Chapter 10

Models used in Macroeconomic Analyses*

The semi-logarithmic[1] and the double logarithmic models are the two models which can be linearized:

- The logarithmic model can be either without free term or with free term.
- The free term model (log-log) is of the dependence form, respectively:

$y_i = \alpha \cdot x_i^{\beta} \cdot \varepsilon_i$

In this model $\alpha \in R_+^*$ and $\beta \in R$. Depending of the sign of the parameter b the properties of the resulting characteristic are set up.

If this parameter is positive, the resulting characteristic has an up warding trajectory. The down warding trajectory of the resulting characteristic is emphasized, in the case of the regression non-linear model, by the negative value of the resulting characteristic exponent.

Applying the logarithms the double logarithmic model results

$\log y_i = \log \alpha + \beta \cdot \log x_i + \log \varepsilon_i$

Using the substitutions

$y_i^* = k = logy_i$, $x_i^* = logx_i \cdot \alpha^* + log\varepsilon_i$, the regression linear model becomes:

$y_i^* = \alpha^* + \beta x_i^* + \varepsilon_i^*$

The free term model (log-log) holds, in addition, a free term and shows under the following form:

$y_i = \alpha_0 \cdot \alpha x_i^{\beta} \cdot \varepsilon_i$

In the case of this model applying the previous procedure of linearization is no more possible. In order to estimate the parameters, one of the following two methods applies:

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* This chapter includes elements presented in the *Using Linear and Non-linear Models in Macroeconomic Analyses*, authors prof. Constantin Anghelache PhD, Ligia Prodan PhD student, Daniel Dumitrescu PhD student, Diana Valentina Soare PhD student, Georgeta Bardaşu (Lixandru) PhD student, RRS Supplement no. 1/2014

[1] Romanian Statistical Review – supliment – December 2013

- when a value of the free term of the model is specified, then, using the notations $\nu_i = y_i - \alpha_0$ and $u_i = x_i$, we get the regression model $y_i = \alpha \cdot x_i^{\beta} \cdot \varepsilon_i$. In this respect, parameters are estimated according to the case of the double logarithmic model;
- then we estimate the three parameters of the model through numerical models. It is possible to transform the model into a linear one using the development of the Taylor series.

The exponential model is used in the case when the points cloud resulting from the graphical representation of the series of values $(x_i, y_i)_{i=\overline{1,n}}$ is directed along the curve of an exponential function.

The exponential model, with the parameters a and b, is defined through the relation:

$$y_i = \alpha \cdot \beta^{x_i} \cdot \varepsilon_i, \alpha, \beta \in R_+^*$$

The estimation of the parameters of the exponential model is made through data transformations by logarithms, following the stages:

- by logarithms applied to the equality terms we get the regression linear model:

$$ln\, y_i = ln\, \alpha + ln\beta \cdot x_i + ln\varepsilon_i$$

The model[2] becomes a linear by the substitution of $u_i = lny_i$, $\eta_i = lnx_i$, $\alpha^* = ln\alpha$ and $\beta^* = \beta$;

- we estimate the parameters of the regression linear model,
- $u_i = \alpha^* + \beta^* x_i + \eta_i$ using the smallest squares method; we get the estimators $\hat{\alpha}^*$ and $\hat{\beta}^*$;
- the estimators of the parameters of the regression non-linear model are established:

  $\hat{\alpha} = e^{\hat{\alpha}^*}$ and $\hat{\beta} = e^{\hat{\beta}^*}$

Finally, we calculate the values adjusted on the basis of the estimates regression non-linear model:

[2] Romanian Statistical Review – supliment – December 2013

$\hat{y}_i = \hat{\alpha}(\hat{\beta})^{x_i}, i = \overline{1,n}$

The exponential model is used when the values of the resulting variable increase in an arithmetic progression while the values of the factorial variable increase in a geometrical progression.

In the case of the exponential model we consider the following situations:

- β is the rate of increasing or decreasing of the characteristic Y as against X;
- if $\beta > 1$, he evolution of the characteristic Y is up warding;
- if: $\beta \in (0,1)$, the characteristic Y records a decrease as against the variable X;
- the values of the characteristic Y are positive only and the parameter a satisfies the positivity property.

We have realized an analysis of the correlation between total production and services branch using Eviews.

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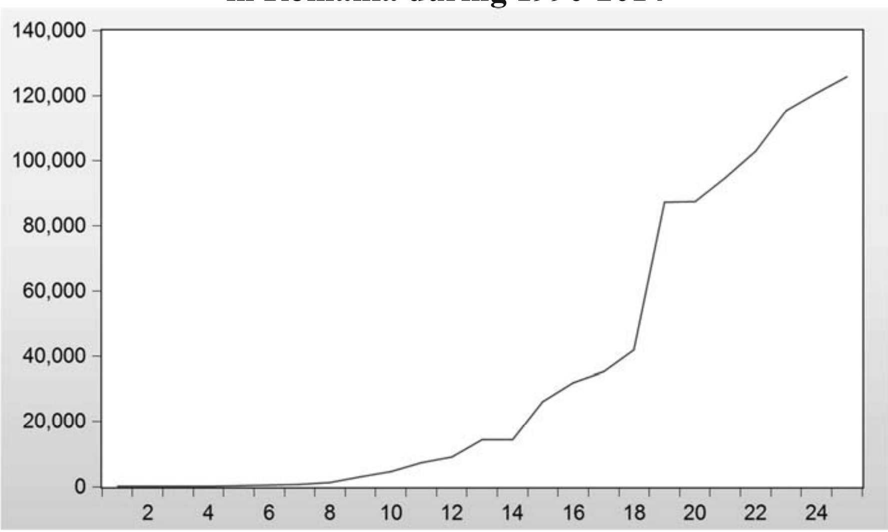
Year	Services	Total production
	x	y
1990	4.6	79.1
1991	12.9	206.4
1992	36.3	606.9
1993	111.2	1906.5
1994	287.1	4700.1
1995	454.6	6746.9
1996	613.2	10197.1
1997	1191.6	23036.5
1998	3109.0	33711.2
1999	4541.1	48888.2
2000	7362.5	71990.9
2001	9165.8	106082.2
2002	14344.7	136922.3
2003	14344.7	166602.3
2004	26088.1	220931.3
2005	32049.6	244676.8

Year	Services	Total production
	x	y
2006	35312.4	289695.6
2007	41950.2	350845.6
2008	87318.2	458535.5
2009	87405.3	450979.1
2010	94723.3	466397.0
2011	102884.4	487733.2
2012	115407.1	512112.2
2013	120652,4	535385.7
2014	125898.0	558659.9

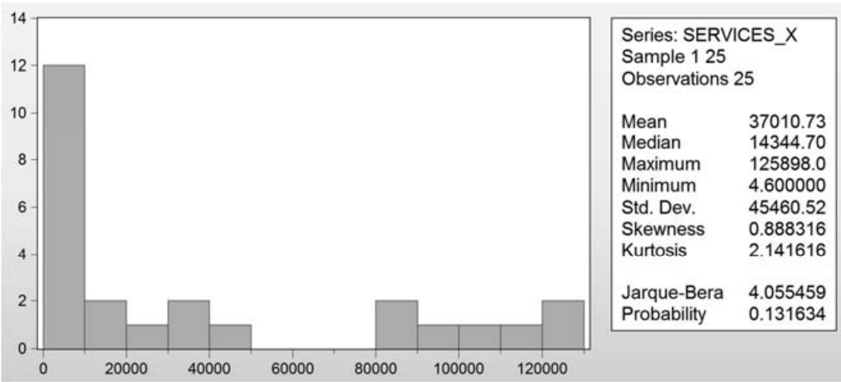
The correlation between services – and the total production highlights that the evolution of two indicators can be related. And from the analysis using software Eviews we can establish that the correlation is significant for the period 2008-2014 when as services register values increasing to establish and increase the total production.

The evolution of the labor in other activities and services branch in Romania during 1990-2014



For an pertinent analyze of the evolution of labor in services it is necessary in a first step of this research to establish the growth of other activities and services in the period under review. To prove this, using the software Eviews, we studied in the first stage, the evolution of this indicator. As can be seen from analyzing the data series under investigation, especially as in the figure shown above, in the period considered, the labor in services branch has registered a steady growth from year to year, except to this rule making the period between 2008 and 2014, when the growth kept getting bigger from an year to another.

Statistical tests performed on the value of labor in services branch in Romania in the period 1990-2014 are graphically represented:



Characteristics of the regression model, as drawn from Eviews, are described in the diagram below:

Sample: 1 25
Included observations: 25
TOTAL_PRODUCTION_Y=C(1)+SERVICES_X*C(2)

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	42878.88	12727.84	3.368904	0.0027
C(2)	4.448068	0.219779	20.23878	0.0000

R-squared	0.946834	Mean dependent var	207505.1
Adjusted R-squared	0.944523	S.D. dependent var	207811.2
S.E. of regression	48947.11	Akaike info criterion	24.51149
Sum squared resid	5.51E+10	Schwarz criterion	24.60900
Log likelihood	-304.3936	Hannan-Quinn criter.	24.53853
F-statistic	409.6083	Durbin-Watson stat	0.259435
Prob(F-statistic)	0.000000		