in the coefficient k, the segment from the previous equation. Considering the correct situation of equilibrium, we appreciate that over the prognosis period the following relation is applied:

$$\Delta y_{T+h} = \gamma + \pi\Delta \mathbf{x}_{T+h} - \alpha[y_{T+h-1} - \beta x_{T+h-1} - \varsigma^*] + \varepsilon_{y,T+h}\Delta$$

$$\mathbf{x}_{T+h} = \varphi + e_{x,T+h}$$

where h = 1, ..., H.

We shall obtain the following prognosis errors, over the -1, for the two studied models:

$$y_{T+1} - \hat{y}_{T+1,EqCM} = -\alpha[\varsigma - \varsigma^*] + e_{y,T+1} \quad (7)$$

$$y_{T+1} - \hat{y}_{T+1,dVAR} = -\alpha[y_T - \beta x_T - \varsigma^*] + e_{y,T+1} \quad (8)$$

Further on, we shall study the interference of the prognosis errors. We note by $bias_{T+1,EqCM}$ and respectively $bias_{T+1,dVAR}$, the interferences of stage 1, defined through the conditioned probability (I_T) of the prognosis errors. The two interferences are established through the relations :

$$bias_{T+1,EqCM} = -\alpha[\varsigma - \varsigma^*]\, bias_{T+1,EqCM} = -\alpha[\varsigma - \varsigma^*] \quad (9)$$

$$bias_{T+1,dVAR} = -\alpha[y_T - \beta x_T - \varsigma^*] \quad (10)$$

If noting by x_t^o the values of stable status of the process x_t, the corresponding values of the stable status f the process y_t, noted y_t^o, are given by the relation $y_t^o = \mu + \beta x_t^o$.

Out of this definition and of the relation of the interference of the correction model, we can reconfigure the formula for the prognosis error of the autoregressive model, in the form:

$$bias_{T+1,dVAR} = -\alpha\left[(y_T - y_T^o) - \beta(x_T - x_T^o) - \frac{\beta\varphi}{\alpha} + (\varsigma - \varsigma^*)\right]$$

$$= -\alpha\left[(y_T - y_T^o) - \beta(x_T - x_T^o) - \frac{\beta\varphi}{\alpha}\right] + bias_{T+1,EqCM} \quad (11)$$

The prognoses of the two models are altered by the modification of the parameter ζ, in ζ*. The correlation between the two interferences, if the deviations of the initial values from the stable status are neglecting, may be described by a simplified form:

If y_T equals its average on long term, we shall state out that the interferences of the prognosis errors of the stage 1 of the two models are identical. The model dVAR un-restricted of the least squares method is justifying this allegation.

The prognoses over the period– h, are set up by applying the following formulas:

$$bias_{T+h,EqCM} = -\alpha\delta_{(h-1)}[\varsigma - \varsigma^*] \quad (12)$$

$$bias_{T+h,dVAR} = \beta\varphi\left(\alpha\psi_{(h-2)} - \delta_{(h-1)}\right) - \alpha\delta_{(h-1)}[(y_T - y_T^o) - \beta(x_T - x_T^o) + (\varsigma - \varsigma^*)] \quad (13)$$

for prognosis horizons h = 2, 3, ...

Since the prognosis horizon h increases to infinite, $\delta_{h-1} \rightarrow 1/\alpha$, hence the interference of the correction model is approaching asymptomatically the size of the modification itself, namely, $bias_{T+h,EqCM} \rightarrow \varsigma - \varsigma^*$.

We can simplify the expression if we consider that $x_T \approx x_T^o$ and $y_T \approx y_T^o$, and the prognosis errors dVAR include an interference term which occurs since xt increases, a term missing from the interference of the correction model.

The term containing $\delta_{(h-1)}$ and $\psi_{(h-2)}$ can be redefined through $[\alpha\psi_{\downarrow}((h-2)) + \delta_{\downarrow}((h-1))] = h$, and we are reaching a linear simple tendency of the future interference of error dVAR for the stage -h in the case that $\varphi \neq 0$, generalizing thus the outcomes of the stage -1 and stage-2.

If there is no automatic increase of x_t ($\varphi = 0$), while y_T and x_T are equal with their values for stable status, the interferences of the prognosis errors of the two models are identical. If we are facing a determinist positive increase of x_t ($\varphi > 0$), and keeping the stable hypothesis, the interference of the model dVAR will be stronger than the interference of the correction model, due to the tendency from the interference dVAR.

Then, we shall analyse the situation where the adjusting coefficient α gets modified into a new value, namely α^*, after having the prognosis prepared for the entire period.

The comparative analysis of the two interferences allows us to notice that the interference of the correction model is proportional with the dimension of the modification, while the interference dVAR is proportional with the magnitude of the new level of the coefficient of the equilibrium correction.

Assuming that $x_T \approx x_T^o$ and $y_T \approx y_T^o$, we can re-configure the expression of the correlation between the two interferences:

$bias_{T+1,dVAR} = \beta\varphi + bias_{T+1,EqCM}$

The difference between the interferences of the two errors of prognosis has been described previously. If we consider the configuration of more periods of prognosis, the interferences of the errors of prognosis of the models get modified accordingly.

Through a similar proceeding, we get the relation: $bias_{T+h,dVAR}$:

$bias_{T+h,dVAR} = \beta\varphi\boldsymbol{h} + bias_{T+h,EqCM}$

We appreciate that a linear tendency in the difference between the interferences of the errors of prognosis between the two models keeps on being maintained due to the inadequate representation of the increase of the value xt in the model dVAR.

As ζ gets modified in T+1, the prognosis for y_{T+2}, it will lead to the following interferences of the prognosis errors for the studied models:

$$bias_{T+2,EqCM}|I_{T+1} = -\alpha[(\varsigma - \varsigma^{\bullet})] \quad (14)$$

$$bias_{T+2,dVAR}|I_{T+1} = -\alpha[y_{T+1} - \beta x_{T+1} - \varsigma^{\bullet}] \quad (15)$$

The modification of parameter is altering the prognosis error of the model EqCM to the same extent as in the previous situations although the effect of the change is incorporated in the initial value y_{T+1}. We notice that the prognoses of the correction models do not correct the events happening prior the prognosis preparation. Excepting the case when a parameter modification is detected and corrections of segment are made, the effect of the modification of parameter previously the prognosis period will influence all the prognoses.

For the autoregressive models, the matter is extremely different.

Using the fact that:

$$y^{\varphi}_{T+1} = \mu^{\bullet} + \beta x^{\varphi}_{T+1} y^{\varphi}_{T+1} = \mu^{\bullet} + \beta x^{\varphi}_{T+1} \quad (16)$$

where

$$\mu^{\bullet} = \varsigma^{\bullet} + \frac{\beta\varphi}{\alpha}$$

The interference of the autoregressive model can be defined through the relation:

$$bias_{T+2,dVAR}|I_{T+1} = -\alpha\left[(y_{T+1} - y^{\varphi}_{T+1}) - \beta(x_{T+1} - x^{\varphi}_{T+1}) - \frac{\beta\varphi}{\alpha}\right] \approx \beta\varphi \quad (17)$$

The model dVAR will be immune against the modification of parameter if φ = 0. For the prognoses dVAR, there is an inherent element of correction of segment, while the modification of parameter before the beginning of the prognosis period will generate an influence on the prognosis of stage – 1 type dVAR. Meantime, the non-zero tendency of x_t will influence on the prognosis of stage -1 autoregressive and the relative accuracy of the prognosis between the two studied models will depend on the size of the tendency connected to the modification size.

If we have no determinist increases in DGP, the prognosis of type dVAR are not influenced for all the values of h.

The prognosis type EqCM does not adjust automatically when the modification $\alpha \rightarrow \alpha^*$ takes place prior to the prognoses preparation, similarly to the average on long term.

The correction of segment automatically induces modifications of parameter taking place prior the prognosis preparation, does not apply to neither of the two prognoses. Because of this fact, the interferences of stage - 1 are similar from functional point of view with the formulas considered for the case when α gets modified after the prognosis preparation. The generalization of the interferences of the errors of multi – stages prognoses is similar to the previous derivations.

The two types of prognosis models being studied are using estimated parameters. Taking into account that the model dVAR is wrongly – specified as against the model EqCM, the estimates for the parameters will be heterogeneous. Abstraction made of the uncertainty of the estimated parameter, the configuration for the model dVAR is given by the relations:

$$\Delta y_t = \gamma^* + \pi^* \Delta x_t + \epsilon^*_{y,t} \quad (18)$$

$$\Delta x_t = \varphi + e_{x,t} \, \Delta x_t = \varphi + e_{x,t} \quad (19)$$

where :

γ^* and π^* mean limits of probability of the estimates of parameters, during the prognosis period $\gamma^* + \pi^* \Delta x_{T+h} = g \neq 0$,

Hence, the prognosis of autoregressive type of y_{T+h} includes an additional determinist tendency which will not necessarily correspond with the tendency from DGP.

Even if the influence of the parameter is reduced, it can accumulate a dominant linear tendency in the interference of the error of prognosis of autoregressive type.

We define the autoregressive model dRIM, as an opposite to the relation of the previous model. The studies being performed presented ample justifications on the way the models of dVAR type can be successfully strengthened as against wrong representations of tendency.

The outcomes of these systems of prognosis, basically simple, are presenting certain features which can be underlined on the basis of the errors of prognosis of the large macroeconomic models.

Conclusions

The performed analysis shows that none of the studied models secures protection against the discontinuities of post-prognosis type. For the situation when the autoregressive model is excluding the increase, the interferences of the errors of prognosis type dVAR contain a tendency component. Taking into consideration this fact as well as the initial conditions, the autoregressive model may be successfully compared with the correction one, by medium horizons of prognosis.

A significant conception on the autoregressive model is the lack of protection against the modifications occurring before the prognosis, as an average on long term. In this case as well, the approaches for the two models differ.

However, these assertions are not always valid in practice. For the ample models, a structural discontinuity in equations may be ignored or might be interpreted as temporary or similar to a down fall. This risk occurs due to the fact that the data available for the model evaluation are preliminary and susceptible of future revisions.

The relative merits of the correction or autoregressive type models for prognosis depend on the modification of parameters pre- and post-prognosis as well as on the length of the prognosis horizon. We shall utilize this perspective in interpreting the outcomes of prognosis out of a large scale model.

The achievement of forecasts based on the version of model with the smallest error of prognosis, irrespectively the model type, is a difficult approach. For the autoregressive models, the prognosis errors are resistant to the modifications of the adjusting coefficient and to the average on long term ζ, while the prognosis error may prove to be larger than the prognosis error for the correction model.

Based on these grounds, we can achieve prognosis on multiple periods from the econometric model RIMINI. These prognoses can be compared with the prognoses of the models based on differentiated data.

The comparison between the models allows us to appreciate that, since all the stochastic equations of the RIMINI model, are of the type EqCM, we can detect a simplified version of the model, by leaving out the equilibrium correction terms from the equation and re-estimating the coefficients of the differential variables. The resulting differential equations become wrongly-specified, by excluding the terms of correction-equilibrium, with auto-correlated residuals and with variables with heteroskedastic dispersions. The theoretic contestation arises from the idea that the model dVAR is wrongly-specified in the polling frame while the error term from the dVAR is auto-correlated provided that there is auto-correlation in the unbalance terms.

If the coefficients change as against the equilibrium correction during the prognosis period, the autoregressive model might get more favourable outcomes comparatively with the one of correction type. There are no disadvantages for a wrongly-specified autoregressive model in comparison with the model type EqCM. The prognoses of autoregressive type may be altered if the element is maintained and the levels are left out. The dRIMc derivate model implies the re-modelling of all the altered equations, in terms of difference, in order to make the residuals of the dVAR equations, empirically, white noise type, while the segment has been maintained for level variables only.

References

1. Alexandru, C., Caragea, N., Dobre, A.M. (2013) – "*Vector Autoregressive Models Using "R",* SEA - Practical Application of Science, Volume (Year): (2013), Issue (Month): 1 (June), pp. 59-67
2. Anghelache, C., Mitruţ, C-tin (coordonatori), Bugudui, E., Deatcu, C. (2009) – *„Econometrie: studii teoretice şi practice"*, Editura Artifex, Bucureşti
3. Cuaresma, J.C., Piribauer, P. (2015) – "*Bayesian Variable Selection in Spatial Autoregressive Models*", Vienna University of Economics and Business, Department of Economics in Department of Economics Working Papers with number wuwp199.
4. Doornik, J. A., Hendry, D. F. (1997) – „*The Implications for Econometric Modelling of Forecast Failure*", Scottish Journal of Political Economy, 44
5. Eitrheim, Ø., Jansen, E., Nymoen, R. (2002) – „*Progress from forecast failure - the Norwegian consumption function*", Econometrics Journal, 5
6. Franses, Ph.H.B.F., Paap, R. (1999) – *"Forecasting with periodic autoregressive time series models"*, Erasmus University Rotterdam, Erasmus School of Economics (ESE), Econometric Institute in Econometric Institute Research Papers with number EI 9927-/A.
7. Hendry, D.F. (1995) – „*Econometrics and business cycle empirics*", Economic Journal, 105
8. Hendry, D.F. (2002) – „*Applied econometrics without sinning*", Journal of Economic Surveys. 16
9. Mitruţ, C., Şerban, D. (2007) – *„Bazele econometriei în administrarea afacerilor"*, Editura ASE, Bucureşti
10. Wang, L., Li, K., Wang, Z. (2014) – "*Quasi maximum likelihood estimation for simultaneous spatial autoregressive models*", University Library of Munich, Germany in MPRA Paper with number 59901.