SOME ASPECTS REGARDING THE EXTENSION OF EDGEWORTH TEST TO NONLINEAR RESTRICTIONS

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Abstract

This paper presents the most important elements of the Edgeworth tests applied to non-linear restrictions. Following a brief introduction, the authors develop on the linear regression with non-linear hypothesis, the variant choice and weight matrix. The authors take into consideration the fact that the popularity of the Wald test is counterbalanced by is lack of invariance to hypothesis modeling.

Key words: *Edgeworth test, linear, hypothesis, restriction, model*

Introduction

Wald test is a popular one, especially due to the fact that is easy to calculate. But, it was proven that Wald test does not represent the best choice in case of non-linear hypotheses, the most frequent mentioned motive being that Walt statistics is not invariant to the algebraic formulation of hypothesis.

Anghelache et. al. (2013), Anghelache and Anghel (2015) develop on the application of regression in economic analyses. Various aspects of Wald tests were studied by Kejriwal et. al. (2013), Qihui and Yu (2013). Park and Philips (1988) have demonstrated that Edgeworth expansion coefficients of Walt statistics depend on formulation. On the other way, Newey and West (1987) have suggested the generalized method of moments (GMM) for the distance of non-linear hypotheses. Thus, in the context of linear regression, their statistics is GMM function evaluated to restrained hypotheses. And when the hypothesis is a linear restriction over parameters, then their test corresponds to Walt statistics. When the hypothesis is non-linear, the two statistics are different. Statistics of distance generalized method of moments is invariant to the algebraic formulation of hypothesis and is robust to heteroskedasticity, contrary to probability rate. There is little known about the GMM statistics behavior and this chapter tries to bring more information by GMM statistics extension in the case considered by Park and Philips (1988), by using explicit matrix to Edgeworth extensions initiated by Park and Philips (1988), pushing forward their approach, using explicit matrix formula for all our expressions, which allows comparisons between statistics.

The result is that the extension of Edgeworth for GMM statistics is strictly simplifying Wald statistics.

Linear regression with non-linear hypothesis

The model is a linear regression of the following form:

$y_i = x'_i\beta + e_i$

E (x_i e_i)= (), i =1,…,n, where x_i and β are each Kx1. Be β_0 the real value of β

The objective is testing the non-linear hypothesis, by testing hypothesis H_0 versus H_1:

H_0:g(β)=0

H_1:g(β)≠0,

where g:R^k→R (1)

Be $\hat{\beta}$ the estimator of β calculated using the method of the smallest squares (OLS)

$\hat{\beta} = (X'X)^{-1}(X'Y)$

and

$V_n = (X'X)^{-1}\Omega_n (X'Y)^{-1}$ (2)

which is the estimator of covariant $\hat{\beta}$ matrix, where Ω_n– estimator of $E(x_i x'_i e_i^2)$.

The statistic test for H_0 is Wald test, but the main weakness of this test is that is not invariant to g hypothesis formulation.

Other applied test is GMM criteria function for the regression model:

$$J(\beta) = (Y-X\beta)'X\Omega_n^{-1}X'(Y-X\beta)$$

The non-restrictive estimator GMM minimizes J (β), β∈R^k, that is

$\hat{\beta}$=argmin J $(\beta) = (X'X)^{-1}(X'Y)$

$\beta \in R^k$

and is identical to estimator OLS. J($\hat{\beta}$)= 0

GMM estimator minimizes J (β) regarding condition(1).

$\breve{\beta}$= argmin J (β)

g(β)=0

Newey-West statistics of distance test is represented by $DM = J(\breve{\beta}) - J(\hat{\beta})$ (4)

This test has more advantages than Wald test, among which, the fact that is invariant to hypothesis formulation (1).

The variant choice and weight matrix

Statistics depend on Ω_n choice. Wald statistics is calculated from unrestricted estimations of $\breve{\beta}$.

A choice for Ω_n is Eicker-White estimator:

$\hat{\Omega}_n = \sum_{i=1}^{n} x_i x_i' \hat{e}_i^2 \hat{\Omega}_n = \sum_{i=1}^{n} x_i x_i' \hat{e}_i^2$ (5)

$\hat{e}_i$ $\hat{e}_i$ $= y_i - x_i'\hat{\beta} y_i - x_i'\hat{\beta}$.

An alternative choice is OLS estimator

$$\ddot{\Omega}_n^0 = X'X\hat{\sigma}^2 \qquad (6)$$

$$\hat{\sigma}^2 = \frac{1}{n-k}\sum_{i=1}^{n}\hat{e}_i^2$$

which is valid under the assumption of heteroscedasticity condition.

$$E\left(e_i^2 \middle| x_i\right) = \sigma^2.$$

Another option would be the calculation of matrix of weights under null hypothesis, which implies a reiterated GMM.

Edgeworth expansions

The objective of this presentation is to use the argument of expansion in order to demonstrate that GMM statistics has Edgeworth approximation for χ_1^2 of Wald superior statistics.

Starting from Park and Philips (1998), we derive the expansion assuming that $e|X \sim N(0, I_n)$ and $X'X = nI_k$, therefore Ω_n=nI_n

So Ω_n=nI_n

We assume that $g(\beta)$ is three times continuum differentiable

$$G_{k\times 1}(\beta) = \frac{\partial}{\partial\beta}g(\beta),\ D_{k\times k}(\beta) = \frac{\partial^2}{\partial\beta\partial\beta'}g(\beta),$$

$$C_{k\times k^2}(\beta) = \frac{\partial}{\partial\beta}((vecD(\beta))',$$

Let us assume that

$$G = G(\beta_0), D = D(\beta_0)$$

And

$$C = C(\beta_0)$$

We define the projection matrixes by the relation:

$$P = G(G'G)^{-1}G'$$
$$\bar{P} = 1 - P.$$

These are defined if $G'G$>0, standard condition for testing the hypothesis.

Let us consider F_W, the cumulative distribution function of W, and F_{DM} the function of DM and F the CDF function of χ^2 distribution.

The asymptotic expansion of W when $n \to \infty$ is given by

$$F_w(x) = F(x - n^{-1}\left(G'G)^{-1}(\alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3)\right) + o(n^{-1}), \qquad (7)$$

Where

$$\alpha_1 = -1/2tr(\bar{P}D\bar{P}D) + 1/4(tr(\bar{P}D))^2$$

$$\alpha_2 = 3/2(tr(PD))\hat{}2 - tr(PDD) - 1/2\,tr(D)tr(PD) - 2/3\,tr(PC \otimes G)$$

and

$\alpha_3 = 1/4(tr(PD))^2$

$$F_{DM}(x) = F(x - n^{-1}(G'G)^{-1}\alpha_1 x) + o(n^{-1}) \quad (8)$$

The asymptotic expansion of DM when n →∞ is given by

Where $\propto_1$ is defined in Theorem 1.

Relation (7) offers a set of a more compact relations, for coefficients $\propto_1$, $\propto_2$, $\propto_3$ which allow a direct comparison with GMM statistics expression. Relation (8) for DM has a new significance:

- The expansion of GMM statistics is a strict simplification of Wald statistics. O(n^{-1}) the expansion of GMM statistics is less modified than χ^2 in comparison to Wald statistics
- Relation (8) shows that just an adjustment is necessary to reach the approximation $O^2(n^{-1})$ at χ^2 distribution
- The invariant $\propto\propto_1$ of formulation (1) is generally true.

Conclusion

I extended the approach of explicit matrix to Edgeworth extensions developed by Park and Philips (1998), I extended the Edgeworth expansion for Wald statistics and I developed a new expansion for GMM statistics. The results limitation is represented by the fact that they were calculated for the restrictive solution of normal regression with a known error of the variant. The simulation report showed an almost perfect performance of DM^{null} statistics.

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