Statistical Indicators of Stock Assessed as a Percentage of GDP, and Consequences of Abusing the Logic of Statistical Thinking

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Abstract

The macroeconomic indicators concern both the flows that are not fully or partially compensated over a time interval, which can be cumulated or integrated, and the stocks or funds identified in a particular moment. These indicators are addressed either as free real and nominal indicators, or as bound or non-autonomous financial indicators. Statistical and economic problems are methodologically distinguished by the need for connection (free real indicators are updated through the time bound indicators, while free nominal macroeconomic indicators are characterized by a certain autonomy in relation to real free indicators), and also by turning flows into stocks and stocks into flows. This paper details three distinct solutions for the transformation of stocks to flow, solutions of different degrees of generalization: economic, statistical and physical, and all of which are detailed below using debt and budget examples.

Key words: *stock, flow, debt, budget deficit, index, continuity equation.*

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1. Introduction: Theoretical and retrospective aspects

The issue of stock and flow indicators, and especially its approach in the light of specific adjustments, goes back a fairly long period in time, in the retrospective of statistical theory, and is solved by means of the restrictive economic thinking, as well as the specifically interrogative and investigative logic of the science of statistics, and finally by taking support on validation through the physical model. The economic process, as a quantified difference between two temporal stocks, and seen as equation of value by Nicolae Georgescu Roegen, also implies an apparently material flux; the solution he identified (in 1978) was entropy itself, but unfortunately it was he who also recognized that it does not enable economic theories to say what exactly will happen in the future, it does not confer prediction and does not seem to facilitate connections and temporal corrections. Periods of time or intervals overlap (extending the moments), while the moment in time or lack of extension become relative, and the real economic process becomes a compromise between these interval limits and moments (a fluctuating range of moments combining moments that are hard to define independently and rigorously in practice), in accordance with Bergson's opinion "what is real is something intermediate between divided extension and total lack of extension” (Bergson,1911), in which Georgescu-Roegen recognized the time interval and the moment of statistical thinking.

The theme of the statistical indicators of stock and flow, graphically expressed, becomes an attempt at separating, through statistical thinking, “the flowing water from the frozen water”, and it seems rather hard to solve without some indications of physical thinking. The first statistician who tried to simplify it was Irving Fisher himself, through his famous formulation, which at the same time also generated various confusions: “*a stock refers to a point in time, and a flow to a time interval*” (Fisher, 1896) in most combinations of data and information that give rise to the concept of relative statistical indicator. The stock-flow antinomy is obvious, and it increases when, instead of the flow, a relative indicator is employed (e.g. flow rate).

2. The transformation stock - flow or flow – stock, from classical economics, through the statistical logic, to the thinking of contemporary physics

Consequently, the flow has come to be defined, in the modern type of statistics, as a difference of stocks in the relation

$$\Delta S = S(t+1) - S(t) \qquad (1)$$

or the stock in time has come to look like the logical consequence of the summation of a stock and a flow

$$S(t+1) = \Delta S + S(t) \qquad (2)$$

If any of the flow coordinates can be derived from the data concerning the stocks, according to the same relationship $\Delta S = S(t+1)-S(t)$, for the flows determined from the stocks, the issues seem somewhat more complicated in $S(t+1) = \Delta S + S(t)$, and the stocks cannot be determined without a baseline and outside an arbitrary constant (a threshold, limit, etc.) or with full knowledge of the value of stocks in some earlier time (John Hicks emphasized, in 1965, the equal contribution stocks and flows to the states of balance, and also the option to derive flows from the coordinates of the stocks, focusing on the stock models).

All the above aspects underline in fact the idea that the basic problem of the transfer flow–stock, and reciprocally, stock–flow, remains as complicated in both statistical and economic terms, because a flow does not necessarily represent an increase or decrease in the stock, the flow being essentially a notion under a qualitative impact, leading to the inclusion of a new variable, namely time, in order to turn the flow into what can be termed *a stock distributed on time*. Analogously, the new pair of variations (ΔS) of the variable S and (Δt) of the time variable or t automatically generates flow rate ($\Delta S / \Delta t$). It was indeed the very grounding of the answer given by Irving Fisher, an answer relatively better adapted to the problem of flow-stock transformation, in keeping with which a stock is not opposed by a flow (ΔS), but rather the flow rate. Similarly, the classic answer given and by Georgescu-Roegen, in whose terminology and signification, the stock and flow are concepts that represent distinct “dimensionalities”, and should be subject to different operations. When the classical logic of statistical thinking is violated, there appear a number of consequences that can radically depreciate the quality of both statistical indicators and temporal and spatial analyses of complex economic processes.

From natural reasons of managing real and financial flows (either autonomous or nominal), these flows are highlighted (observed, recorded, processed, stored, interpreted) by indicators, as quantitative (numerical) expressions of qualitative determinations conducted on a certain economic process. Hence, the flows are subsequently designated by the agency of the process categories or variables defining the business processes, and significant quantitative expressions become statistical indicators.

The stock of a given economic variable is described following a verification of attributes considered sufficient: I) the attribute invariance qualitative over time of the stock (a stock defines a temporally indiscriminate accumulation of a process, or defining variable of the process, and any change of the stock, having the sense of time variation, does not change its economic identity); II the attribute of qualitative invariance of the stock, in space or territory (there are no temporal strata in the structure of the stock, which is homogeneous in terms of quality); III) the attribute of causality of the stock relative to the flows (the initial stock cannot be defined by means of flows, or at least involves a limiting constant, in other words the initial stock can be considered “virgin” or describing at best a limit, a threshold or a reserve fund); IV) the attribute of artificial variation of the stock in the absence of any flow (any stock denominated in a currency that rises against another currency “grows” artificially or nominally, in much the same way as, when a currency depreciates, it decreases as artificially or nominally).

An example meant to clarify the difficulty of analyzing these developments in the complex economic universe is provided by the succession: Budget deficit (ΔBudget as a flow indicator) => Public or external debt (DEBT as a stock indicator) => Gross Domestic Product (GDP as an indicator of flow).

Governments hold annual budgets of revenues and expenditures, positive flows and opposite, rarely fully compensated and generating deficits or negative flows.

Budget deficit (ΔB) = Income (VB) - Expenses (CB) (3)

Annually, trade balances of the national economies as flows partially offset by export and import can generate, and do generate, deficits:

Trade balance deficit (ΔBC) = Exports (X) - Imports (M) (4)

Analogously, one can find current account deficits of the balance of payments, the notations being slightly different and the essence of the phenomenon analyzed remaining virtually always the same.

Current account deficit of the balance of payments (ΔCCBPE) = Proceeds (A) – Payments (P)

(5)

A public deficit, be it a trade deficit or a current account deficit, is covered by loans, which can be seen in the evolution of the stock of (public, external, etc.) debt. To understand more quickly the consequences of abusing the logic of statistical thinking, one can reduce everything to the budget deficit, rewriting a defining relationship of the flow-stock and stock-flow type, between the budget deficit and the debt (by changing the notation in equations (1) and (2).

$$\Delta Budget = DEBT(t+1) - DEBT(t) \qquad (6)$$

the stock in time of public debt becoming the consequence of the summation of an initial stock and a flow called budget deficit.

As economic convergence is a fundamental process of integrating the economies of EU countries and the convergence indicator concerning the maximum or limiting value of public debt to GDP is 60%, one finally reaches the threshold analysis based on a double error: a) a stock indicator is incorrectly constructed, uncorrected and unconnected by means of a flow indicator; b) a stock indicator is related to a another indicator, this time a flow indicator, and the resulting relative flow is then expressed as a percentage. By abandoning the logic of statistical thinking, all of the above errors conduce to indicators of impaired informational content.

$$DEBT(t+1) = \Delta Budget + DEBT(t) \qquad (7)$$

The convergence criterion of public debt = $[DEBT\ (t+1):GDP]\times 100 = \{[\Delta Budget + DEBT\ (t)]:GDP\}\times 100$ (8)

What does an accurate relationship look like, which also takes into account the logic of statistical thinking? This question allows for three distinct solutions of different degrees of generalization: economic, statistical and physical, all of which are detailed below.

I. The economic solution is based on the budgetary restrictions

Starting from the variation of debt as an indicator of stock between two points in time (t+1) and (t), assuming budget restrictions:

$$DEBT_{(t+1)} - DEBT_{(t)} = i_t \times DEBT_t + \Delta Budget_t \qquad (9)$$

where: i_t is the nominal interest rate in the short term; following successive processing, one gets the relationship

$$DEBT_{(t+1)} = (1 + i_t)\, DEBT_t + \Delta Budget_t \qquad (10)$$

which is then divided by GDP_t to express percentages of the GDP_t in keeping with the standard of the convergence indicator:

$$[DEBT_{(t+1):}\ GDP_t] = [(1+ i_t) \times DEBT_t]: GDP_t + [\Delta Budget_t: GDP_t] \qquad (11)$$

and as the pace of real GDP connects $GDP_{(t)}$ and $GDP_{(t-1}$ in keeping with the relation for calculating real GDP index

$$GDP_{(t)} = (1+Rate\ I\ real\ GDP) \times PIB_{(t-1)} \qquad (12)$$

A new relationship is obtained:

$$[DEBT_{(t+1):}\ PIB_t] = [\Delta Budget_{t:}\ GDP_t] + [(1+i_t):(1+I\ real\ GDP)]\times[DEBT_{t:}\ GDP_{(t-1)}] \qquad (13)$$

$$[DEBT_{(t+1)}:GDP_t] = [\Delta Budget_t:GDP_t]+[Index_{short\ term\ interest}:Index_{economic\ growth}]\times[DEBT_t:GDP_{(t-1)}] \qquad (14)$$

As can be seen, a factor emerged from the careful analysis of the economic process, a factor that connects the two concepts (flow and stock), a correction factor of flow-to-stock and stock-to-flow transformation, which when the public budget is balanced ($\Delta Budget_t = 0$),

$DEBT_{(t+1)} > DEBT_{(t)}$, if short-term interest rate > rate of real GDP;
$DEBT_{(t+1)} = DEBT_{(t)}$, if short-term interest rate = real GDP index;
$DEBT_{(t+1)} < DEBT_{(t)}$ if short-term interest rate < real GDP index. (15)

This approach is accurate and realistic, which, according to the traditional transformation from flow to stock or from stock to flow did even not represent a hypothesis for analysis.

II. The statistical solution directly introduces an index of correction (by linking temporally and really the real stock-flow phenomenon), starting directly from the equation (7):

Debt(t+1) as % of available GDP (t) = ΔBudget + DEBT(t) as % of available GDP (t-1)

(16)

Thus, the first solution is given by the correction of the value DEBT(t) with β = 1/[k], where *k* is the ratio of real GDP index (an ascending or favourable dynamics of macro-results in connection decreases the value DEBT(t), while a decreasing or unfavourable trend increases the same value) and the index of short-term interest rate or the infra-annual index (things are now completely reversed: obviously, increasing the rate increases the amount of debt).

A second solution involves decomposing the real GDP index in two other indices, i.e. starting from the coefficient β = 1/[k], where *k* is equal to a triple ratio of indices (I real GDP is substituted by the ratio of the calculation I real GDP = I nominal GDP: GDP deflator), namely:

$$\frac{DEBT_{(t+1)}}{GDP} \qquad \frac{\Delta Budget_t}{GDP_t} \qquad \frac{Index_{\text{short term interest}}}{\dfrac{Index_{\text{nominal GDP}}}{Deflator_{\text{implicit for PIB}}}} \qquad \frac{DEBT_t}{GDP_{(t-1)}} \qquad (17)$$

The correction, or connection from flow to stock, is now made by the agency of the interest index (directly correlated) of the nominal GDP index (correlated indirectly) and the deflator (directly correlated). They actually represent a specific macroeconomic phenomenon of connection of the triple indexing of the stock, subsequently aggregated with a macroeconomic flow that highlights three types of adjustments or connections generating errors if not used correctly.

III. The generalized approach, through the physical model, generalizes, simplifies, and confirms the need for correction.

Economic space and time are actually redefined by economic results and inflation (GDP becoming the macroeconomic space, and price, exchange rate or interest redefining economic *time*; analyzes are relaxed by the instrumental use of the indices such as real GDP index and price index or interest index, etc.)

A physical flow may be defined by various parameters (mass, volume, power, speed, pressure, etc.), and the continuity equation is like:

$\nabla(\rho v) - (d\rho):(dt) = 0$ (19)

And to turn the flow into what can be called *a stock distributed over a time interval*, then the continuity equation becomes

$\rho_{(t+dt)} = d\phi + (1/k) \times \rho_{(t)}$ (20)

Specifically, for the variables q and t between times q (t) and q(t+1) the continuity relationship is identified

$\Delta q_{(t+dt)} = q_{(t+dt)} - 1/_{(k)} \times q_{(t)}$ (21)

benefiting from a correction of the form 1/k, or otherwise written as final variation

$\nabla(\rho) - (dq):(dt) = 0$ (22)

A physical flow may be defined by various parameters (mass, volume, power, speed, pressure, etc.) and its continuity equation is given by:

$(\partial\rho):(\partial t) + \mathrm{div}(\rho v) = 0$ (19')

However, ρ being a spatial and temporal function:

$\rho = \rho(x, y, z, t)$

equation (19) is converted to:

$(1/\rho) \times (d\rho/dt) + \mathrm{div}(v) = 0$ (20')

and speed may vary as a result of external constraints (flow) or as a result of volume deformation (deformation time, similar deformation or phenomenon such as the budget deficit in the economy).

The relationship between the final volume (τ_1) following initial volume deformation (τ_0) is:

$(\tau_1) = D(x_1, y_1, z_1):D(x_0, y_0, z_0) = 1- [(Du:Dx) + (Dv:Dy) + (Dw:Dz)]dt+(Du:Dx)+(Dv:Dy)+(Dw:Dz)]dt^2$

(21')

where: $D(x_1, y_1, z_1)$ şi $D(x_0, y_0, z_0)$ are the transfer functions from one domain to another, and u , v and w are velocity components caused by deformation.

The final part $[(Du:Dx) + (Dv:Dy) + (Dw:Dz)]\ dt^2$ as derived from 2nd order tends to zero and the previous equation change into:

$(\tau_{1} - \tau_0): (\tau_0) = [(Du:Dx) + (Dv:Dy) + (Dw:Dz)]\ dt = \mathrm{div}(v)\ dt$ (22')

Hence, the first relationship that characterizes the flow

$(1/\tau_0) \times [(d\tau) : (dt)] = \mathrm{div}(v)$ (23)

and a second connection on the stock:

$(1/\rho) \times [(d\rho) : (dt)] = \mathrm{div}(v)$ (24)

From the obvious equality of the two previous relationships (23) and (24) to get the conversion stock flow or what might be called *a stock distributed to time* when continuity equation becomes

$(1/\tau_0) \times [(d\tau) : (dt)] = (1/\rho) \times [(d\rho) : (dt)] = \mathrm{div}(v)$ or $\nabla(v)$ (25)

Following a correction by a factor $\beta = (1 / k)$, and the general relation becomes:

$$(1/k) \times (1/\tau_0) \times [(d\tau) : (dt)] = (1/ \rho) \times [(d\rho): (dt)] \quad (26)$$

It may be noted that this general relationship for analyzing debt (Debt) as a stock in relation to the budget deficit (ΔBudget) is immediately deductible by a simple change of notation benefiting from the correction (1 / k):

$$(1/k) \times (1/Budget_0) \times [(dBudget) : (dt)] = (1/Debt_0) \times [(dDebt) : (dt)] \quad (27)$$

A final remark

The initiation and the completion of such an argument, focused on successive replacements of calculation relations and process economic investigation of the classic type, to which are also added a few corrections, adjustments or dynamic connections (capitalizing the method of the indices) is under the impact of the logic of statistical thinking, and generalizing the reasoning, based on the thinking of physics, leads to optimal solutions for transforming contemporary indicators from stock into flow and conversely.

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