Chapter 3

Econometric models utilized for the portfolio selection*

In order to make the choice of the regression function, the points $(R_t, \Delta i_t)$ for all the periods t are graphically represented, in the Cartesian of axis, the points $(R_t, \Delta i_t)$ for all the periods t. A points cloud is thus generated which stands at the basis of forming the dependence between the two variables.

If these points aligned along a line, then the dependence between the two variables is a linear one:

$$R_t = \alpha + \beta \Delta i_t + \varepsilon_t$$
 where:

- the parameter α is quantifying the component of the total yield of the independent equity as against the fluctuation of the index of the exogenous characteristic from the regression linear model;
- the parameter β is fixing the extent to which the alteration by one per cent of the index of the exogenous characteristic is generating the increase or the decrease of the equity yield;
- ε_t is representing the residual variable of the regression linear model, which quantifies the alleatory fluctuation of the equity yield under the influence of factors other than the recorded one.
- through the intermediary of this model, the factors acting on the equities yield are divided into two classes: macroeconomic factors, acting to a larger or a smaller extent on all the equities: the inflation rate of the economy, the performance indicators of the economic environment or financial markets etc. Out of these factors, the choice goes

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^{*} Aspects inserted in this chapter were described in the article *The model of W.F. Sharpe and the model of the global regression utilized for the portfolio selection* RRS supplement no. 7/2014, prof. Constantin Anghelache PhD, lecturer Mădălina Anghel PhD

to that one depending on which we want to define the regression linear model and microeconomic factors, acting on the yield of an equity or group of equities. These factors are quantified in the frame of the model through the residual variable.

In order to estimate the two parameters of the regression line, the method of the least squares is resorted to. For defining the estimators, we have to keep in mind that the residual variable (ε_t) is satisfying the following hypotheses¹:

- each residual has a null mathematical expectation so that $E(\varepsilon_t) = 0$;
- the variables ε'_t şi ε_t are not correlated since the hypothesis of non-correlation for the specific risks of the equities, taken two by two, is admitted;
- the residual variables are normally distributed: $\varepsilon_t \in N(0, \sigma_{\varepsilon}), (\forall (t) = \overline{1, m}).$

Following the application of the least squares method, an estimator of the volatility coefficient (β) is established through the relation:

$$\hat{\beta} = \frac{cov(R_{t,}\Delta i)}{\sigma_{\Delta}^2} = \rho_{t\Delta} \frac{\sigma_t}{\sigma_{\Delta}}$$

where:

- $\rho_{t\Delta}$ is the linear coefficient of correlation calculated in order to measure the linear dependence between the equity yield and the market index:

- σ_t represents the standard deviation calculated in the case of the equity yield;

 $-\sigma_{\Delta}$ is the standard deviation of the financial market index;

Depending on the value of the parameter β , the following types of equities are identified²:

¹ Anghel M.G. (2014) – Econometric Model Applied in the Analysis of the Correlation between Some of the Macroeconomic Variables, Revista Română de Statistică – Supliment Nr. 1

² Anghel, M.G. (2010) – "*Utilizarea modelelor econometrice în analizele economice*", Simpozionul ştiințific internațional "Necesitatea reformei economico – sociale a României în contextul crizei globale", Editura Artifex, București

- if β < 0, then the equity yield is developing differently as against the general tendency of the financial market;
- in the situation of $\beta \in (0,1)$, then the equity has a low volatility. In this case, the variation of the equity yield is lower as against the changes of market index from one period to another. Thus, an example of equity of low volatility is given by the case when the market index changes by 8% in the conditions of a variation of the equity yield of 5% only;
- in case that $\beta = 1$ the equity is neutral. For a neutral equity the yield changes to the same extent as the financial market index. For instance, if the index of the financial market is recording a fluctuation of 2% then the equity yield will record the same increase;
- if $\beta > 1$, then the equities bear a high volatility. The equities of this class have a high sensitivity to the overall fluctuations of the financial market. These equities are of an increased interest for the speculators on the financial markets. In this case, the equity risk is extremely high. For instance, for a fluctuation of 3% of the market price, a fluctuation of the yield higher than 3% is recorded for the equities of this class.
- In the financial practice, the outcomes given by this method are regarded with certain reluctance since, while the parameter β is calculated on the basis of a data series from the past, the risk refers to a period in the future.

This method is substantiated on the hypothesis that the yield of equity is fluctuating as against the global yield of the market where it is transacted or as against the overall performances of the economic environment.

Through the intermediary of the global regression, the number of the operations involved by the calculation of the global yield and the total risk is getting significantly reduced.