Essential aspects regarding the optimal prevention

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Abstract

It is recurrently very likely for decision-makers to wrongly assess risk and consequently to ineffectively put effort in diminishing or avoiding it. Preventing losses or self-protection represent an effort we make in order to reduce the impact of a probable accident. The question is: what would be the level that maximizes the estimated utility of an economic agent? In many cases, the cost-benefit an analysis of prevention is analyzed based on the risk-neutrality assumption. This means that only losses of average size matter. There is no intention of reducing the variability of losses if it doesn’t involve a loss above average. The extreme measure of risk prevention is giving up any type of risk.

Key words: prevention, assets, risk, aversion, optimization

Introduction

Optimality of activities of prevention against loss was initially examined by Ehrlich and Becker (1972). They called this type of activity “self-protection” and proved that insurance and prevention can be either complementary or substitute each other. Effects of potential risks in activities of risk prevention were examined by Briys, Schlesinger si Shulenburg (1991). Dionne si Eeckhoudt (1995) demonstrated that an increase in risk aversion has an ambiguous effect because prevention does not generate a risk reduction as Rothschild si Stiglitz stated. Julien, Salanie si Salanie (1999) showed that an increase in risk aversion led to an increase in optimal level of effort if and only if the initial optimal probability of loss is smaller than the dependent threshold of utility. Chiu (2000) showed for the first time that the third derivative of utility function had an important part in determining the threshold. Jewitt (1989) and Athey (2002) examined the effect of risk aversion on optimal decision for general issues, including prevention as a special case.
2. Aspects regarding prevention in case of risk neutrality

Let us consider the case in which a risk-neutral agent may have the risk of losing a quantity $L$ with probability $p$. The agent may invest in preventive measures to reduce the loss probability. If $e$ is the amount of money invested in prevention, the probability of loss $L$ is $p(e)$. Let us assume that $P\ p$ is a twice differentiable, decreasing and convex function $p' < 0$ and $p'' \geq 0$. The convexity condition is that the prevention activity should have a decreasing marginal productivity. The risk-taker should select $e$ so as to minimize the expected net cost of risk, taking into account the cost of prevention. This can be written:

$$e^* \in \arg\min_{e \geq 0} C(e) = e + p(e)L$$ (1)

Since $C$ is convex in $e$, the first order condition is necessary and sufficient for a minimum. In the end, we assume that $C'(0) < 0$, so as the constraint $e \geq 0$ is not binding.

$$C'(e^*) = 0$$

The preventive optimal investment $e^*$ for a risk neutral agent is defined by:

$$-p'(e^*)L = 1$$ (2)

This equation is the classical optimization condition according to which the marginal cost is equal to marginal benefit. This gives a positive probability of loss, since a total elimination of risk is usually extremely expensive. It is possible to obtain a situation with risk 0 only from the technical point of view, but not from the economic point of view.

Risk aversion and optimal prevention

The hypothesis of risk neutrality is of good approximation when risk is low or can be diversified on the market. Let us consider an estimated utility maximize with an welfare degree $w_0$ who risks to lose quantity $L$ with with $p(e)$. The decision can be written:

$$e^* \in \arg\max_{e \geq 0} V(e) = p(e)u(w_0 - e - L) + (1 - p(e))u(w_0 - e)$$ (3)

With probability $p(e)$, final wealth is $w_0 - e - L$ otherwise is $w_0 - e$. When $u$ is linear, programs (1) and (3) are equivalent, since $V(e) = a-bC(e)$ for $(a,b > 0)$. Comparing optimal prevention $e^*$ of estimated utility maximizer with $e^*$, the optimal prevention of risk neutral decision maker, we observe that risk-aversion agents invest more in risk-prevention and risk takers invest less.
Condition type 2 is not necessarily fulfilled even in the case of risk-aversion.

We have:

\[ V''(e) = -p'\left[u(w_0 - e) - u(w_0 - e - L)\right] + 2p'\left[u'(w_0 - e) - u'(w_0 - e - L)\right] + Eu'', \]

\[ Eu'' = pu''(w_0 - e - L) + (1 - p)u''(w_0 - e) \]

Where the first term is negative, the second and the third term is positive, respectively negative, in case of risk aversion. Consequently, we can not take for a certain fact that \( V \) is concave without establishing supplementary restrictions to \( u \) and \( p \). Assessing this derivative, we come to the conclusion that:

\[ V'(e^n) = -p'(e^n)[u(w_0 - e) - u(w_0 - e - L)] - Eu', \]

where

\[ Eu' = pu'(w_0 - e - L) + (1 - p)u'(w_0 - e). \]

Using the condition (2) we observe that \( V'(e^n) \) only if

\[ \frac{u'(z) - u'(z - L)}{L} \geq p(e^n)u'(z - L) + (1 - p(e^n))u'(z), \]

where \( z = w_0 - e^n \)

The equation can be written:

\[ -p(e^n)[u'(z) - u'(z - L)] \leq \frac{u'(z) - u'(z - L)}{L} - u'(z). \]

The right part of this inequality is positive under risk aversion. Risk aversion increases optimal investment in prevention if and only if the loss probability which is optimal for risk neutral agent is smaller than a critical threshold where:

\[ \overline{p} = \left(\frac{1}{L}[a(z) - a(z - L)] - a'(z)\right)[a'(z - L) - a'(z)]^{-1}. \]

Risk aversion \( \overline{p} \) does not necessarily increase the optimal investment in prevention. More prevention would lead to a reduction in wealth in both cases and risk aversion agents would not like the decreasing welfare.

The critical point is \( \frac{1}{2} \) if the utility function is quadratic, for instance if prudence degree is 0. The quadratic agent measures risk by its variant with a maximum at \( p = 1/2 \).
If \( p^n \) is smaller than \( \frac{1}{2} \), an increase in loss prevention reduces \( p \) and \( \sigma^2 \), which is desirable by risk-aversion quadratic agents. If \( p^n \) is bigger than \( \frac{1}{2} \), an increase in loss prevention reduces \( p \) but increases \( \sigma^2 \).

**Aspects regarding prudence and optimal prevention**

A generally accepted theory is that prudent people invest more in prevention. Yet, this is contradicted by practice. In order to isolate the effect of prudence \((u^{''}>0)\) or imprudence \((u^{'''}<0)\), we limit the analysis to the case in which risk-neutral agents select \( p^n=1/2 \). Risk aversion does not influence optimal investment in prevention for quadratic utility, since quadratic preferences show prudence \( 0, u^{''''}=0 \). In such a situation, a prudent agent would invest more in prevention than a risk-neutral agent? More prevention is optimal if condition (5) is fulfilled, for instance

\[
\frac{u'(z-L)+u'(z)}{2} \leq \frac{u(z)-u(z-L)}{L},
\]

(6)

We assume that agents are imprudent. \((u^{''''}<0)\). Using Jensen’s inequality for each possible value of the integrand below, we get

\[
u(z)-u(z-L) \geq \frac{1}{2}L[u'(z-L)+u'(z)]
\]

If prudence \((u^{''''}>0)\), we obtain the opposite result.

So, in case we assume that a risk-neutral agent takes effort \( e^n \) so as the loss probability should be \( \frac{1}{2} \), all prudent agents will choose an effort smaller \( p^n \) while all imprudent agents choose an effort bigger than \( e^n \).

Prudence increases the marginal value of welfare so it reduces the willingness to invest in financing prudence. So, a prudent agent will save more cautiously as a protection against loss than an imprudent agent.

**Conclusions**

Risk approach is important when analyzing financial decision set. Any decision is based on certain evaluations of the probability of risk emergence. Especially financial decisions need a particular framing given the efforts of altering the decisional circumstances and the nature of risk itself. Investments for risk reducing alter risk distribution as opposed to insurance, which finally alter risks’ consequences financing, normally accepted as loss control. The way the distribution is altered represents a rather complex process. Loss prevention is a certain type of loss control, known also as “self-
prevention”, which is the amount of effort made with the view to reduce the probability of an unwanted event.

References