SOME THEORETICAL ASPECTS REGARDING NON-LINEAR ECONOMETRIC MODELS UTILIZED IN ECONOMIC ANALYSES

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Abstract

This paper describes the possibilities to use non-linear econometric models in economic analyses. Using this type of models can complete the analyses made with the help of linear model, generating added value for information achieved by applying econometric models.

Key words: *linear, non-linear, logarithm, log-log, exponential*

The evolution of the economic phenomena is not always observing linear trajectories, as they can be non-linear as well.[1].

The analysis of the correlations between the macroeconomic variables can be made also through non-linear functions which are linearized through transformations. We proceed in this way in order to submit the non-liner model in an equivalent form, simple and easy for interpreting the parameters values or for their estimation.[2].

Thus, if the dependence between two variables is represented by the non-linear regression model, $y_i = a^{x_i}\varepsilon_i$, through logarithms method we get the linear regression model $\ln y_i = \ln b + \ln a \cdot x_i + \ln \varepsilon_i$.

As to estimating the parameters of a non-linear regression model, we proceed to the estimation of the parameters by applying the least squares method. Then, through transformations, we linearize the non-linear function and estimate the parameters by applying the least squares method. Finally, we set up the parameters through numerical methods.

From econometric point of view, the models which can be linearized through logarithms/antilogarithms methods, irrespectively of their form, can be with or without free term able[3].

- **Variations concerning the log-log model**

The free term model (log-log) is of the form of the dependence given by the relation:

$$y_i = a x_i^b \varepsilon_i \tag{1}$$

1. Hendry, D.F. (1995) – „Econometrics and business cycle empirics", Economic Journal, 105

2. Clements, M. P., Hendry, D. F. (1993) – "On the Limitations of Comparing Mean Squared Forecasts Errors", Journal of Forecasting, 12

3. Anghelache, C., Lilea, F.P.C., (2012) - „Econometrie", Editura Artifex, Bucureşti

In this model $a \in R^*_+$ while $a \in R$. The properties of the resulting characteristic are established depending on the sign of the parameter *b*. If this parameter is positive, the resulting characteristic has an up warding trajectory. The down warding tendency of the resulting characteristic is emphasized, through the non-linear regression model, by the negative value of the resulting characteristic exponent.[1].

Applying the logarithms method to the above relation, the outcome is the double logarithmic model, respectively:

$\log y_i = \log a + b \log x_i + \log \varepsilon_i$ (2)

Using the substitutions $y_i^* = k = \log y_i, x_i^* = \log x_i a^* = \log \varepsilon_i$, the linear regression model becomes:

$$y_i^* = a^* + bx_i^* + \varepsilon_i^*$$

We estimate the two parameters of the linear regression model and establish the parameter „a" which appears in the non-linear regression model.

$$\hat{a} = 10^{\hat{a}^*}$$

The free term model (log-log) has an additional free term and shows the following form[2]:

$$y_i = a_0 + a x_i^b \varepsilon_i \quad (3)$$

In the case of this model, the utilization of the previous linearization method is no more possible. For the estimation of the parameters, the following two stages are to be run over:

- If the is a value specified for the free term of the model, then by using the notations $v_i = y_i - a_0$ and $u_i = x_i$, we shall get the free term regression model. For this one the parameters are estimated similarly to the case of the double logarithmic model.

- Then, we estimate the three parameters of the model through numerical methods. We can resort to the transformation of the model in a linear one by using the Taylor series development.

We submit now a number of properties of the parameters required for the interpretation of the model parameters and of the characteristics of the factorial variable as against the parameters values. The interpretations are achieved in the context of utilizing the linearized model, respectively:

- if *b < 0,* the log-log function is down warding as against the factorial characteristic.

In this case $\lim_{x \to \infty} y_i(x_i) = 0$; in the situation of the free term model, $\lim_{x \to \infty} y_i(x_i) = a_0$;

1. Eitrheim, Ø., Jansen, E., Nymoen, R. (2002) – "Progress from forecast failure—the Norwegian consumption function", Econometrics Journal, 5

2. Mitruţ, C., Şerban, D. (2007) – „Bazele econometriei în administrarea afacerilor", Editura ASE, Bucureşti

- if $b > 0$, the non-linear function is up warding and $\lim_{x\to\infty} y_i(x_i) = \infty$;

- irrespectively of the sign of the parameter b, this one is equal to the elasticity of the resulting variable calculated in connection with the factorial variable, namely:

$$b = \frac{\partial y_i}{\partial x_i} : \frac{y_i}{x_i}; \qquad (4)$$

Under these circumstances, if the second degree differential is $\frac{\partial^2 y_i}{\partial x_i^2} = ab(b-1)x_i^{b-2}$, it is resulting: $b \in (0,1)$. The analytical function is up warding and concave; $b = 1$, the regression model is nothing else but the simple linear model, without free term; $b > 1$, the function is up warding and convex[1].

Further on we shall submit a number of possibilities for bringing to linearity of certain specific functions of regression, applicable in the macroeconomic analyses, such as the models: exponential, hyperbolic, parabolic, polynomial and multiplicative.

• General notions concerning the exponential model

The exponential model is utilized in the case the points cloud resulting as a consequence of the graphical representation of the values series $(x_i, y_i)_{i=\overline{1,n}}$ is directed along the curve of an exponential function[2].

The exponential model, with the parameters a and b, is defined through the relation:

$$y_i = a \cdot b^{x_i} \varepsilon_i, a, b \in R_+^* \qquad (5)$$

The estimation of the parameters of the exponential model is achieved through data transformations through logarithmic method, going over the following stages:

- through the logarithmic method applied to the equality terms we get the linear regression model:

$$\ln y_i = \ln a + \ln b \cdot x_i + \ln \varepsilon_i \qquad (6)$$

The model becomes linear by substituting $u_i = \ln y_i, \eta_i = \ln x_i, a^* = \ln a$ and $b^* = b$;

- we estimate the parameters of the linear regression parameters, $u_i = a^* + b^* x_i + \eta_i$ applying the least squares method; we get the estimators $\hat{a}^*$ and $\hat{b}^*$;

- the estimators of the non-linear regression model parameters are set up: $\hat{a} = e^{\hat{a}^*}$ and $\hat{b} = e^{\hat{b}^*}$

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2. Bardsen, G. şi colaboratorii (2005) - "The Econometrics of Macroeconomic Modelling", Oxford University Press

Finally, we calculate the adjusted values on the basis of the estimated non-linear regression model:

$$\hat{y}_i = \hat{a}\left(\hat{b}\right)^{x_i}, i = \overline{1,n}$$

The exponential model is utilized when the values of the resulting variable are increasing with an arithmetic progression and the values of the factorial variable are increasing with a geometrical progression.[1].

In order to interpret the significance of the parameter *b* we start from the relation:

$$b = \frac{1}{y} \cdot \frac{\partial y}{\partial x} \tag{7}$$

It is noticeable that the parameter *b* is defining the rate of the increase of the resulting characteristic depending on the factorial variable *X*.

In the exponential model there are the following situations to separate:

- *b* is the rate of increase or decrease of the characteristic *Y* as against *X;*
- if $b > 1$, the evolution of the characteristic *Y* is up warding;
- when $b \in (0,1)$, the characteristic *Y* is recording a decrease as against the variable *X;*
- the values of the characteristic *Y* are positive only and the parameter *a* is satisfying the positivity property.

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