Estimation and Variance Decomposition in a Small-size DSGE Model

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Abstract

The purpose of this study is to make a grounded estimation-based analysis of the Romanian economy, considering several central economic variables, aggregated at macroeconomic level. By resorting to a basic dynamic stochastic general equilibrium model, the final equations of which are briefly described herein, and to the Bayes theorem imposing the setting of priors for the parameters to be estimated, the paper reveals the output obtained, in compliance with the methodological steps previously rendered, the main results, relating to the check of plots, the posterior distribution and the variance decomposition of the model variables being presented and discussed from an economic perspective.

Key words: *dynamic stochastic general equilibrium model, Bayes theorem, parameter estimation, posterior distribution, variance decomposition*

JEL codes: *C11, C13, C52, C68*

1. Introduction

The intention of the study is to estimate a last generation dynamic stochastic general equilibrium model prototype by resorting to the Bayesian analysis. The small model called in this paper takes into account two representative rational economic agents, households and firms, acting on a monopolistic market and aiming at optimising their characteristic functions, maximisation of satisfaction for the first one and minimisation of costs for the second one.

The model deals with 8 endogenous variables, among which 3 state variables, 4 jumpers, 3 static variables and one observable, namely the gross domestic product. The latter, used in logarithm and differentiated, is associated with quarterly data taken from NIS, ranging form 2000Q1 to 2014Q2. The analysis, specific to the real business cycle approach, considers also the occurrence of 2 technology level-related structural shocks which impact on the model variables, determinant for their fluctuation in time: the permanent technology level shock and the investment-specific technology level shock, as in Greenwood et al. (2000).

The paper is structured according to the following pattern: section 2 - Model and Data - renders the model selected equations transposed into the Dynare code to be implemented in Matlab, as well as some preliminary information on the model data, section 3 - Methodology, Results and Discussions - starts by listing several steps in undergoing a dynamic stochastic general equilibrium estimation of parameters and continues with the description of the posterior distribution of parameters, with the representation of the log-posterior and log-likelihood kernel check plots and with the analysis of the variance decomposition of the model variables, all of them construed accordingly, and section 4 - Conclusions - synthesises the main results obtained.

2. Model and Data

The model approached in this paper is a small standard RBC (Real Business Cycle) model, similar to the one rendered by Griffoli (2010), but reconsidered from the perspective of the real wage and real interest rate equations. For concision reasons, only the final equations, prepared for implementation in Dynare (Matlab), are presented below:

- F.O.C. related to consumption, as one solution to the household problem:

$$\frac{1}{c_t} = \beta \times \mathrm{E}_t\left[\frac{1}{c_{t+1}} \times \alpha \times k^{\alpha-1} \times \left(e^{z_{1,t+1}} \times l_{t+1}\right)^{1-\alpha} + (1-\delta) \times e^{\left(z_{2,t} - z_{2,t+1}\right)}\right] \qquad [1]$$

where c represents the household consumption, β, the subjective discount factor, E_t, the value expected at time t, α, the capital share in total production, k_t, the capital stock held, l_t, the labour force provided by household, δ, the capital depreciation rate and z_t, the technology level, captured in two instances (permanent technology level and investment-specific technology level)

- F.O.C. related to labour force, as the complementary solution to the same problem above:

$$\zeta \times \frac{c_t}{1-l_t} = (1-\alpha)\times k_{t-1}^{\alpha} \times \left(e^{z_{1,t}}\right)^{1-\alpha} \times l_t^{-\alpha} \quad [2]$$

where ζ represents the spare time utility parameter

- The Cobb-Douglas production function of firms:

$$y_t = k_{t-1}^{\alpha} \times \left(e^{z_{1,t}} \times l_t\right)^{1-\alpha} \quad [3]$$

where y_t represents the production obtained by firms

- The equation of capital stock used by firms:

$$k_t = (1-\delta)\times k_{t-1} + e^{z_{2,t}} \times i_t \quad [4]$$

where i_t represents the investments

- The aggregate demand-supply equilibrium on the market of goods and services:

$$y_t = c_t + i_t \quad [5]$$

- The technology level-related AR(1) processes:

$$z_{1,t} = \rho_1 \times z_{1,t-1} + e_{1,t} \quad [6]$$

$$z_{2,t} = \rho_2 \times z_{2,t-1} + e_{2,t} \quad [7]$$

If several endogenous variables are deduced within the model, the level of production is previously given, under the differentiated logarithmic form of the 2000 quarterly-based average prices expressed in million RON, for fourteen years and a half since the baseline time, the related data being provided by NIS. Also, the prior distribution of the model parameters is established according to the literature in the matter, considering the previously obtained results specific to our country. The technology level shocks are deemed to represent an essential element of the analysis, being the main generating source of fluctuations for the model variables.

3. Methodology, Results and Discussions

The method used to estimate the model parameters, based on the Bayesian analysis, involves the establishment of the log-likelihood:

$$\ln L\left(\Theta_M \middle| X_T, M\right) = -\frac{T\times n}{2}\times \ln(2\pi) - \frac{1}{2}\ln\left|\Sigma_{X_T}\right| - \frac{1}{2}\times\left(X_T - \bar{X}_T\right)' \times \Sigma_{X_T}^{-1} \times \left(X_T - \bar{X}_T\right)$$

where $L(\Theta_M | X_T, M)$ is the log-likelihood funion, X_T, the observable endogenous variables, n, the number of observables, $\bar{X}_T$, the expected value of X_T and Σ_{X_T}, its var-covariance matrix and the determination of the posterior log-kernel:

$$\ln K(\Theta_M|X_T, M) = \ln L(\Theta_M|X_T, M) + \ln p(\Theta_M|M)$$

where $P(\Theta_M | M)$ is the prior distribution of the model parameters

The posterior-log kernel is then maximized using an optimization numeric routine, resulting in the related Hessian matrix and posterior distribution mode (*Table 1*):

Table 1. Results from posterior maximisation

parameters	prior mean	mode	s.d.	prior	pstdev
α	0.323	0.3589	0.0026	beta	0.0200
β	0.992	0.9940	0.0002	beta	0.0020
δ	0.030	0.0264	0.0006	beta	0.0030
ζ	1.750	1.7485	0.0023	gamma	0.0200
ρ_1	0.850	0.9939	0.0014	beta	0.0500
ρ_2	0.500	0.4862	0.0022	beta	0.0500
$\sigma_{e,1}$	0.007	0.0032	0.0051	invg	Inf
$\sigma_{e,2}$	0.007	0.0032	0.0039	invg	Inf

To get the posterior distribution, the MH (Metropolis-Hastings) algorithm is called. This MCMC (Monte Carlo Markov Chains) type method, which pursues to simulate such distribution by generating Markov chains denoting possible estimations of the model parameters, thereafter selected from the perspective of their relevance, based on filtering rules, is implemented in Dynare, a specific tool of the well-known Matlab software.

Subsequent to the model implementation, various results have been generated, these ones being presented and discussed below.

Table 2 renders the posterior distribution and confidence interval for the model parameters:

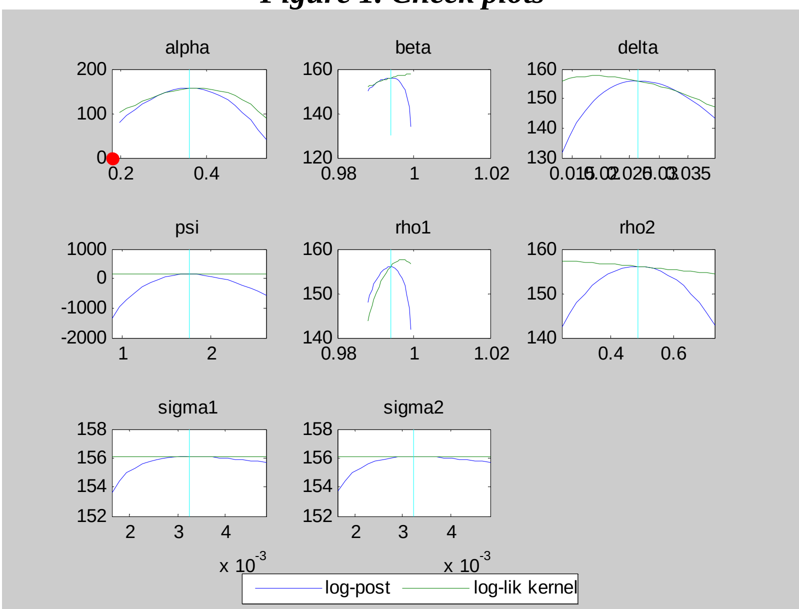
Table 2. Estimation results

parameters	prior mean	post mean	conf. interval		pstdev
α	0.323	0.3542	0.3356	0.3785	0.0200
β	0.992	0.9947	0.9936	0.9960	0.0020
δ	0.030	0.0261	0.0219	0.0302	0.0030
ζ	1.750	1.7456	1.7178	1.7698	0.0200
ρ_1	0.850	0.9940	0.9919	0.9963	0.0500
ρ_2	0.500	0.4912	0.4492	0.5436	0.0500
$\sigma_{e,1}$	0.007	0.0064	0.0017	0.0118	Inf
$\sigma_{e,2}$	0.007	0.0082	0.0013	0.0160	Inf

The estimates indicate a classical value, as per the related literature, of the subjective discount factor (β), remaining fixed to 0.99, an imperceptibly higher capital share in total production (α), of 0.35, and a lower rate of capital depreciation (δ), decreased to 0.02, the last two elements inducing the increase of capital stocks. The spare time utility parameter (ζ) reaches a value of 1.74, lower than the one established as prior, but a little bit superior to those obtained in previous personal studies, being, overall, in line with the results encountered in the field. Concerning the technology level autoregressive parameters (ρ_1, ρ_2), we observe a very high persistence of the fist one, of 0.99, its current value being finally influenced by its past value, other determining factors having a quite insignificant influence, and, as expected, reduced persistence of the second one, amounting to 0,49. As for the standard deviation of the technology structural shocks ($\sigma_{e,1}, \sigma_{e,2}$), although having started from the same prior value, have slowly differentiated from one another, reaching 0.006, respectively 0.008, therefore suggesting some informative characteristics of data in determining the estimation results. Economically speaking, such values denote a reduced degree of volatility, with positive effects on the anticipative analysis capacity.

An aggregate representation of both log-posterior and log-likelihood kernel check plots is rendered in *Figure 1*.

Figure 1. Check plots



In the graph above, the green line reflects the informative level of data, while the transition to the blue line indicates the influence on the same of the previously established prior distributions. If the likelihood is quite flat, the prior distribution will clearly show up in the posterior one.

Going forward, to the model endogenous variables, we undertake to underline several aspects, such as their variance and the variance decomposition depending on the considered shocks.

Table 3. Variance of model variables and its decomposition (%)

variable	s.d.	var.	var. dec. (e_1)	var. dec. (e_2)
y	0.1273	0.0162	99.71	0.29
c	0.0852	0.0073	99.42	0.58
k	1.4443	2.0860	99.62	0.38
i	0.0463	0.0021	92.80	7.20
l	0.0041	0.0000	61.47	38.53
y/l	0.3583	0.1284	99.86	0.14
z_1	0.0940	0.0088	100.00	0.00
z_2	0.0080	0.0001	0.00	100.00

Table 3 presents, beyond the standard deviation, respectively its square value denoting the variance of the model variables, also the contribution of the model structural shocks, namely the permanent technology level shock and the investment-specific technology level shock, evidencing the main driving forces impacting on the real economy.

As it can be seen, the permanent technology level sock greatly accounts for the variation of the variables considered. Save for the case of the investment-specific technology level, obviously wholly influenced by its related shock, there are just two variables for which this second shock is to be considered: investments (*7.20%*) and labour force (*38.53%*), it being reasonably explainable from the economic perspective, as a modification of the investment-related technology generates changes both as concerns the level of the investments and especially the number of employees needed by firms.

4. Conclusions

The small-size standard model approached, encompassing eight endogenous variables and two exogenous ones, estimated by resorting to a

Bayesian approach, revealed a slightly perceptible, still existing, informative characteristic of gross-domestic product-related data, the prior distributions having a significant role in influencing the output, this being mainly due to the setting of the same in compliance with previously obtained results. Regarding the posterior distributions of the model parameters, they indicated, among others, an increased capital stock level and a reduced volatility of the technology-related structural shocks. Finally, but not at least, the variance decomposition of the model variables suggested the significant impact of the permanent technology shock on the fluctuation of the same, the investment-specific technology shock being relevant exclusively for the investment and labour force items.

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