Portfolio Management and Predictability

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Abstract

Future developments of states and states of nature of a system are predictable. Portfolio management needs predictability techniques In order to benefit of opportunities. In theory, predictability has no time dimension. Practically, as opportunity is embedded stochastically, there may appear changes of state that are predictable, as in the correlation between returns and stocks. A certain resource reversion might be possible with regard to returns and stock. The predictability of the optimal portfolio management becomes the objective of any investor who follows a flexible strategy based on optimal exposure to risk. Thus, investors will try to anticipate the possible shocks affecting the opportunity set of their investment. More precisely, they will admit the possibility to hedge any bad news concerning the future opportunity set, the so called "myopia" relative to time horizon when predictability is possible. This circumstance is part of the relative risk aversion. We can affirm that predictability has the same effect as a reduction of risk aversion.

Key words: *Portfolio management, predictability, risky assets, hedging demand, planning horizon, conditional distribution, marginal value of wealth.*

Introduction

Predictability represents the ability of an entity or system to value the future developments of the states of nature of a system. Especially with regard to portfolio management, predictability is important as a possibility to detect future laws of the decision process, but also measures to be taken in order to re direct the evolution of management process. Predictability is closely related to opportunity concept, and in correlation with management portfolio it has to do with time management too.

Theoretically, a problem of portfolio management will deal with a settled opportunity which is non-variable in time. In the real world, the opportunity is settled in a stochastic manner, with some changes of state being predictable. For example, predictability is possible in case of the correlation between incomes and stocks. For example, there is a kind of reversibility of resources in case of returns relative to stocks and it was lately accepted. Thus, a large cash-in generated by a risky portfolio implies a smaller cash-in tomorrow. In other words, good news of today will bring bad news in the future regarding opportunities.

Predictability and the optimal dynamic portfolio

When exposing themselves to risky decisions, investors follow a flexible strategy. This is part of the process of opportunity organizing. They will try to anticipate any shock inside the opportunity set. More precisely, they will admit the possibility to hedge any bad news concerning the future opportunity set. This process is not a difficult one if shifts made are statistically correlated with the current returns. The demand for stocks due to this anticipation is called "hedging demand" for stocks. As stocks are considered safer on long terms than on short terms, intuition suggests that an investor with a longer planning horizon will take more risks early in life than one with a shorter planning horizon.

In order, to simplify this picture we will limit our analysis to the constant relative aversion to risk γ with a time horizon of two periods. Constant relative risk aversion implies myopia relative to time horizon when predictability is not possible.

Suppose the economy has one risk-free asset with zer return, and one risky asset in period *t* with return denoted by $\tilde{x}_t$, $t = 0,1$. The opportunity set in the second period comes from the supposition that distribution of $\tilde{x}_1$ is correlated with $\tilde{x}_0$. Suppose that $E\tilde{x}_0 > 0$. Investors invest only with regard to retirement at the end of the second period, with no intermediary consumption.

In order to evaluate the first period optimal demand for the risky asset, particularly the hedging component, we have to follow a certain method. We will begin by solving the problem that confronted the investors in the second period for each possible situation. The news are related to not only the wealth *z* accumulated by that time, but also by the return generated by the risky asset x_0 in the first period. More exactly, the value function *v* is defined by

$$v(z, x_0) = \max_{\alpha} E\left[\frac{(z + \alpha\tilde{x}_1)^{1-\gamma}}{1-\gamma} \,\middle|\, x_0\right] \tag{1}$$

The optimal solution for this program is a separable function $\alpha_1(z,x_0) = a(x_0)z$. This implies that the value function is separable, with $v(z,x_0) = h(x_0) z^{1-\gamma}/(1-\gamma)$, where

$$h(x_0) = E[(1 + a(x_0)\tilde{x}_1)^{1-\gamma} \mid x_0].$$

Let's get back to the first period decision problem. This can be written as

$$\alpha_0^* = \arg\max_{\alpha} H(\alpha) = E\left[h(\tilde{x}_0)\frac{(w_0 + \alpha\tilde{x}_0)^{1-\gamma}}{1-\gamma}\right]. \tag{2}$$

In order to determine the hedging component for risky assets demand, we compare α_0* to the demand for risky assets when there is no predictability, for example when $\tilde{x}_1$ is independent of $\tilde{x}_0$. In this specific case, we know that myopia is optimal. Thus, in the absence of any predictability, investors have to solve the following:

$$\alpha_0^m = \arg\max_{\alpha} E\left[\frac{(w_0 + \alpha\tilde{x}_0)^{1-\gamma}}{1-\gamma}\right]$$

When returns are somehow predictable, the hedgind demand is α_0*-α_0^m. The hedging demand will be positive if the derivative of *H* evaluated at α_0^m is positive. In other words,

$$H'(\alpha_0^m) = E[\tilde{x}_0 h(\tilde{x}_0)(w_0 + \alpha_0^m \tilde{x}_0)^{-\gamma}] \geqslant 0$$

anytime $E\,[\tilde{x}_0\,(w_0 + \alpha_0^m \tilde{x}_0)^{1-\gamma}]=0$.

In order to evaluate a specific type of predictability, let us examine the case of an increase in x_0 which deteriorates the distribution of $\tilde{x}_1$ in sense of first order stochastic dominance (FSD). We have a special case when the stochastic process $(\tilde{x}_0, \tilde{x}_1)$ indicates a mean-reversion. Suppose the conditional distribution of $\tilde{x}_1$ may be written as $\tilde{x}_1 | \tilde{x}_0 = -kx_0 + \tilde{\varepsilon}$, where $\tilde{\varepsilon}$ is considered to be independent of $\tilde{x}_0$ and where k is a positive scalar. As any shift of FSD in $\tilde{x}_1$ diminishes final wealth EU, this assumption implies that $\partial v/\partial x_0$ is negative. Since $v(z,x_0) = h(x_0)z^{1-\gamma}/(1-\gamma)$, *it* results that *h'* should be negative when $\gamma < 1$, and positive when $\gamma > 1$.

Suppose that relative risk aversion γ is larger than unity. As *h'* should be positive in this case, it follows that for all x_0,

$$x_0 h(x_0)(w_0 + \alpha_0^m x_0)^{-\gamma} \geqslant x_0 h(0)(w_0 + \alpha_0^m x_0)^{-\gamma}.$$

Considering the expectations for both sides, it follows in turn that

$$H'(\alpha_0^m) \geqslant h(0)E[\tilde{x}_0(w_0 + \alpha_0^m \tilde{x}_0)^{-\gamma}] = 0.$$

Thus, the hedging demand is positive when the relative risk aversion is larger than unity. In case we have a relative risk aversion less than unity, $\gamma < 1$, than *h'* is negative and the inequality is reversed. This result is rezumed in the following proposition:

Suppose the increase of return in the first period deteriorates the distribution of return of the second period in the sense of the first-order *stochatic dominance. Then, the hedging demandf or risky asset is positive (respective negative) if constant relative risk aversion is larger (respectively smaller) than unity.*

An other interpretation of this result is the following: when the relative risk aversion is constant and larger than unity, a longer time horizon should induce the wish of investors to take more risks. The contrary is true also if the relative risk aversion is smaller than unity. Note the fact that when investors have a logarithmic utility function ($\gamma = 1$) myopia is still optimal in the presence of predictability. The choice of an initial risk portofolio is dictated by the fall of the marginal value of wealth at the end of the first period. This wealth marginal value depends on the future opportunities set. In case predictability reduces the wealth marginal value in its abundence states, making it increase where it is low, then predictability has the same effect as a reduction of the risk aversion: it raises the risk optimal level in the portfolio. as consequence, we observe that the next step of the analysis is to determine the effect of the FSD deteriorating-shift in the return of the risky asset will have on the marginal value of wealth. In the special case of mean reversion, we have two different effects of x_0 increase. The first effect is the effect of wealth: as the return expected in the second period becomes smaller , the same is going with wealth, which becomes smaller. This event raises the wealth marginal value, while v is concave in z. The second effect is a precautionary effect: investors will invest less in the risky asset thus reducing the risk exposure. Under prudence, this reduces the wealth marginal value. The global effect of an increase in x_0 of the marginal value of wealth is ambiguous. When the relative risk aversion is constant and larger unity (and this happens if and only if the absolute prudence is smaller than twice the absolute risk aversion, that explains why this condition implies the fact that precautionary effect is dominated by the wealth effect), then the wealth effect will be always dominant against the precautionary effect, and the hedging demand is positive. When the relative risk aversion is less then unity, the wealth efect is dominated by the precautionary effect.

Conclusions

Risky decision regarding portfolio are implied in management activities with regard to stocks. Let's consider that an opportunity set is important for any investor who manages risky assets. In theory, predictability was not an aspect of interest given the lack of any time horizon assumption. In every day life and experience, opportunity and its management is possible and it explains the changes of states of any process in stochastic expression. Thus some changes of state might be predictable with regard to return and stocks'volume correlation. The predictability of the optimal portfolio management becomes the objective of any investor who follows a flexible strategy based on optimal exposure to risk. Thus, investors will try to anticipate the possible shocks affecting the opportunity set of their investment. More precisely, they will admit the possibility to hedge any bad news concerning the future opportunity set, the so called "myopia" relative to time horizon when predictability is possible. This circumstance is part of the relative risk aversion. When relative risk aversion is larger than unity, hedging demand for risk assets is positive, that means that the time horizon is larger making investors to wish to invest more risky. When the relative risk aversion is less than unity, hedging demand for risky assets is negative, while time horizon is smaller. In presence of predictability, myopia is the optimal solution of the investors. Considering all these, we can affirm that predictability has the same effect as a reduction of risk aversion.

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